REINFORCED CONCRETE DESIGNER'S HANDBOOK

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PREFACE TO SIXTH EDITION

THE arrangement generally and the content of this edition are substantially the same as in previous editions in so far that the descriptive text is in Part I and the tables, with examples applying to the tables, are given in Part II. Notes relating to the tables are now given mainly on the pages facing the tables. Some examples involving the use of several tables and typical details of some common structural parts are given in an Appendix, as are also notes on the effect of various materials on concrete. Specifications and quantities are now included in Part I.

Advantage has been taken of the necessity to reprint the entire book to re-cast and re-number the tables and to re-write and reset the descriptive text in a more convenient form. The result is that without increasing the bulk of the book, the amount of data and the number of tables have been increased. Most of the formulæ appearing in Part I of previous editions are now given in the tables. Additional data in the tables include moment-distribution applied to continuous beams and beams with splays, bending-moment diagrams, non-rectangular panels of slabs, portal frames, metric properties of reinforcement bars, load-factor method applied to beams and slabs and to columns subjected to bending, bending in two planes, types of bridges, and formulæ abstracted from the original Part I.

The notation throughout has been brought into line with the British Standard Codes where applicable, and effect is given to the recommendations given in B.S. Codes Nos. 114 (buildings) and 2007 (liquid-containers), and other codes and regulations where appropriate.

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INTRODUCTORY NOTE

A STRUCTURE is an assembly of members each of which is subjected to bending or to direct force (either tensile or compressive) or to a combination of bending and direct force. These primary influences may be accompanied by shearing forces and sometimes by torsion. Secondary effects due to changes in temperature, to shrinking and creep of the concrete, and the possibility of damage resulting from abrasion, vibration, frost, chemical attack, and similar causes may also have to be considered. Design includes the calculation of, or other means of assessing and providing resistance against, the moments, forces, and other effects on the members. An efficiently designed structure is one in which the members are arranged in such a way that the weight, loads and forces are transmitted to the foundations by the cheapest means consistent with the intended use of the structure and the nature of the site. Efficient design means more than providing suitable sizes for the concrete members and the provision of the calculated amount of reinforcement in an economical manner. It implies that the bars can be easily placed, that reinforcement is provided to resist the secondary forces inherent in monolithic construction, and that resistance is provided against all likely causes of damage to the structure. Experience and good judgment may do as much towards the production of safe and economical structures as calculation. Complex mathematics should not be allowed to confuse the sense of good engineering. Where possible, the same degree of accuracy should be maintained throughout the calculations; it is illogical to consider, say, the effective depth of a member to two decimal places if the load is over-estimated by 25 per cent. On the other hand, in estimating loads, costs, and other numerical quantities, the more items that are included at their exact value the smaller is the overall percentage of error due to the inclusion of some items the exact magnitude of which is unknown.

Where the assumed load is not likely to be exceeded and the specified quality of concrete is fairly certain to be obtained, high working stresses can be employed. The more factors allowed for in the calculations the higher may be the working stresses, and vice versa (see pages 57 and 59). If the magnitude of a load, or other factor, is not known precisely it is advisable to study the effects of the probable largest and smallest values of the factor and provide resistance for the most adverse case. It is not always the largest load that produces the greatest stresses in all parts of a structure.

Much design is to-day controlled by regulations or codes, but even within such bounds the designer must exercise judgment in his interpretation of the requirements, endeavouring to grasp the spirit of the requirements rather than to design to the minimum allowed by the letter of a clause. In Great Britain the detail design of reinforced concrete is based largely on the British Standard codes relating to buildings, principally those for "Loading" (Code No. 3, Chapter V), the "Structural Use of Normal Reinforced Concrete in Buildings" (Code No. 114), and "Design and Construction of Reinforced and Prestressed Concrete Structures for the Storage of Water and other Aqueous Liquids" (Code No. 2007). In addition there are such documents as the Memorandum on the design of bridges issued by the Ministry of Transport, and the by-laws of local authorities, such as the London Building By-laws (1952).

The tables given in Part II enable the designer to reduce the amount of arithmetical work. The use of such tables not only makes for speed but also eliminates inaccuracies if the tables are thoroughly understood and their bases and limitations realised. In the succeeding sections of Part I and in the supplementary information given on the pages facing the tables, the basis of the tabulated material is described Some general information is also tabulated; for example, *Table* 108 gives fractions of an inch expressed in decimals of a foot; *Table* 109 is a conversion table for the metric system. *Table* 110 gives trigonometrical functions of angles to a degree of accuracy sufficient for design purposes; in conjunction with this table, fundamental trigonometrical formulæ and other mathematical formulæ and useful data are given.

Economical Structures.

The cost of a reinforced concrete structure is obviously affected by the prices of concrete, steel, shuttering, and labour. Upon the relation between these prices depend the economical proportions of the quantities of concrete, reinforcement, and shuttering. There are possibly other factors to be taken into account in any particular case, such as the use of available steel shuttering of standard sizes. In Great Britain, economy generally results from the use of simple shuttering even if this requires more concrete compared with a design requiring more complex and more expensive shuttering.

Some of the factors which may have to be considered are whether less concrete of a rich mixture is cheaper than a greater volume of a leaner concrete; whether the cost of higher-priced bars of long lengths will offset the cost of the extra weight used in lapping shorter and cheaper bars; whether, consistent with efficient detailing, a few bars of large diameter can replace a larger number of bars of smaller diameter; whether the extra cost of rapid-hardening cement justifies the saving made by using the shuttering a greater number of times; or whether uniformity in the sizes of members saves in shuttering what it may cost in extra concrete.

There is also the wider aspect of economy, such as whether the anticipated life and use of a proposed structure warrant the use of a higher or lower factor of safety than is usual; whether the extra cost of an expensive type of construction is warranted by the improvement in facilities; or whether the initial cost of a construction of high quality with little or no maintenance cost is more economical than less costly construction combined with the expense of maintenance.

The wording of a contract and the experience of the contractor, the position of the site and the nature of the available materials, and even the method of measuring the quantities, together with numerous other points, all have their effect, consciously or not, on the designer's attitude towards a contract. So many and varied are the factors to be considered that only experience and the study of the trend of design can give any reliable guidance. Attempts to determine the most economical proportions for a given member, based only on inclusive prices of concrete, steel, and shuttering, are often misleading. It is nevertheless possible to lay down certain principles.

For equal weights, combined material and labour costs for reinforcement bars of small diameter are greater than those for large bars, and within wide limits long bars are cheaper than short bars if there is sufficient weight to justify special transport charges and handling facilities.

The lower the cement content the cheaper the concrete but, other factors being equal, the lower is the strength and durability of the concrete. Taking compressive strength and cost into account a concrete rich in cement is more economical than a leaner concrete. In beams and slabs, however, where much of the concrete is in tension and therefore neglected in the calculations, it is less costly to use a lean concrete than a rich one. In columns, where the concrete is in compression, the use of a rich concrete is more economical, since besides the concrete being more efficient, there is a saving in shuttering consequent upon the reduction in size of the column.

The use of steel in compression is always uneconomical when the cost of a single member is being considered, but advantages resulting from decreasing the depth of beams and by reducing the size of columns may offset the extra cost of the individual member. A doubly-reinforced beam that contains the smallest amount of reinforcement is less costly than a beam of the same size containing a greater amount of steel, even although in the latter case the tensile steel is stressed to the maximum permissible value. The most economical doubly-reinforced beam is that in which the compressive stress in the concrete is the maximum permissible stress and the tensile stress in the steel is that which gives the minimum combined weight of tensile and compressive reinforcement. Tee-beams and slabs with compression reinforcement are seldom economical. When the cost of mild steel is high in relation to that of concrete, the most economical slab is that in which the proportion of tensile reinforcement is well below the so-called "economic" proportion. (The "economic" proportion is that at which the maximum permissible stresses in the steel and concrete are simultaneously attained.) Tee-beams are cheaper if the rib is made as deep as practicable; here again

the increase in headroom consequent on reducing the depth may offset the small extra cost of a shallower beam. It is rarely economical to design a tee-beam for the maximum permissible stress in the concrete.

Bent-up bars are more economical than binders for resisting shearing force, and this may be true even if bars have to be inserted specially for this purpose.

Shuttering is obviously cheaper if angles are right-angles, if surfaces are plane, and if there is some repetition of use. Therefore splays and chamfers are omitted unless structurally necessary or essential to durability. Wherever possible architectural features in cast-insitu work should be formed in straight lines. When the cost of shuttering is considered in conjunction with the cost of concrete and steel, the introduction of complications in the shuttering may sometimes lead to more economical construction; for example, large continuous beams may be more economical if they are haunched at the supports. Cylindrical tanks are cheaper than rectangular tanks of the same capacity if many uses are obtained from one set of shuttering. In some cases domed roofs and tank bottoms are more economical than flat beam-and-slab construction, although the unit cost of the shuttering may be doubled for curved work. When shuttering can be used several times without alteration, the employment of steel shuttering should be considered and, because steel is less adaptable than wood, the shape and dimensions of the work may have to be determined to suit. Generally steel shuttering for beam-and-slab or column construction is cheaper than timber shuttering if twenty or more uses can be assured, but for circular work half this number of uses may warrant the use of steel. Timber shuttering for slabs, walls, beams, column sides, etc., can generally be used four times before repair, and six to eight times before the cost of repair equals the cost of new shuttering. Beam-bottom boards can be used at least twice as often.

The use of precast construction usually reduces considerably the amount of shuttering and temporary supports required, and the moulds may be used a greater number of times. Except in some obvious cases, the loss of structural rigidity due to the absence of monolithic construction may offset the economy otherwise resulting from precast construction. To obtain the economical advantage of precasting and the structural advantage of cast-insitu construction, it is convenient to combine both types of construction in some structures.

In many cases the most economical design can be determined only by comparing the approximate costs of different designs. This is particularly true in border-line cases and is practically the only way of determining, say, when a simple cantilevered retaining wall ceases to be more economical than one with counterforts; when a solid-slab bridge is more economical than a slab-and-girder bridge; or when a cylindrical container is cheaper than a rectangular container. Although it is usually more economical in floor construction for the main beams to be of shorter span than the secondary beams, it is sometimes worth while investigating different spacings of the secondary beams, to determine whether a thin slab with more beams is cheaper or not than a thicker slab with fewer beams.

An essential aspect of economical design is an appreciation of the possibilities of materials other than concrete. The judicious incorporation of such materials may lead to substantial economies. Just as there is no structural reason for facing a reinforced concrete bridge with stone, so there is no economic gain in constructing a 4-in. cast-insitu reinforced concrete wall panel if a $4\frac{1}{2}$-in. or 9-in. brick wall is cheaper and will serve the same purpose. Other common cases of the consideration of different materials are the installation of timber or steel bunkers when only a short life is required, the erection of light steel framing for the superstructures of industrial buildings, and the provision of pitched steel roof trusses. Included in such economic comparisons should be such factors as fire-resistance, deterioration, depreciation, insurance, appearance, speed of construction, and structural considerations such as the weight on the foundations, convenience of construction, and scarcity or otherwise of materials.

Drawings.

The methods of preparing drawings vary considerably, and in most drawing offices a special practice has been developed to suit the particular class of work done.

The following observations can be taken as a guide when no precedent exists. One of the principal factors is to ensure that on all drawings for any one contract the same conventions are adopted and uniformity of appearance and size should be aimed at, thereby making the drawings easier to read.

In the preliminary stages a general drawing of the whole structure is generally prepared to show the principal arrangement and sizes of beams, columns, slabs, walls, foundations, and other members. Later this, or a similar drawing, is utilised as a key to the working drawings, and should show precisely such particulars as the setting-out of the structure in relation to adjacent buildings or other permanent works, and the level of, say, the ground floor in relation to a datum. All principal dimensions such as the distance between columns and overall and intermediate heights should be indicated, in addition to any clearances, exceptional loads, and other special requirements. The most convenient scale for these general drawings is usually $\frac{1}{8}$ in. to 1 ft. It is often of great assistance if the general drawing can be used as a key to the detailed working drawings by incorporating reference marks for each column, beam, slab panel, or other member.

The working drawings should be large-scale details of the members shown on the general drawing. A suitable scale is $\frac{1}{2}$ in. to 1 ft., although plans of slabs and elevations of walls are often conveniently prepared to a scale of $\frac{1}{4}$ in. to 1 ft., while sections through beams and columns with complicated reinforcement are preferably drawn to a scale of 1 in. to 1 ft. Separate sections, plans, and elevations should be shown for the details of the reinforcement in slabs, beams, columns, frames, and walls, since it is not advisable to show the reinforcement for more than one such member in a single view. An indication should be given, however, of the reinforcement in slabs and columns in relation to the reinforcement in beams or other intersecting reinforcement. Sections through beams and columns showing the detailed arrangement of the bars should be placed as closely as possible to the position where the section is taken.

In reinforced concrete details it is best to indicate the outline of the concrete by a bold line and the reinforcement by a thinner full line representing the centre-line of the bar and, wherever clearness is not otherwise sacrificed, the line representing the bar should be placed in the exact position intended for the centre-line of the bar, proper allowance being made for the amount of cover. Thus the reinforcement as shown on the drawing should represent as nearly as possible the appearance of the reinforcement as fixed on the site, all hooks and bends being drawn to scale.

The dimensions given on the drawing should be arranged so that the primary dimensions connect column and beam centres or other leading setting-out lines, and so that secondary dimensions give the detailed sizes with reference to the main setting-out lines. The dimensions on working drawings should also be given in such a way that the carpenters making the shuttering have as little calculation to do as possible. Thus, generally, the distances between breaks in any surface should be dimensioned. Disjointed dimensions should be avoided by combining as much information as possible in a single line of dimensions.

Marks indicating where cross-sections are taken should be bold and, unless other considerations apply, the sections should be drawn as viewed in the same two directions throughout the drawing; for example, they may be drawn as viewed looking towards the left and as viewed looking from the bottom of the drawing. Consistency in this makes it easier to understand complicated details.

Any notes on general or detailed drawings should be concise and free from superfluity in wording or ambiguity in meaning. Notes which apply to all working drawings can be reasonably given on the general arrangement with a reference to the latter on each of the detail drawings. Although the proportions of the concrete, the cover of concrete over the reinforcement, and similar information are usually given in the specification or bill of quantities, the proportions and covers required in the parts of the work shown on a detail drawing should be described on the latter, as the workmen rarely see the specification. If the bar-bending schedule is not given on a detail drawing, a reference should be made to the page numbers of the bar-bending schedule relating to the details on that drawing.

Notes that apply to one view or detail only should be placed as closely as possible

to the view or detail concerned, and only those notes that apply to the drawing as a whole should be collected together. If a group of notes is lengthy there is a danger that individual notes will be read only cursorily and an important requirement be overlooked.

Factor of Safety.

The factor of safety of a reinforced concrete structure is not the ratio of the strength of the materials to the design stress, mainly because of yielding of the reinforcement at a stress less than the tensile strength. The true factor of safety is the ratio of the greatest load that the structure can carry to the design load. The greatest load, which is that causing collapse or so much distortion that the structure fails to fulfil its purpose, cannot be calculated exactly. The ultimate moment of resistance of a rectangular beam or the failure-load of a column can be calculated with reasonable accuracy, but the bending moments and forces acting on an overloaded monolithic reinforced concrete structure are indefinite, as overstressing at one part (say, at the support of a continuous beam) may be relieved by a reserve of strength at another part (say, at midspan). The distribution of bending moments is therefore different from that assumed under the design load. The factor of safety with stresses not exceeding half the yield-point stress of the reinforcement and one-third of the crushing strength of concrete cubes at 28 days is probably not less than three.

A more definite factor of safety can be applied to the lateral stability of a structure. Generally the factor of safety against overturning of a cantilevered retaining wall is about one and a half. For bridges, B.S. No. 153 (Part 3A) requires the stabilising effect to be 1·1 times the overturning effect due to dead loads plus 1·4 times the overturning effect due to other loads.

NOTATION

The symbols adopted in the B.S. Code of Practice No. 114 (1957) are given in the following and are used in the tables and elsewhere in this book. The signification of these and other symbols used are given in the appropriate tables and texts.

A_b, Equivalent area of helical reinforcement (volume of helix per unit length of column).
A_c, Cross-sectional area of concrete, excluding any finishing material and reinforcement.
A_k, Cross-sectional area of concrete in column core, excluding area of longitudinal reinforcement.
A_{sc}, Cross-sectional area of steel in compression.
A_{st}, Cross-sectional area of steel in tension.
A_w, Cross-sectional area of stirrup.
b, Breadth of a rectangular beam; breadth of flange of a T-beam or L-beam.
b_r, Breadth of the rib of a T-beam or L-beam.
D, Diameter generally.
d, Overall depth.
d_1, Effective depth to the tensile reinforcement in a beam.
d_2, Depth to the compression reinforcement in a beam.
d_n, Depth of concrete in compression in a beam (= depth to neutral plane).
d_s, Depth of slab forming the flange of a T-beam or L-beam.
e, Eccentricity of a load on a column.
K_b, Stiffness of beam.
K_{b1}, Stiffness of beam on one side of a column.
K_{bb}, Stiffness of beam on the opposite side of a column.
K_l, Stiffness of lower column.
K_u, Stiffness of upper column.
L, Length of a column or beam between adequate lateral restraints; in flat slabs, L is the average of L_1 and L_2.
L_1, Length of panel of flat slab in the direction of span considered.
L_2, Width of panel of flat slab at right-angles to direction of span considered.
l, Effective span of beam or slab; effective height of column.
l_a, Lever arm of the resistance moment.
l_x, Length of shorter side of slab spanning in two directions.
l_y, Length of longer side of slab spanning in two directions.
M, Bending moment (suffixes as required).
M_e, Bending moment at end of beam framing into a column, assuming fixity at both ends of the beam.
M_{es}, The maximum difference between the moments at the ends of two beams framing into opposite sides of a column, each calculated on the assumption that the ends of the beams are fixed and that one of the beams is not subjected to live load.
M_r, Moment of resistance of a section to bending.
M_x and M_y, Maximum bending moments, for spans l_x and l_y respectively, on strips of unit width in slabs spanning in two directions.
m, Modular ratio.
o, Sum of perimeters of the bars in the tensile reinforcement.
P, Permissible load on a short column subject to load and bending.
P_b, Eccentric load on a column.
P_o, Axial load permissible on a short column.
p_{cb}, Permissible compressive stress in concrete in bending.
p_{cc}, Permissible stress in concrete in direct compression.
p_{sc}, Permissible compressive stress in the reinforcement.
p_{st}, Permissible tensile stress in the reinforcement.
Q, Total shearing force across a section.
q, Shearing stress at a section of a beam or slab.
R, Modulus of rupture.
s, Spacing or pitch of stirrups.
u, Cube crushing strength of concrete.
$u_p = u$ for preliminary test.
$u_w = u$ for works test.
W, Total load on beam or slab.
W_d, Dead load.
W_s, Imposed load.
w, Total load per unit area of slab or per unit length of beam.
α_x and α_y, Bending-moment coefficients for the short and long spans respectively of slabs spanning in two directions and freely supported on four sides.
β_x and β_y, Bending-moment coefficients for the short and long spans respectively of rectangular panels supported on four sides and with provision for torsion at the corners.

PART I

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PART I

SECTION 1

LOADS AND PRESSURES

THE loads on a structure are permanent (or dead) loads and transient (or imposed or live) loads. Dead loads include the weights of the structure itself and any permanent fixtures, partitions, finishes, superstructures, and the like. Data for computing dead loads are given in *Tables* 1 and 2.

Live loads include any external loads imposed upon the structure when it is serving its normal purpose, and include the weight of stored materials, furniture and movable equipment, cranes, vehicles, snow, wind, and people. The accurate assessment of the actual and probable loads is an important factor in the production of economical and efficient structures. Some live loads, such as the pressures and weights due to contained liquids, can be determined exactly; less definite, but capable of being calculated with reasonable accuracy, are the pressures of retained granular materials. Other loads, such as those on floors, roofs, and bridges, are generally specified at safe values. Wind forces are much less definite, and marine forces are among the least determinable.

Imposed Loads.

Floors.—For buildings in most towns the loads imposed on floors, stairs, and roofs are specified in codes or regulations issued by local authorities. The loads given in *Table* 3 are based on the London By-laws (1952) and the B.S. Code No. 3, Chapter V (1952). For floors and roofs with small intensities of loading the beams are sometimes designed for a smaller imposed load than the slabs, since it is unlikely that the entire floor will be loaded at one time. It is the practice to design short-span slabs and beams for a minimum total imposed load irrespective of the span; the values of the minimum loads are given in *Table* 3. The limiting span for most slabs is 8 ft., below which length the minimum total imposed loads control the design of the slab. For spans in excess of this limiting value, the normal intensity of imposed loading is the controlling factor. In the case of slabs, the minimum total imposed load is expressed in lb. per foot width of slab, whereas for beams the minimum load is the total load on the beam. In accordance with the London By-laws and B.S. Code, the minimum load on beams applies when the area of floor supported by the beam is not greater than 64 sq. ft. When a beam supports more than 500 sq. ft. of a level floor, it is permissible to reduce the specified imposed load by 5 per cent. for every 500 sq. ft. of floor supported, the maximum reduction being 25 per cent.; this reduction does not apply to floors used for storage. The minimum total imposed loads do not apply to cantilevers. The imposed loads on balconies, corridors, stairs and landings are also given in *Table* 3.

The loads on floors of warehouses and garages are dealt with on pages 11 and 13. In all cases of floors in buildings it is advisable, and in some localities it is compulsory, to affix a notice indicating the imposed load for which the floor is designed. Floors

of industrial buildings where machinery and plant are installed should be designed not only for the load when the plant is in running order, but for the probable load during erection and the testing of the plant, as in some cases this load may be more severe than the working load. The weights of any machines or similar fixtures should be allowed for if they are likely to cause effects more adverse than the specified minimum imposed load. Any reduction in the specified imposed load due to multiple stories or to floors of large area should not be applied to the gross weight of the machines or fixtures. The approximate weights of some machinery such as conveyors and screening plants are given in *Table* 2. The effect on the supporting structure of passenger and goods lifts and the forces in colliery pit-head frames of the A-frame type are given on the page facing *Table* 5. The support of heavy safes requires special consideration, and the floors should be designed not only for the safe in its permanent position but also for the condition when the safe is being moved into position, unless temporary props or other means of relief are provided during installation.

Structures subject to Vibration.—For floors subjected to vibration from such causes as dancing, drilling, and gymnastics, the imposed loads specified in *Table* 3 are adequate to allow for the dynamic effect. For structural members subjected to continuous vibration due to machinery, crushing plant, centrifugal driers, and the like, allowance for dynamic effect can be made by reducing the working stresses by, say, 25 per cent. or by increasing the total dead and live load by this amount; the advantage of the latter method is that ordinary stresses and standard tables and charts are applicable.

Balustrades and Parapets.—The balustrades of stairs, landings, and balconies should be designed for a horizontal load acting at the level of the handrail or coping. This load may be 15 lb. per linear foot for light access stairs, 25 lb. per linear foot for stairs in residential buildings, and 50 lb. per linear foot for other cases, including roof parapets. On the parapets of balconies, roofs, bridges, or elsewhere where crowds can assemble a horizontal load of 200 lb. per foot should be allowed. B.S. No. 153 (Part 3A) specifies the load on bridge parapets as 25 lb. to 100 lb. per linear foot depending on conditions of probable loading.

Roofs.—The imposed loads on roofs given in *Table* 3 are additional to all surfacing materials and include snow and other incidental loads but exclude wind pressure. Freshly-fallen snow weighs about 5 lb. per cubic foot, but compact snow may weigh 20 lb. per cubic foot, which should be considered in districts subject to heavy snowfalls. For sloping roofs the snow load decreases with an increase in the slope. According to the B.S. Code the imposed load is zero on roofs sloping at an angle exceeding 75 deg., but a sloping roof with a slope of less than 75 deg. must be designed to support an imposed load of 15 lb. to 20 lb. per square foot or such minimum total imposed load as is given in *Table* 3.

If a flat roof is used for purposes such as a café, playground, or roof garden, the appropriate imposed load for such a floor should be allowed. The possibility of converting a flat roof to such purposes or for use as a floor in the future should also be anticipated.

Columns, Walls, and Foundations.—Columns, walls, and foundations of buildings should be designed for the same imposed loads as the beams of the floors they support. In the case of buildings of more than two stories, and which are not warehouses, garages, or stores and are not factories or workshops the floors of which are designed for less than 100 lb. per square foot, the column and foundation loads

can be reduced thus. Consider the roof as being fully loaded. If two floors are supported, reduce the imposed load on both floors by 10 per cent. If three floors, reduce the imposed load on the three floors by 20 per cent., and so on in 10-per-cent. reductions down to five or more floors for which the imposed load is reduced by not more than 40 per cent. These requirements are in accordance with the London By-laws and the B.S. Code and appropriate factors are given in *Table* 3. If the unit load on a beam is reduced because of the large area supported the columns or other supporting members can be designed for either this reduced load or for the reduction due to multiple stories.

Storage Loads.

Warehouses.—Floor slabs, beams, columns, and foundations of buildings of the warehouse class should be designed for a minimum specified imposed load, but the purpose for which each floor of the warehouse is to be used should be considered and, if necessary, heavier loads should be assumed. For example, paper stores and printing works are usually designed for an imposed load of 3 cwt. per square foot. The minimum imposed loads for warehouses for various conditions as given in the B.S. Code and the London By-laws are given in *Table* 3.

Containers.—Wherever possible the actual weights of the materials to be stored in such container structures as tanks, bunkers, and silos should be ascertained. The weights of some of the materials commonly stored in concrete containers are given in *Table* 4. The horizontal pressures due to such materials are considered on page 19. Where material is floating or submerged in water or other liquid the loads or vertical pressures on the horizontal bottom of the container can be calculated from the expressions and data given in *Table* 4.

Live Loads due to Vehicles and Moving Equipment.

Structures such as bridges, gantries, and buildings used for garages are designed for the loads imposed by vehicles, trains, cranes, and other moving equipment. Reliable data, relating to the weights and other loads, thrusts and other forces, to which suitable allowances for impact should be added, should be obtained from the maker (or elsewhere) of the vehicle or equipment.

Road Bridges.—The live load on public road bridges in Britain is specified by the Ministry of Transport, and comprises a uniformly-distributed load combined with a knife-edge load. The intensity of the uniformly-distributed load depends on the "loaded length" (that is the length of the member which must be loaded to produce the greatest stress in that member), and is given in *Table* 6. Consideration of the "loaded length", and notes on the application of the normal live loads, are given on the page facing the table. Similar loads on reinforced concrete decks of steel bridges are specified in B.S. No. 153, Part 3A; particulars of the normal uniformly-distributed load and alternative wheel-loads for certain slabs are given in *Table* 6, and notes on their application are on the page facing this table. Reference should be made to B.S. No. 153 for the requirements for stiffening the edges of slabs and for other secondary matters.

On some bridges the effects of an abnormal load must be considered as an alternative to the normal loads; particulars of this load are also given in *Table* 6, and is the same in B.S. No. 153 as that prescribed by the Ministry of Transport. For the design of slabs this load can be converted to an equivalent distributed load as follows: On slabs spanning in one direction, the equivalent load is the normal uniformly-distributed load

plus the knife-edge load; if the slab spans more than 20 ft. in the direction of the longitudinal axis of the bridge, this equivalent load must be increased by 25 per cent. but need not be greater than the load producing a bending moment of 294,000 in.-lb. per foot width. On slabs spanning in two directions the abnormal load is equivalent to the two-wheel loads specified in *Table* 6.

If the standard load is excessive for the traffic likely to use the bridge (having regard to possible increases in the future), the load from ordinary and special vehicles using the bridge, including the effect of the occasional passage of steam-rollers, heavy lorries, and abnormally heavy loads should be considered. Axle loads (without impact) and other data for various types of road vehicles are given in *Table* 6. The actual weights and dimensions vary with different types and manufacturers; notes on weights and dimensions are given on page 150 and include aircraft.

The effect of impact of moving loads is usually allowed for by increasing the static load by an amount varying from 10 per cent. to 75 per cent., depending on the type of vehicle, the nature of the road surface, the type of wheel (whether rubber or steel-tyred), and the speed and frequency of crossing the bridge. An allowance of 25 per cent. on the actual maximum wheel loads is specified in B.S. No. 153, Part 3A for road bridges. A road bridge that is not designed for the maximum loads common in the district should be indicated by a permanent notice stating the maximum loads permitted to use it, and a limitation in speed, and possibly weight, should be enforced on traffic passing under or over a concrete bridge during the first few weeks after completion of the concrete work.

Road bridges may be subjected to forces other than dead load and live loads (including impact); these include wind forces (see page 18) and longitudinal forces due to friction of bearings (*Table* 92). There is also a longitudinal force due to tractive effort and braking which is assumed to act at the level of the road surface on an area 10 ft. wide and 30 ft. long (or the length of the bridge, if less) so disposed as to have the most adverse effect on the member; this force for bridges designed for the normal loading is $10 + 0{\cdot}5\ (L - 10)$ tons (L ft. is the span of the bridge) with a minimum of 10 tons, and 45 tons for bridges of any span to which the abnormal load applies.

Footpaths on road bridges should be designed to carry pedestrians and accidental loading due to vehicles running on the path. A load of 100 lb. per square foot is sufficient to allow for crowds, and an alternative of a single-wheel load of 4 tons, including impact, should be allowed for accidental loading where this can occur. An increase of 50 per cent. in the maximum permissible stresses is sometimes allowed for such accidental loading. If it is probable that the footpath may be converted into a road in the future, the slab and supports should be designed for the same load as the road.

Railway Bridges.—The live load for which a main-line railway bridge should be designed is generally specified by the railway authority and may be a standard load such as that in B.S. No. 153, Part 3A. For light railways, sidings, colliery lines, and the like, smaller loads might be assumed and may be a percentage of the standard load, which assumes a number of heavy locomotives to be on the bridge at one time, but for secondary lines the probability of there being only one locomotive and a train of vehicles of the type habitually using the line should be considered in the interest of economy. The weights of some typical rail vehicles are given in *Table* 6. In addition to dead and live load (including impact), railway bridges have to be designed to resist the effects of lurching and nosing, centrifugal and longitudinal forces, wind pressure, and temperature changes. Data on these effects is given in B.S. No. 153,

Part 3A, for steel bridges and, in the absence of other information, can be used as a guide for reinforced concrete bridges, at least for preliminary designs; the data is summarised on the page facing *Table* 4.

Railway Sleepers.—The load on a railway sleeper is the load transmitted through the chair or other rail-seating, and depends on the type of traffic and whether the sleeper is near or remote from a joint. In B.S. No. 986 (a war-time B.S.) different design loads are specified for ordinary reinforced concrete sleepers and prestressed concrete sleepers. The loads specified for the design of reinforced concrete sleepers are given in *Table* 5.

Structures supporting Cranes.—Cranes and other hoisting equipment are commonly supported on columns in factories or similar buildings, or on gantries. The wheel loads and other particulars for typical overhead travelling cranes are given in *Table* 5. It is important that a dimensioned diagram of the actual crane to be installed is obtained from the makers to ensure that the necessary clearances are provided and the actual loads taken into account. Allowances for the secondary effects on the supporting structure due to the operation of overhead cranes are given on the page facing *Table* 5.

For jib cranes running on rails on supporting gantries, the load to which the structure is subjected depends on the disposition of the weights of the crane. The wheel loads are generally specified by the maker of the crane and should allow for the static and dynamic effects of lifting, discharging, slewing, travelling, and braking. The maximum wheel load under practical conditions may occur when the crane is stationary and hoisting the load at the maximum radius with the line of the jib diagonally over one wheel.

Garages.—The floors of garages are usually considered in two classes, namely those for cars and other light vehicles and those for heavier vehicles. Floors in the lighter class are designed for specified uniformly-distributed imposed loads, or alternative minimum loads which depend on whether the floor is to be used as an all-purpose garage or for parking only. In the design of floors for vehicles in the heavier class, the bending moments and shearing forces should be computed for the probable weights and the most adverse disposition of the heaviest vehicles. Actual wheel loads should be increased by 50 per cent. to allow for dynamic effect. The requirements of the B.S. Code and the London By-laws are given in *Table* 5. A load of 1·5 times the maximum actual wheel load (but not less than 2000 lb.) is assumed to be distributed over an area 2 ft. 6 in. square. For convenience of design, there are given in *Table* 5 the equivalent uniformly-distributed loads for the calculation of the bending moments due to a load of one ton distributed over an area 2 ft. 6 in. square; these equivalent loads are tabulated for various conditions of continuity for slabs and beams and should be multiplied by one-and-a-half times the actual load (in tons). With a wheel directly over a beam and spread on an area 2 ft. 6 in. square, the load is partially carried by adjacent beams. To allow for this effect, reduction factors are given in *Table* 5, the factor being unity for wide beams and for widely-spaced beams; since the thickness of the slab must also be considered in an accurate computation, the tabulated factors are approximate.

Dispersal of Concentrated Loads.

A load from a wheel or similar concentrated load bearing on a small but definite area of the supporting surface (called the contact area) is considered as being further

dispersed over an area dependent upon the combined thicknesses of the road or other surfacing material, filling, concrete slab, and other constructional material. The width of the contact area of the wheel on the slab is equal to the width of the tyre, which may be 1 ft. 6 in. for the heaviest wheels; a minimum of 6 in. is usually assumed. The length of the contact area depends on the type of tyre and the nature of the road surface, and is nearly zero for steel tyres on steel plate or concrete. The maximum contact is probably obtained with an iron wheel on loose metalling or a pneumatic tyre on a tar-macadam surface. A maximum length of 12 in. is reasonable. The dispersal of a concentrated load through the total thickness of the road formation and concrete slab is generally considered as acting at an angle of 45 deg. from the edge of the contact area to the centre of the lower layer of reinforcement, as is shown in the diagrams in *Table* 6.

In the case of a pair of wheels, on one axle, on two rails supported on sleepers it can be considered that the load from the wheels in any position is distributed transversely over the length of a sleeper and that two sleepers are effective in distributing the load longitudinally. The dispersal is assumed as 45 deg. through the ballast and deck below the sleepers, as indicated in *Table* 6. When a rail bears directly on concrete the dispersion may be four to six times the depth of the rail. These rules apply to slowly-moving trains; fast-moving trains may cause a "mounting" surge in front of the train such that the rails, and sleepers, immediately in front of the driving wheels may tend to rise and therefore impose less load in front, but more behind, on the supporting structure.

Marine Structures.

The forces acting upon wharves, jetties, dolphins, piers, docks, sea-walls, and similar marine and riverside structures include those due to the wind and waves, blows and pulls from vessels, the loads from cranes, railways, roads, stored goods and other live loads imposed on the deck, and the pressures of earth retained behind the structure.

In a wharf or jetty of solid construction the energy of impact due to blows from vessels berthing is absorbed by the mass of the structure, usually without damage to the structure or vessel if fendering is provided. With open construction, consisting of braced piles or piers supporting the deck in which the mass of the structure is comparatively small, the forces resulting from impact must be considered, and these forces are dependent upon the weight and speed of approach of the vessel, on the amount of fendering, and on the flexibility of the structure. A large vessel has generally a low speed of approach and a small vessel a higher speed of approach. Some examples are a 500-tons trawler berthing at a speed of 1 ft. per second; a 4000-tons vessel at 6 in. per second; a 10,000-tons vessel at 2 in. per second; and a 26,000-tons vessel at 5 in. per second. The kinetic energy of a vessel of 1000-tons displacement moving at a speed of 1 ft. per second and of a vessel of 25,000 tons moving at 2·4 in. per second is in each case about 16 ft.-tons. The kinetic energy of a vessel having a displacement of W tons approaching at a velocity of V ft. per second is $0{\cdot}016WV^2$ ft.-tons. If the direction of approach is normal to the face of the jetty, the whole of this energy must be absorbed upon impact. More commonly a vessel approaches at an angle of θ deg. with the face of the jetty and touches first at one point about which the vessel swings. The kinetic energy then to be absorbed is $0{\cdot}016W[(V \sin \theta)^2 - (\rho\omega)^2]$, where ρ is the radius of gyration (ft.) of the vessel about the point of impact and ω is the angular

velocity (radians per second) of the vessel about the point of impact. The numerical values of the terms in this expression are difficult, if not impossible, to assess and can vary considerably under different conditions of tide and wind and with different vessels and methods of berthing.

The kinetic energy of approach is absorbed partly by the resistance of the water, but most of it will be absorbed by the fendering, elastic deformation of the structure and the vessel, movement of the ground, and by the energy "lost" upon the impact. The proportion of energy lost upon impact (considered as inelastic impact), if the weight of the structure is W_s, does not exceed $\frac{W_s}{W + W_s}$ approximately. It is advantageous to make W_s approximately equal to W. The energy absorbed by the deformation of the vessel is difficult to assess, as is also the energy absorbed by the ground. It is sometimes recommended that only about one-half of the total kinetic energy of the vessel be considered as being absorbed by the structure and fendering.

The force to which the structure is subject upon impact is calculated by equating the product of the force and half the elastic horizontal displacement of the structure to the kinetic energy to be absorbed. The horizontal displacement of an ordinary reinforced concrete jetty may be about 1 in., but probable variations from this amount combined with the indeterminable value of the kinetic energy absorbed results in the actual value of the force being also indeterminable. Ordinary timber fenders applied to reinforced concrete jetties cushion the blow, but may not substantially reduce the force on the structure. A spring fender or a suspended fender can, however, absorb a large portion of the kinetic energy and thus reduce considerably the blow on the structure. Timber fenders independent of the jetty are sometimes provided to relieve the structure of all impact forces.

The combined action of wind, waves, currents, and tides on a vessel moored to a jetty is usually transmitted by the vessel pressing directly against the side of the structure or by pulls on mooring ropes secured to bollards. The pulls on bollards, due to the foregoing causes or during berthing, vary with the size of the vessel. A pull of 15 tons acting either horizontally outwards or vertically upwards or downwards is sometimes assumed. A guide to the maximum pull is the breaking strength of the mooring rope, or the power of capstans (when provided) which vary from 1 ton up to more than 20 tons at a large dock.

The effects of wind and waves acting on a marine structure are much reduced if an open construction is adopted and if provision is made for the relief of pressures due to water and air trapped below the deck. The force is not, however, directly related to the proportion of solid vertical face presented to the action of the wind and waves. The magnitude of the pressures imposed is impossible to assess with accuracy, except in the case of sea-walls and similar structures where there is such a depth of water at the face of the wall that breaking waves do not occur. In this case the pressure is merely the hydrostatic pressure which can be evaluated when the highest wave level is known, or assumed, and an allowance is made for wind surge, which, in the Thames estuary for example, may raise the high-tide level 5 ft. above the normal head.

A wave breaking against a sea-wall induces a shock pressure additional to the hydrostatic pressure and reaches its maximum value at about mean water-level and diminishes rapidly below this level and less rapidly above it. The shock pressure may be ten times the hydrostatic pressure and maximum pressures up to 6 tons per square foot are possible with waves 15 ft. to 20 ft. high. The shape of the face of

the wall, the slope of the foreshore, and the depth of the water at the wall affect the maximum pressure and the distribution of pressure. All the possible factors that may affect the stability of a sea-wall cannot be taken into account by calculation, and there is no certainty that the severity of the worst recorded storms may not be exceeded in the future.

Wind Forces.

Velocity and Pressure of Wind.—The force due to wind on a structure depends on the velocity of the wind and the shape and size of the exposed members. The velocity depends on the district in which the structure is erected, the height of the structure, and the shelter afforded by buildings or hills in the neighbourhood. In Britain the velocity of gusts may exceed 100 miles per hour but such gusts occur mainly in coastal districts. Gusts of upwards of 80 miles per hour, although infrequent, may be experienced in inland districts. The classification of winds according to the Beaufort scale and the corresponding velocities are given in *Table* 7. The velocity of wind usually increases with the height above the ground. A relationship between velocity and height specified by the Meteorological Office is given in *Table* 7; this expression is based on the velocity at a height of 33 ft. (10 metres).

The pressure due to wind varies as the square of the velocity V, and on a flat surface the theoretical pressure is $0{\cdot}0025V^2$ lb. per square foot. It is necessary, however, to combine the effect of suction on the leeward side of an exposed flat surface with the positive pressure on the windward side. The resultant pressure can be calculated from the expression given in *Table* 7, in which are also tabulated the pressures for certain velocities.

The distribution and intensity of the resultant pressures due to wind depend on the shape of the surface upon which the wind impinges. For example, on a vertical cylindrical surface the intensity of pressure varies as shown on the diagram in *Table* 7. The ratio of height to diameter seriously affects the intensities of the pressures; the greater this ratio, the greater is the pressure. In practice it is usual to allow for the distribution and variation in intensity of the pressure by applying a factor to the normal specified or estimated pressure acting on the projected area of the structure. Such factors are given in *Table* 7 for cylindrical, octagonal, and square " solid " structures with various ratios of height to width; the corresponding factors for open-frame (unclad) structures and for chimneys and sheeted towers are also given. The factors are in accordance with B.S. Code No. 3 (Chapter V).

The wind pressure to be used in the design of any particular structure should be assessed by consideration of relevant conditions, and especially should be based on local records of velocities.

Buildings.—The effect of the wind on buildings is very complex but most building by-laws simply specify a static pressure. For example, local by-laws in Britain sometimes specify that the wind pressure on a building in any horizontal direction shall be assumed to be not less than 15 lb. per square foot on the upper two-thirds of the vertical projected surface of the building with an additional pressure of not less than 10 lb. per square foot on all projections above the general level of the roof. In any particular case it is necessary to determine the requirements of the local authority.

The B.S. Code No. 3 deals with wind forces in more detail. The intensity of external pressure is calculated from the velocity of the wind and the height of the building. The assumed velocity depends on the locality and degree of exposure of

the building. Neglect of the effect of wind on the building as a whole is permitted when, in a building stiffened by walls and floors, the height does not exceed twice the width. The basic pressures recommended in the B.S. Code for buildings of various heights and exposures are given in *Table* 8; see also the notes relating to *Table* 8 on the page facing *Table* 7.

The probable variation and distribution of pressures and suctions on buildings without permanent openings in the walls are shown in *Table* 7, but for design purposes, these effects are simplified as follows.

The design of a building to resist wind should take into consideration (i) the stability of the building as a whole and the pressures and forces on the foundations; (ii) the design of the walls; (iii) the design of the roof; and (iv) projections above the roof. For stability and foundations, the primary force is generally the pressure on the vertical faces of the building. The basic intensity of pressure p (as given in *Table* 8), is considered to be a pressure of $0{\cdot}5p$ on the windward face and a suction of $0{\cdot}5p$ on the leeward face. These pressures are assumed to act uniformly on the whole height of the vertical face of the building and must be combined with the horizontal components of the pressures and suctions on the roof which are assumed to act at right-angles to the slope of the roof. On a building with a flat roof, the windward half of the roof is subjected to a suction of p and the leeward half to a suction of $0{\cdot}75p$. On a pitched roof the pressures and suctions on the windward and leeward slopes depend on the degree of slope and appropriate values are given in *Table* 8; these pressures and suctions apply to the roof as a whole, but for the design of the roof covering and purlins, or other supports, greater local pressures and suctions, as also given in *Table* 8, should be considered. In calculating the stability the total pressure on any projection above the general level of the roof must also be taken into account, the value of p depending upon the total height of the projection above the ground. In the design of the walls, to the external pressure and suction of $0{\cdot}5p$ an allowance must be added for internal pressure and suction, depending on the area of openings in the walls. In ordinary buildings it is recommended that the alternatives of an internal pressure or an internal suction should be assumed; therefore a wall as a whole should be designed to resist an inward or outward pressure of the amounts given in *Table* 8, but individual panels of the wall should be designed to resist the greater pressure stated in the table. The walls and roofs of projections above the general roof level should be designed as described for the walls and roof of the main building, the basic intensity of pressure being greater to allow for the greater height. An example of the application of the recommendations in *Table* 8 is given on the page facing the table.

Curved roofs should be divided into segments as described in *Table* 3. B.S. Code No. 3 should be referred to for roofs of multiple spans and the effect of wind " drag ".

Chimneys and Towers.—Since a primary factor in the design of chimneys and similarly exposed isolated structures is the force of the wind, careful consideration of each case is necessary to avoid either under-estimating this force or making an unduly high assessment. Where records of wind velocities in the locality are available an estimate of the probable wind pressures can be made. Due account should be taken of the susceptibility of narrow shafts to the impact of a gust of wind. Some by-laws in Britain specify the intensities of horizontal wind pressure to be used in the design of circular chimney shafts for factories. The total lateral force is the product of the specified pressure and the maximum vertical projected area, and a factor of safety of at least 1½ is required against overturning. In some instances specified pressures

are primarily intended for the design of brick chimneys, and in this respect it should be remembered that the margin of safety is greater in reinforced concrete than in brickwork or masonry owing to the ability of reinforced concrete to resist tension, but a reinforced concrete chimney, like a steel chimney, is subject to oscillation under the effect of wind. The pressures recommended in the B.S. Code No. 3 are given in *Table* 8; these recommendations allow for a variable pressure increasing from a minimum at the bottom to a maximum at the top of the chimney (or tower). A factor, as given in *Table* 7, to allow for the shape of the structure, can be applied to allow for the relieving effect of curved and polygonal surfaces of chimneys, and of the tanks and the supporting structures of water towers. For cylindrical shafts with fluted surfaces a higher factor than that given in *Table* 7 should be applied. Local meteorological records should be consulted to determine the probable maximum wind velocity at a height of 40 ft. The chimney, or other structure, can be divided into a number of parts and the average pressure on each part is taken from *Table* 8.

Chimneys extending above the roof of a building can be designed for the wind forces recommended in the B.S. Code or can be dealt with as described in the foregoing for projections above the general roof level.

Bridges.—The forces due to the wind can be neglected on an ordinary reinforced concrete bridge that does not project far above the level of the surrounding land and the width of which is about the same as its height. For high bridges and those erected in exceptionally exposed positions the probable wind forces should be investigated. If other intensities of wind pressure are not specified, and if greater pressures due to exceptional conditions of exposure are not deemed likely, the pressure and rules for calculating the exposed area as given in *Table* 8 can be adopted.

Poles and Posts.

Transmission-line Poles.—A transmission-line pole is subject to the weights and pulls of the conductors suspended from it. Due to the pressure of the wind on the conductors, a horizontal force acts transversely to the line at the point at which the conductors are attached. In computing the weight of the conductor and the area exposed to the pressure of the wind, the diameter is considered to be increased by adhering ice, the increase in Britain generally being assumed to be about $\frac{3}{8}$ in., although some regulations require the increase to be 1 in. for high-tension conductors and $\frac{1}{2}$ in. for low-tension conductors. The wind pressure on a high pole must also be taken into account, this being assumed to act at the midpoint of the part of the pole above ground. Formulæ for the wind forces on the pole and the conductors are given in *Table* 9 and suitable wind pressures are given in *Table* 8. Notes on the design of transmission-line poles are given in *Table* 9.

The wind force on a transmission-line pole is mainly that transferred to the pole by wind pressure on the conductors. In Britain it is usual to assume that a net wind pressure of 8 lb. per square foot (p_c in the formulæ in *Table* 9) acts on the projected area of the ice-covered conductor; this pressure includes the allowance for the shape of the conductor. If a pole carries several conductors no allowance should be made for one conductor shielding another, since high-tension conductors are not closely spaced. If 8 lb. per square foot is allowed for the pressure on the conductors of assumed circular section, reasonable values for the pressure on the exposed face of the pole (p_p in *Table* 9) are 15 lb. per square foot for a rectangular section (as in

Table 8) and 8 lb. per square foot for a circular section. Greater pressures may be necessary for poles that are perforated.

Lamp Posts.—The load on a lamp post includes, in addition to the weight of the post, the weights of and wind pressure on the bracket, the lantern, and the raising and lowering gear. The weights of the lantern and of the moving part of the raising and lowering gear should be increased by 50 per cent. to allow for dynamic effects. A factor of safety of $2\frac{1}{2}$ on these loads is usually specified. For the calculation of wind pressures on tall lamp posts B.S. No. 1308 specifies a wind velocity of 70 miles per hour for which the corresponding pressures at various heights are given in *Table* 8.

Active Pressures of Retained and Contained Materials.

The value of the horizontal pressure exerted by a contained material or by earth or other material retained by a wall is uncertain, except when the contained or retained material is a liquid. The formulæ, rules and other data in *Tables* 10 to 14 are given as practical bases for the calculation of such pressures. Reference should also be made to Code No. 2, " Earth-Retaining Structures ", of the Institution of Structural Engineers.

Liquids.—At any depth h ft. below the free surface of a liquid, the intensity of pressure p lb. per square foot normal to a surface subject to pressure from the liquid is equal to the intensity of vertical pressure which is given by the simple hydrostatic expression $p = wh$, where w is the unit weight of the liquid in lb. per cubic foot.

Granular Materials.—When the contained material is granular, for example, dry sand, grain, small coal, gravel, or crushed stone, the pressure normal to a retaining surface can be expressed conveniently as a fraction of the equivalent fluid pressure, thus $p = kwh$, where k is a measure of the " fluidity " of the contained or retained material and varies from unity for perfect fluids to zero for materials that stand unretained with a vertical face. The value of k depends also on the physical characteristics, water content, angle of repose, angle of internal friction, and the slope of the surface of the material, on the slope of the wall or other retaining surface, on the material of which the wall is made, and on the surcharge on the contained material. The value of k is determined graphically or by calculation, both methods being usually based on the wedge theory or the developments of Rankine or Cain. The total pressure normal to the back of a sloping or vertical wall of height H can be calculated from the formulæ in *Table* 10 for various conditions.

Friction between the wall and the material is usually neglected, resulting in a higher calculated normal pressure which is safe. Friction must be neglected if the material in contact with the wall can become saturated and thereby reduce the friction by an uncertain amount or to zero. Only where dry materials of well-known properties are being stored may this friction be included. Some values of the coefficient of friction μ are given in *Table* 14. When friction is neglected (that is $\mu = 0$), the pressure normal to the back of the wall is equal to the total pressure and there is, theoretically, no force acting parallel to the back of the wall.

Generally, in the case of retaining walls and walls of bunkers and other containers, the back face of the wall is vertical (or nearly so) and the substitution of $\beta = 90$ deg. in the general formulæ for k gives the simplified formulæ in *Table* 10. Values of k_1 (maximum positive slope or surcharge), k_2 (level fill) and k_3 (maximum negative slope) for various angles of internal friction (in degrees and gradients) are given in *Table* 11; the value of such angles for various granular materials are given in *Tables* 11 and 14.

For a wall retaining ordinary earth with level filling k_2 is often assumed to be 0·3 and, with the average weight of earth as 100 lb. per cubic foot, the corresponding intensity of horizontal pressure is 30 lb. per square foot per foot of height. The formulæ assume dry materials. If ground-water occurs in the filling behind the wall, the modified formula given on the page facing *Table* 10 applies.

The intensity of pressure normal to the slope of an inclined surface is considered on the page facing *Table* 10 and in *Table* 11.

Effect of Surcharge (Granular Materials).—The effects of various types of surcharge on the ground behind a retaining wall are evaluated in *Table* 13, comments on which are given on the page facing the table.

Theoretical and Actual Pressures of Granular Materials.—In general practice, horizontal pressures due to granular materials can be determined by the purely theoretical formulæ of Rankine, Cain, and Coulomb. Many investigators have made experiments to determine what relation actual pressures bear to the theoretical pressures, and it appears that the Rankine formula for a filling with a level surface and neglecting friction between the filling and the back of the wall gives too great a value for the pressure. Thus retaining walls designed on this theory should be on the side of safety. The theory assumes that the angle of internal friction of the material and the surface angle of repose are identical, whereas some investigators find that the internal angle of friction is less than the angle of repose and depends on the consolidation of the material. The ratio between the internal angle of friction and the angle of repose has been found to be approximately between 0·9 to 1. For a filling with a level surface the horizontal pressure given by $p = wh\left(\frac{1 - \sin\theta}{1 + \sin\theta}\right)$ agrees very closely with the actual pressure if θ is the angle of internal friction and not the angle of repose. The maximum pressure seems to occur immediately after the filling has been deposited, and the pressure decreases as settling proceeds. The vertical component of the pressure on the back of the wall appears to conform to the theoretical relationship $V = P \tan \mu$. A rise in temperature produces an increase in pressure of about 1 per cent. per 10 deg. F.

The point of application of the resultant thrust on a wall with a filling with a level surface would appear theoretically to be at one-third of the total height for shallow walls, and rises in the course of time and with increased heights of wall. According to some investigators the point of application is at the third-point for fillings the surface of which slopes downwardly away from the wall and rises as the slope increases upward.

Loads imposed on the ground behind the wall and within the plane of rupture increase the pressure on the wall, but generally loads outside the wedge ordinarily considered can be neglected. The increase of pressure due to transient imposed loads remains temporarily after the load is removed. If the filling slopes upwards, theory seems to give pressures almost 30 per cent. in excess of actual pressures.

Cohesive Soils.—Cohesive soils include clays, soft clay shales, earth, silts, and peat. The active pressures exerted by such soils vary greatly; due to cohesion, pressures may be less than those due to granular soil, but saturation may cause much greater pressure. The basic formula for the intensity of horizontal pressure at any depth on the back of a vertical wall retaining a cohesive soil is that of Mr. A. L. Bell (derived from a formula by Français). Bell's formula is given in two forms in *Table* 10. The cohesion factor is the shearing strength of the unloaded clay at the surface.

Some typical values of the angle of internal friction and the cohesion C for common cohesive soils are given in *Table* 12, but actual values should be ascertained by test.

According to this formula there is no pressure against the wall down to a depth $\frac{2C}{w\sqrt{k_2}}$ below the surface if the nature of the clay is prevented from changing, but as this condition is unlikely to exist owing to the probability of moisture changes, it is essential that hydrostatic pressure should be assumed to act near the top of the wall. Formulæ for the pressure of clays of various types and in various conditions are given in *Table* 12, together with the properties of these and other cohesive soils. In general, friction between the clay and the back of the wall should be neglected.

Passive Resistance of Granular and Cohesive Materials.

The remarks in the previous paragraph relate to the active horizontal pressure exerted by contained and retained materials.

If a horizontal pressure in excess of the active pressure is applied to the vertical face of a retained bulk of material, the passive resistance of the material is brought into action. Up to a limit, determined by the characteristics of the particular material, the passive resistance equals the applied pressure; the maximum intensity that the resistance can attain for a granular material with a level surface is given theoretically by the reciprocal of the active-pressure factor. The passive resistance of earth is taken into account when considering the resistance to sliding of a retaining wall, when dealing with the forces acting on sheet-piles, and when designing earth anchorages, but in these cases consideration must be given to those factors, such as wetness, that may reduce the probable passive resistance. Abnormal dryness may cause clay soils to shrink away from the surface of the structure, thus necessitating a small but most undesirable movement of the structure before the passive resistance can act.

For a dry granular material with level fill the passive resistance is given by the formula in *Table* 10; expressions for the passive resistance of water-logged ground are given on the page facing the table. It is not easy to assess the passive resistance when the surface of the material is not level, and it is advisable never to assume a resistance exceeding that for a level surface; when the surface slopes downwards the passive resistance should be neglected.

For ordinary saturated clay the passive resistance is given by the formula in *Table* 10 and the corresponding formulæ for clay in other conditions are given in *Table* 12.

Horizontal Pressures of Granular Materials in Liquid.

The effect of saturated soils is considered in preceding paragraphs. The notes on the page facing *Table* 10 and the numerical values of some of the factors involved for certain materials as given in *Table* 11 apply to granular materials immersed in or floating in liquids.

Deep Containers (Silos).

The foregoing observations relating to the pressures on walls relate only to retaining walls and the walls of shallow containers. In deep containers, termed silos, the arching effect of the contained material considerably relieves the wall of some of the horizontal pressure, so that the principle that the pressure is proportional to the depth of filling is no longer true. The limiting condition that determines whether any particular container shall be considered as " shallow " or " deep " is that if the plane of rupture

strikes the opposite wall before reaching the free surface the container can be treated as "deep"; if otherwise the container must be considered as "shallow". These conditions are shown in the diagrams in *Table* 14, in which also is given the formula for determining the theoretical minimum depth of a "deep" container. For grain the minimum depth is theoretically equal to about one-and-a-quarter times the breadth of the container, and for cement it is equal to about one-and-a-half times the breadth. During the emptying process the arching action on which the reduction of pressure depends is partially or completely destroyed, and pressures appreciably in excess of the theoretically determined pressures are obtained. It would seem therefore that for absolute security no container that is not at least twice as deep as it is wide should be treated as a deep container.

Pressures in Silos.—The value and variation of the pressures in silos is usually computed by either Janssen's or Airy's formula. Janssen's formula gives pressures slightly lower than Airy's formula, and in its general form is given in *Table* 14. The horizontal pressure p_h depends on the weight w per cubic foot of material in the silo, the ratio R of plan area A to the perimeter of the silo, the angle of friction μ between the contained material and the wall of the silo, the depth h of material above the plane considered, and a factor involving, among other terms, the ratio k of the horizontal to the vertical pressure; this ratio equals $k_2 \left(= \frac{1 - \sin\theta}{1 + \sin\theta}\right)$ for unconfined granular materials, but values for confined materials are given in *Table* 14, together with other properties of materials commonly stored in silos, and values of R for silos of common shapes.

The intensity of vertical pressure p_v on any horizontal plane of the material is $\frac{p_h}{k}$. Therefore the total pressure on any horizontal plane is $\frac{p_h A}{k}$, and the load transferred by friction to the walls of the silo is $\left(wh - \frac{p_h}{k}\right)R$ per unit length of wall.

For materials such as grain the value of k is usually about 0·5, and the value of $\tan\mu$ for concrete walls is about 0·444. On the page facing *Table* 14 the magnitude of $\frac{p_h}{w}$ for various values of R and h, with these values of k and $\tan\mu$, are given.

The value of k equal to 0·5 commonly adopted for reinforced concrete grain silos is on the safe side for the calculation of the horizontal pressure and load carried by the walls, but for the calculation of the vertical pressure on the bottom of the silo, a value of k equal to $\frac{1}{3}$ is advisable. For ordinary coarse cement common values are $w = 90$ lb. per cubic foot, $k = 0{\cdot}53$ (assuming an angle of repose of 18 deg.), and $\tan\mu = 0{\cdot}577$. In silos where the cement is pneumatically agitated, the friction between the wall and the cement, on which the reduced pressure depends, does not operate, and the walls of such silos must be designed for "fluid" pressures assuming a weight of, say, 75 lb. per cubic foot.

From the data on the page facing *Table* 14 it is seen that the pressure on the wall increases very little below a depth of three times the least width of the container. Tests indicate that the maximum pressures occur in silos during filling, and are somewhat greater for a silo filled rapidly than one filled slowly. The pressures p_h and p_v, which vary with the depth of the filling (see the diagram in *Table* 14) and attain theoretical maximum values at infinite depth, approach the maximum values of $p_{max.} = \frac{wR}{\tan\mu}$ and $p_{v max.} = \frac{wR}{k\tan\mu}$. A practical and safe method is therefore to

design the upper part of the wall of a silo for a pressure that increases according to the formula for a shallow container, $p_h = k_2wh$, until a depth is reached where $p_h = p_{max.}$; below this depth the wall is designed for the pressure $p_{max.}$ as shown in the diagram in *Table* 14.

Grain will not flow down a slope less steep than 33 deg. to the horizontal; therefore the angle in the valley of a pyramidal bottom of a grain silo should be not less than, say, 30 deg. The corresponding valley angle for clean coal is 45 deg. A method of calculating the valley slope is given on page 342.

Since the weight of barley is about 25 lb. per cubic foot the pressures due to this material are only about half the pressures due to wheat; therefore a silo designed to contain barley must not be filled with other grain. Likewise, any container designed to hold coal should not be filled with stone, and a silo designed to hold static cement may be insufficiently strong to hold cement fluidised by the injection of air. It is important also to ensure that the coefficient of friction between the stored material and the wall is not over-estimated when designing the bottom or under-estimated when designing the wall; some smooth linings installed after the silo has been constructed may invalidate assumptions made in the design.

SECTION 2

BENDING MOMENTS AND FORCES

THE bending moments and shearing forces on freely-supported beams are determined readily from the simple statical rules but, for continuous beams and statically-indeterminate frames, one of several methods of analysis should be applied. The most suitable method depends mainly on the type of problem and, to a certain extent, on the appeal a certain method may have to the designer if two or more methods are otherwise equally suitable. The methods given in this section include mathematical methods such as the Theorem of Three Moments and Slope-deflection, and practical methods such as Moment distribution. Many of the tabulated results, the derivation of which are not necessarily given, are obtainable by application of more than one method.

The basic relation between the shearing force, bending moment, slope and deflection caused by a load in a structural member are given in *Table* 15, in which is also given typical diagrams of bending moments and shearing forces for cantilevers, propped cantilevers, freely-supported beams, and beams fixed or continuous at both supports.

In practice, the deflection of a reinforced concrete beam is restricted to a reasonable amount by limiting the ratio of the span to the depth of the member; the ratios for various types of beams and slabs are given in *Table* 15. Adoption of these or smaller ratios ensures that in general the efficiency of a structure is not impaired and that cracks are not likely to occur in applied finishes.

Cantilevers and Beams of One Span.

In *Table* 16 are tabulated the coefficients for maximum shearing forces (that is the reactions of the supports), the coefficients for the maximum bending moments, and the coefficients for the maximum deflections produced by common types of load on cantilevers and freely-supported beams of one span. Similar data are given in *Table* 17 (general cases) and *Table* 17A (special cases) for beams rigidly fixed at both ends and beams with one end fixed and one end freely supported (that is propped cantilevers). Notes on the derivation of the factors for propped cantilevers and fixed beams are given on the page facing *Table* 18.

The bending-moment factors for beams of one span fixed at both supports are the basis of the load-factors used in calculations in some methods of analysing statically-indeterminate structures, and such load-factors (which should not be confused with load-factors used in determining the resistances of members by ultimate-load methods) and notes relating to the methods to which they apply are given in *Table* 18.

Continuous Beams.

There are several methods of determining the bending moments and shearing forces on beams continuous over two or more spans; some of these methods are dealt with in the following and in the corresponding tables.

Calculated Bending Moments and Shearing Forces.—The bending moments on a beam continuous over two or more spans can be calculated by the Theorem of Three Moments which in its general form for any two contiguous spans is expressed

by the general and special formulæ given in *Table* 19. Notes on the use of the formulæ and calculation of the shearing forces are considered on the page facing *Table* 20; an example is given on the page facing *Table* 19. The formulæ establish the values of the bending moments negative at the supports; the positive bending moment in the span can be obtained graphically or, in the case of uniformly-distributed loads, from the formulæ given in *Table* 20.

It should be noted that the loading producing the greatest negative bending moments at the supports is not necessarily that producing the greatest positive bending moments in the span. The incidence of live load to give the greatest bending moments is illustrated in *Table* 20 and comments are given on the page facing the table. Some dispositions of live load may produce negative bending moments in adjacent unloaded spans and approximate formulæ for calculating such bending moments are given in *Table* 20.

The moment of inertia of a reinforced concrete beam of uniform depth may vary throughout its length because of variations in the amount of reinforcement. It is common, however, to neglect these variations for beams of uniform depth and for beams having small haunches at the supports. Where the depth of a beam varies considerably, neglect of the variation of moment of inertia when calculating the bending moments leads to results that differ widely from the probable bending moments. Methods of dealing with beams of non-uniform moment of inertia are given in *Table* 19 and on the page facing *Table* 20. If the method of moment distribution is used, the coefficients and formulæ in *Table* 33 are applicable.

Another method of determining the bending moments at the supports of beams continuous over equal or unequal spans is given in *Table* 25; explanatory notes of this method are given on the page facing *Table* 24 and an example of the use of *Table* 25 is given on the page facing the table. A graphical method based on "fixed points" is given in *Table* 24 and described on the page facing the table.

The shearing forces on a span of a continuous beam can be calculated from the basic formulæ given in *Table* 20.

Coefficients for Bending Moments and Shearing Forces for Equal Spans.— For beams continuous over a number of equal spans, calculation of the maximum bending moments from basic formulæ is unnecessary since the moment and shearing forces can be tabulated. For example, in *Table* 21 are given the values of the bending-moment coefficients for the middle of each span and at each support for two, three, four, and five or more continuous equal spans carrying identical loads on each span, which is the usual disposition of the dead load on a beam. The coefficients for the maximum bending moments at midspan and support for the most adverse incidence of live loads are also given; the alternative coefficients assuming only two spans to be loaded in the case of the bending moments at the supports are given in brackets. It should be noted that the maximum bending moments do not occur at all sections simultaneously. The types of load considered are a uniformly-distributed load, a single load concentrated at midspan, and equal loads at the two third-points of the span. The bending-moment diagrams for beams continuous over two or more spans are given in *Tables* 22 and 23. The theoretical bending moments may be adjusted to reduce the peak negative bending moments which can be reduced by 15 per cent. if the numerical value of the reduction is added to the positive bending moments in the two adjacent spans. This adjustment, which is convenient to reduce the inequality between the negative and positive moments, conforms to the recommendations in

B.S. Code No. 114; these recommendations permit an increase or decrease of 15 per cent. with consequent adjustment of the bending moments in the adjacent spans, the amount of the adjustment being different for interior spans and end spans; the adjusted bending-moment coefficients and diagrams are given in *Tables* 22 and 23. The basis of the diagrams in these tables is described below.

It is generally assumed that an ordinary continuous beam is freely supported on the end supports (unless fixity or other condition of restraint is specifically known), but in most cases the beam is constructed monolithically with the support, thereby producing some restraint; in *Table* 23 are given coefficients for this case.

The shearing forces produced by a uniformly-distributed load when all spans are loaded and the greatest shearing forces due to any incidence of live load are given in *Table* 21 for beams continuous over equal spans.

Approximate Bending-moment Coefficients.—The precise determination of the theoretical bending moments on continuous beams may involve much mathematical labour, except in cases which occur often enough to warrant tabulation. Having regard to the general assumptions of unyielding knife-edge supports and uniform moment of inertia, the probability of the theoretical bending moments being greater or less than those actually realised should be considered. The effect of variation of the moment of inertia is given on page 174. The following factors cause a decrease in the negative bending moment at a support: settlement of the support relative to adjacent supports causing an increase in the positive bending moments in the adjacent spans and may be sufficient to convert the bending moment at that support into a positive bending moment; supports of considerable width; support and beam constructed monolithically. The settlement of one or both of the supports on either side of a given support causes an increase in the negative bending moment at the given support and consequently affects the positive bending moments in adjacent spans.

The indeterminate nature of the actual bending moments occurring leads in practice to the adoption of approximate bending-moment coefficients for continuous beams and slabs of about equal spans with uniformly-distributed loads. Such coefficients are given in the lower part of *Table* 20; notes on the use of the coefficients are given on the page facing the table.

When the bending moments are calculated with the spans assumed to be equal to the distance between the centres of the supports, the critical bending moment in monolithic construction can be considered as that occurring at the edge of the support. When the supports are of considerable width the span can be considered as the clear distance between the supports plus the effective depth of the beam, or an additional span introduced equal to the width of the support minus the effective depth of the beam. The load on this additional span can be considered as the reaction of the support spread uniformly along the part of the beam over the support. When a beam is constructed monolithically with a very wide and massive support the effect of continuity with the span or spans beyond the support may be negligible, in which case the beam should be treated as fixed at the support. Other cases of beams being constructed monolithically with a support of moderate size, such as in ordinary beam and column construction, are considered in *Tables* 31 and 32, but in these cases the bending moment on the beam at the interior supports does not exceed the calculated bending moment, assuming knife-edge supports.

Bending-moment Diagrams for Equal Spans.—The basis of the bending-moment diagrams in *Tables* 22 (two and three spans) and 23 (four or more spans) is

as follows. The theoretical bending moments are calculated to obtain the coefficients for the bending moments near the middle of each span and at each support for a uniformly-distributed load, a central load, and loads concentrated at the third-points of each span. The condition of all spans loaded (for example, dead load) and conditions of incidental (or live) load producing the greatest bending moments are considered. As the coefficients are calculated by the Theorem of Three Moments (and are therefore not approximate), the decrease or increase of 15 per cent. of the maximum negative bending moments at the supports, as recommended in the B.S. Code, is permissible. The coefficients are reduced by 15 per cent. to establish the reduced bending moments at the supports, and are increased by 15 per cent. to derive the reduced positive bending moments in the spans. *Tables* 22 and 23 also give the coefficients for the positive bending moments at the supports and the negative bending moments in the spans which are produced under some conditions of live load; it is not generally necessary to take these small bending moments into account as they are generally insignificant compared with the bending moments due to dead load.

The method of calculating the adjusted coefficients is that the theoretical bending moments are calculated for all spans loaded (dead load), and for each of the four cases of live load that produce maximum bending moments, that is at the middle of an end span (positive), at an inner support (negative), at the middle of the interior span (positive), and at an inner support (positive). For each case, the theoretical bending-moment diagram is adjusted as described in the following. For the diagram of maximum negative bending moments, the theoretical negative bending moments at the supports are reduced by 15 per cent. and the positive bending moments are increased accordingly. For the diagram of maximum positive bending moments in the spans, the theoretical negative bending moments at the supports are increased by 15 per cent. and the positive bending moments are reduced accordingly. The diagrams for live load are combined to give the resultant diagrams for the theoretical bending moments without adjustment and with the greatest degree of adjustment.

Moment Distribution applied to Continuous Beams.

The bending moments at the supports of beams continuous over any number of equal or unequal spans, with any type of loading, and with different moments of inertia in each span, can be calculated directly by moment-distribution methods. The general principles of moment-distribution are given in *Table* 47, but the different sign convention used in *Table* 26 should be noted. Basic formulæ for the bending moments at particular supports in systems of two spans and upwards are given in *Table* 26. The fundamental formula, which is that for any interior support T in four or more spans, is based on two distributing operations, and the results are reasonably accurate compared with those obtained by more tedious methods. All terms substituted in the tabulated formulæ are positive numerical values. If the resultant sign of the support bending moment is negative, tension in the top edge of the beam is indicated. The formulæ are applicable to all spans loaded, or to one or more spans only loaded.

Two conditions of end support are allowed for, namely, complete fixity and nominal free support. For any other conditions, such as a beam cantilevered beyond the end support or continuous with an exterior column, the support moments due to the loaded spans are first calculated assuming free support at the end support, and an adjustment is made to the penultimate support bending moment (as indicated in *Table* 26) for the externally-applied bending moment at the end support. The

externally-applied bending moment is the cantilever bending moment in the case of a beam extending beyond an end support. If the beam is monolithic with an interior column, the externally-applied bending moment can be calculated from *Table* 32, the bending moment at the end support being the externally-applied bending moment.

The stiffness of a beam freely supported at one support and fixed at the other is less than that of a beam fixed at both supports. In moment-distribution operations it is common to allow for the difference, but in the formulæ in *Table* 26 this difference has been neglected, thereby allowing the same expressions to be used for all distribution factors and fixed-end bending moments. Neglect of the theoretically reduced stiffness also makes allowance for the partial restraint which in most cases exists at nominally free supports and for restraint less than complete fixity at supports over which a beam is continuous.

Formulæ for special conditions, such as equal spans, uniform moment of inertia, symmetrical inequality of spans, and the like for two, three, four and five or more spans are given in *Tables* 27, 28, 29 and 30. The effects of bending moments applied at end supports are given in *Tables* 31 and 32, and the effects of splays adjacent to the supports of beams are given in *Table* 33. Explanatory notes and the method of using *Tables* 26 to 33 are given on the pages facing *Tables* 27, 31, 32 and 33; some examples are given on the pages facing *Tables* 28, 29 and 30.

An example of the direct application of the principles of moment distribution to the calculation of the bending moments on a continuous beam, with the minimum use of the tables, is given on the page facing *Table* 30.

Since the bending moments given by the formulæ in *Tables* 26 to 33 are approximate, as are in fact most calculations by moment-distribution except in the case of two continuous spans, the decrease of some peak moments (and the consequent increase of others) is not applicable.

End Restraint.—If, as is common in column-and-beam construction, the end of a continuous beam is subject to a restraining moment due to monolithic construction, the positive and negative bending moments and shearing forces are affected throughout the beam, but principally in the first two spans. The bending moments resulting from such monolithic action can be calculated from the formulæ (based on moment distribution) given in *Table* 32, and the effects of the bending moments can be calculated from the formulæ in *Table* 31, in the upper part of which are given the comprehensive, but approximate, formulæ in accordance with the method of moment distribution. In the lower part of *Table* 31 are given more exact expressions for equal spans. Coefficients are given by which the bending moments and shearing forces can be adjusted when a bending moment is applied at one end or at both ends of the beam.

Moving Loads on Continuous Beams.

Bending moments caused by moving loads, such as those due to vehicles traversing a series of continuous spans, are most easily calculated by the aid of influence lines. An influence line is a curve with the span of the beam as a base, the ordinates of the curve at any point being the value of the bending moment produced at a particular section of the beam when unit load acts at the point. The data given in *Tables* 34 to 37 enable the influence lines for the critical sections of beams continuous over two, three, four, and five or more spans to be drawn. By plotting the position of the load on the beam (drawn to scale), the bending moments at the section being considered

are derived as explained in the example given on the page facing *Table* 34. The curves in the tables for equal spans are directly applicable to equal spans but the corresponding curves for unequal spans should be plotted from the data tabulated.

The bending moment due to a load at any point is the ordinate of the influence line at that point multiplied by the product of the load and the span, the length of the shortest span being used when the spans are unequal. The influence lines in the tables are drawn for symmetrical inequality of spans. Coefficients for span-ratios not plotted can be interpolated. The symbols on each curve indicate the section of the beam and the ratio of spans to which the curve applies.

Slabs spanning in One Direction.

Uniformly-distributed Load.—The bending moments on slabs supported on two opposite sides are calculated in the same way as for beams, account being taken of continuity. For slabs carrying uniformly-distributed loads and continuous over nearly equal spans, the coefficients for dead and live load as given in *Table* 20 for slabs without splays conform to the recommendations of B.S. Code 114. Other coefficients, allowing for the effect of splays on the bending moments, are also tabulated. Spans are considered to be equal if the difference in length of adjacent spans does not exceed 15 per cent. of the shorter span.

If a slab is nominally freely supported at an end support, it is advisable to provide resistance to a probable negative bending moment at a support with which the slab is monolithic. If the slab carries a uniformly-distributed load, the value of the negative bending moment should be assumed to be not less than $\frac{1}{24}wL^2$.

Although a slab may be designed as though spanning in one direction, it should also be reinforced in a direction at right-angles to the span, as described on page 268.

Concentrated Load.—When a slab supported on two opposite sides only carries a load concentrated on a part only of the slab, such as a wheel load on the deck of a bridge, there are two principal methods of determining the bending moments. The first method is to assume that a certain width of the slab carries the entire load, and in one such method the contact area of the load is first extended by dispersion through the thickness of the slab as shown in *Table* 6, giving the dimensions of loaded area as v at right-angles to the span and u parallel to the span L. The width of slab carrying the load may be assumed to be $\frac{2}{3}(L + u) + v$. The total concentrated load is then divided by this width to give the load carried on a 1-ft. width of slab for the purpose of calculating the bending moments. The width of slab assumed to carry a concentrated load according to the recommendations of B.S. Code No. 114 is as illustrated in the lower part of *Table* 43.

Another method is to extend to slabs spanning in one direction the theory of slabs spanning in two directions. For example, the curves given in *Table* 41 for a slab infinitely long in the direction L_L can be used to evaluate directly the bending moments in the direction of, and at right-angles to, the span of a slab spanning in one direction and carrying a concentrated load; an example of this application is given on the page facing *Table* 42.

Slabs spanning in Two Directions.

When a slab is supported otherwise than on two opposite sides only, the precise amount and distribution of the load taken by each support, and consequently the

magnitude of the bending moments on the slab, are not easily calculated if assumptions resembling practical conditions are made. Therefore approximate analyses are generally used. The method applicable in any particular case depends on the shape of the panel of slab, the condition of restraint at the supports, and the type of load. The principal bases of the analyses of slabs spanning in two directions are the theory of plates and the yield-line theory.

Distinction must be made between the conditions of free support, fixity, partial restraint, and continuity, and it is essential to establish whether the corners of the panel are free to lift or not. Free support occurs rarely in practice, since in ordinary reinforced concrete beam-and-slab construction, the slab is monolithic with the beams and is thereby partially restrained and is not free to lift at the corners. The condition of being freely supported may occur when the slab is not continuous and the edge bears on a brick wall or on un-encased structural steelwork. If the edge of the slab is built into a substantial brick or masonry wall, or is monolithic with concrete encasing steelwork or with a reinforced concrete beam or wall, partial restraint exists. Restraint is allowed for when computing the bending moments on the slab but the supports must be able to resist the torsional and other effects induced therein; the slab must be reinforced to resist the negative bending moment produced by the restraint. Since a panel of slab freely supported along all edges but with the corners held down is uncommon (because corner restraint is generally due to edge-fixing moments), bending moments for this case are of interest mainly for their value in obtaining coefficients for other cases of fixity along or continuity over one or more edges. A slab can be considered as fixed along an edge if there is no change in the slope of the slab at the support irrespective of the incidence of the load. This condition is assured if the polar moment of inertia of the beam or other support is very large. Continuity over a support generally implies a condition of restraint less rigid than fixity, that is the slope of the slab at the support depends upon the load not only on the panel under consideration but on adjacent panels.

Rectangular Panel with Uniformly-distributed Load.—Empirical formulæ and approximate theories have been put forward for calculating the bending moments in the common case of a rectangular panel of slab supported along four edges (and therefore spanning in two directions mutually at right-angles) and carrying a uniformly-distributed load. The bending moments depend on the ratio of the length of the sides of the panel. The " exact " theory of the bending of plates spanning in two directions was established by Lagrange and Navier in the nineteenth century. Pigeaud and others later completed the analysis of panels freely supported along all four edges. Because most theoretical expressions are complex, curves or close arithmetical approximations are generally adopted in practice. Westergaard has combined theory with the results of tests and his work is the basis of the bending-moment coefficients which are recommended in B.S. No. 114 and which are represented by the curves in *Table* 39; these curves apply to freely-supported panels and to panels where there is continuity on one or more sides.

The simplified analysis of Grashof and Rankine can be applied when the corners of a panel are not held down; the bending-moment coefficients are given in *Table* 38 and the basic formulæ are given on the page facing the table. If corner restraint is provided, coefficients based on more exact analyses should be applied; such coefficients for a panel freely supported along four sides are given in *Table* 38. It has been shown by Marcus that for panels, the corners of which are held down, the midspan bending

moments obtained by the Grashof-and-Rankine method can be converted to approximately those obtained by more exact theory by multiplying by a simple factor, and this method is applicable not only for conditions of free support along all four edges but for all combinations of fixity on one to four sides with free support along the other edges; the bending moments at the supports are calculated by an extension of the Grashof-and-Rankine method but without the adjusting factors. The Marcus factors for a panel fixed along four edges are given in *Table* 38 and these and the Grashof-and-Rankine coefficients are substituted in the formulæ given in the table to obtain the midspan bending moments and the bending moments at the supports.

If the corners of a panel are held down, reinforcement should be provided to resist the tensile stresses due to the torsional strains. The amount and position of the reinforcement required for this purpose, as recommended in B.S. Code No. 114, are given in *Table* 38. No reinforcement is required at a corner formed by two intersecting supports if the slab is monolithic with the supports.

At a discontinuous edge of a slab monolithic with its support, resistance to negative bending moment must be provided, and the expressions given in the centre of *Table* 38 give the magnitude, in accordance with the B.S. Code No. 114, of this bending moment, which is resisted by reinforcement at right-angles to the support. The Code also recommends that no reinforcement is required in a narrow strip of slab parallel and adjacent to each support; particulars of this recommendation are also given in *Table* 38, the coefficients for use in which are taken from *Table* 39.

Examples of the use of *Tables* 38 and 39 are given on the page facing *Table* 39.

If a panel is truly freely supported along one or more edges, the approximate method described on the page facing *Table* 41 may be applied; this is based on the use of the basic coefficients given in *Table* 38.

The shearing forces on rectangular panels spanning in two directions and carrying a uniformly-distributed load are considered briefly on the page facing *Table* 39.

Rectangular Panels with Triangularly-distributed Loads.—In the design of rectangular tanks, storage bunkers and some retaining structures, cases occur of walls spanning in two directions and subjected to triangularly-distributed pressures. The intensity of pressure is uniform at any given level, but vertically the pressure varies from zero near the top to a maximum at the bottom. If there is a support along the top edge of the panels, the probable maximum bending moments on vertical and horizontal strips of unit width are given by the coefficient and expressions in the lower half of *Table* 40. For fixity along the top, the curves are derived from basic data given by Timoshenko for the exact theory of plates with Poisson's ratio equal to 0·3. For free support along the top, the curves are interpolated from the basic data. The approximate positions of the maximum bending moments are also indicated. If there is no support along the top of the panel, the bending moment can be determined from the curves in the upper half of *Table* 40; these curves are based on an analysis by Buchi.

If Poisson's ratio is assumed to be less than 0·3 the bending moments would be slightly less, but the effect of corner splays, which is to increase slightly the negative bending moments, is ignored. In deriving the curves for positive bending moments it has been assumed that only two-thirds full fixity exists at edges over which the slab is continuous. Further comments on the curves and data and an example are given on the page facing *Table* 40.

Beams supporting Rectangular Panels.

When designing the beams supporting a panel freely supported along all four edges or with the same degree of fixity along all four edges, it is generally accepted that each of the beams along the shorter edges of the panel carries the load on an area having the shape of a 45-deg. isosceles triangle with a base equal to the length of the shorter side, that is each beam carries a triangularly-distributed load; half the remaining load, that is the load on a trapezium, is carried on each of the beams along the longer edges. In the case of a square panel, each beam carries one-quarter of the total load on the panel, the load on each beam being distributed triangularly. The diagram and expressions in the top left-hand corner of *Table* 41 give the amount of load carried by each beam. Bending-moment coefficients for beams subjected to triangular and trapezoidal loading are given in *Tables* 16, 17, and 17A. The formulæ for equivalent uniformly-distributed loads given on the page facing *Table* 41 apply only to the case of the span of the beam being equal to the width or length of the panel.

An alternative method is to divide the load between the beams along the shorter and longer sides in proportion to K_B and K_L (*Table* 38) respectively. Thus the load transferred to each beam along the shorter edges is $\frac{1}{2}K_L w L_B L_L$, triangularly distributed, and to each beam along the longer edges is $\frac{1}{2}K_B w L_B L_L$, trapezoidally distributed. For square panels the loads on the beams by both methods are identical.

When the panel is fixed or continuous along one, two, or three supports and freely supported on the remaining edges, the subdivision of the load to the various supporting beams can be determined from the diagrams and expressions on the left-hand side of *Table* 41. Alternatively the loads can be calculated approximately as follows. For the appropriate value of the ratio K_e of the equivalent spans (see page facing *Table* 41), determine the corresponding values of K_B and K_L from *Table* 38. Then the load transferred to each beam parallel to the longer equivalent span is $\frac{1}{2}K_B w L_B L_L$ and to each beam parallel to the shorter equivalent span is $\frac{1}{2}K_L w L_B L_L$. Triangular distribution can be assumed in both cases, although this is a little conservative for the load on the beams parallel to the longer actual span. For a span freely supported at one end and fixed at the other, the foregoing loads should be reduced by about 20 per cent. for the beam along the freely-supported edge and the amount of the reduction added to the load on the beam along the fixed or continuous edge.

If the panel is unsupported along one edge or two adjacent edges, the loads on the beams supporting the remaining edges are as given on the right-hand side of *Table* 41.

Rectangular Panels with Concentrated Load.

The curves in *Table* 42, based on M. Pigeaud's theory, can be used directly for calculating the bending moment on a panel freely supported along all four edges with restrained corners and carrying a load uniformly distributed over a defined area symmetrically disposed upon the panel. Wheel loads and similarly highly-concentrated loads are dispersed through the road finish (if any) down to the surface of the slab, or farther down to the reinforcement, as shown in *Table* 6, to give the dimensions u and v and thence the ratios $\frac{u}{L_B}$ and $\frac{v}{L_L}$, for which the bending moments m_1 and m_2 for unit load are read off the curves for the appropriate value of the ratio

of spans k. For a total load of W on the area u by v, the bending moments in ft.-lb. per foot width of slab are given by the expressions in *Table* 42, in which the value of Poisson's ratio is assumed to be 0·15. The positive bending moments calculated from *Table* 42 for the case of a uniformly-distributed load over the whole panel (that is $u = v =$ unity) do not coincide with the bending moments based on the corresponding coefficients K'_B and K'_L given in *Table* 38 unless Poisson's ratio is assumed to be zero, as is commonly recommended.

The curves in *Table* 42 are drawn for $k = 1{\cdot}0$, $1{\cdot}25$, $\sqrt{2}$ (1·4 approximately), 1·67, 2·0, 2·5, and infinity. For intermediate values of k, the values of m_1 and m_2 can be interpolated from the values above and below the given value of k. The curves for $k = 1{\cdot}0$ apply to a square panel.

The curves for $k = \infty$ apply to a panel of great length (L_L) compared with the short span (L_B) and can be used for determining the transverse (main reinforcement) and longitudinal (distribution reinforcement) bending moments on a long narrow panel supported on the two long edges only. Alternatively the data at the bottom of *Table* 43 can be applied to this case which is really a special extreme case of a rectangular panel spanning in two directions and subjected to a concentrated load.

When there are two concentrated loads symmetrically disposed or an eccentric load, the resulting bending moments can be calculated from the rules given for the various cases in *Table* 43. Other conditions of loading, for example, multiple loads the dispersion areas of which overlap, can generally be treated by combinations of the particular cases considered. Case I is an ordinary symmetrically-disposed load. Case VI is the general case for a load in any position, from which the remaining cases are derived by simplification.

The bending moments derived directly from *Table* 42 are those at midspan of panels freely supported along all four edges but with restraint at the corners. If the panel is fixed or continuous along all four edges, M. Pigeaud recommends that the midspan bending moments are reduced by 20 per cent. The estimation of the bending moment at the support and midspan sections of panels with various sequences of continuity and free support along the edges can be dealt with by applying the following rules, which possibly give conservative results when incorporating Poisson's ratio equal to 0·15; they are applicable to the common conditions of continuity with adjacent panels over one or more supports, and monolithic construction with the supports along the remaining edges. Find m_1 and m_2 from the curves in *Table* 42 for the appropriate value of $k_e = f_1 \dfrac{L_L}{L_B}$, where f_1 is obtained from *Table* 43. For similar conditions of support on all four sides, that is Cases (a) and (j), or for a symmetrical sequence as in Case (f), $f_1 = 1{\cdot}0$; therefore the actual value of $\dfrac{L_L}{L_B}$ is used in these cases. If in Cases (b), (d), and (h) the value of $f_1 \dfrac{L_L}{L_B}$ is less than unity, L_L and L_B (and consequently u and v) should be transposed throughout the calculation of m_1 and m_2. Having found the bending moments in each direction with the adjusted values of $\dfrac{L_L}{L_B}$, the bending-moment reduction factors for continuity given in *Table* 43 are applied to give the bending moments for the purpose of design.

Examples of the use of *Tables* 42 and 43 are given on the pages facing *Tables* 42, 43 and 44.

The maximum shearing forces Q per unit length on a panel carrying a concentrated load are given by M. Pigeaud as follows:

$$u > v.\text{—At the centre of length } u, \; Q = \frac{W}{2u + v}.$$

$$\text{,, ,, ,, ,, ,, } v, \; Q = \frac{W}{3u}.$$

$$v > u.\text{—At the centre of length } u, \; Q = \frac{W}{3v}.$$

$$\text{,, ,, ,, ,, ,, } v, \; Q = \frac{W}{2v + u}.$$

To determine the load on the supporting beams, the rules given for a uniformly-distributed load over the entire panel are sufficiently accurate for a load concentrated at the centre of the panel, but this is not always the critical case for live loads, such as a load imposed by a wheel on a bridge deck, since the maximum load on a beam occurs when the wheel is passing over the beam, in which case the beam carries the whole load.

Non-rectangular Panels.

When a panel which is not rectangular is supported along all its edges and is of such proportions that main reinforcement in two directions seems desirable, the bending moments can be determined approximately by the rules given in *Table* 44, which apply to a trapezoidal panel approximately symmetrical about one axis, to a panel which in plan is an isosceles triangle (or very nearly so), and to panels which are regular polygons or are circular. The case of a triangular panel continuous or partially restrained along three edges occurs in pyramidal hopper bottoms (*Table* 99); the reinforcement calculated by the expressions for this case should extend over the entire area of the panel, and provision must be made for the negative moments and for the direct tensions which act simultaneously with the bending moments.

If the shape of a panel approximates to a square, the bending moments for a square slab of the same area should be determined. A slab having the shape of a regular polygon with five or more sides can be treated as a circular slab the diameter of which is the mean of the diameters of the inscribed and circumscribed circles; the mean diameters for regular hexagons and octagons are given in *Table* 44.

For a panel which is circular in plan and is freely supported along the circumference and carries a load concentrated symmetrically about the centre on a circular area, the total bending moment which should be provided for across each of two diameters mutually at right-angles is given by the expression in *Table* 44. If the panel is restrained around the circumference, the positive bending moment can be reduced to about two-thirds of this value if the load approximates to a "point" load, or to 80 per cent. if the load covers practically the entire panel. The negative bending moment provided for around the edge should be not less than one-third of the "free" positive bending moment. The foregoing are the bases of the corresponding expressions in *Table* 44. If the degree of restraint is equivalent to complete fixity, the average and maximum positive bending moment can be reduced to two-thirds or less of the "free" positive bending moment if the load covers a considerable area of the entire panel, but the negative bending moment provided for should be not less than two-thirds of the "free" positive bending moment.

If the load covers the entire panel, the bending moments provided for should be not less than those given by the expressions given at the bottom of *Table* 44. The

data for the various conditions are based on the following analysis. The total positive bending moment across a diameter of a freely-supported panel of diameter D ft., carrying a uniformly-distributed load of intensity w lb. per square foot, is $\frac{wD^3}{24}$ ft.-lb. Therefore the average positive bending moment is $\frac{wD^2}{24}$ ft.-lb. per foot of diameter. The positive bending moment, however, varies from zero at the extremities of the diameter to a maximum at the centre of the panel, where it may be, say, one-and-a-half times the average bending moment. If the panel is partially restrained around the edges, the average and maximum positive bending moments may be reduced to about 80 per cent. of the corresponding " free " positive bending moment, but provision should be made for a negative bending moment around the edge of not less than one-third of the average " free " bending moment. If the panel is completely fixed around the edge, the average and maximum positive bending moments should be not less than one-third of the corresponding " free " positive bending moments, and the negative bending moment provided for around the circumference should be not less than two-thirds of the average " free " positive bending moment.

A circular panel can therefore be designed by one of the following methods. (i) Design for the maximum positive bending moment at the centre of the panel and reduce the amount of reinforcement or the thickness of the slab towards the circumference; if the panel is not truly freely supported, provide for the negative bending moment acting around the circumference. (ii) Design for the average positive bending moment across a diameter and retain the same thickness of slab and amount of reinforcement throughout the entire panel; if the panel is not truly freely supported around the circumference, provide for the appropriate negative bending moment.

The reinforcement required for the positive bending moments in both the preceding methods must be provided in two directions mutually at right-angles; the reinforcement for the negative bending moment should be provided by radial bars normal to, and equally spaced around, the circumference, or reinforcement equivalent thereto should be provided.

Flat Slabs.

The design of flat slabs, that is beamless slabs or mushroom floors, is based primarily on empirical considerations. The principles which follow and the data summarised in *Table* 45 and on the page facing the table are in accordance with the empirical method described in B.S. Code No. 114. This type of floor can incorporate drop-panels at the columns or the slab can be of uniform thickness throughout. The tops of the columns may be plain or may be provided with a splayed head having the dimensions indicated in *Table* 45.

The lengths (or widths) of adjacent panels should not differ by more than 10 per cent. of the greater length (or width). The ratio of the longer to the shorter dimension of a panel which is not square should not exceed $1\frac{1}{3}$. The length of the drop in any direction should be not less than one-third of the length of the panel in the same direction.

For the purpose of determining the bending moments, the panel is divided into " middle strips " and " column strips " as shown in the diagram on the page facing *Table* 45, the width of each strip being half the length or width of the panel. If drop-panels of less width than half the panel length or width are provided, the width

of the column strip can be reduced to the width of the drop-panel and the middle strip increased accordingly.

The thicknesses of the slab and the drop-panels must be sufficient to provide resistance to the shearing forces and bending moments, but should be not less than 5 in. nor less than the thickness conforming to the span-thickness ratios on the page facing *Table* 45.

Bending Moments.—For the calculation of the bending moments, the effective spans are $L_1 - \frac{2}{3}D$ and $L_2 - \frac{2}{3}D$, where L_1 and L_2 are the shorter and longer dimensions respectively of the panel and D is the diameter of the column or column-head if one is provided. The total bending moments to be provided for at the principal sections of the panel are given in *Table* 45 and are functions of the effective spans.

Walls and other concentrated loads must be carried on beams, and beams should be provided around openings other than small holes; the Code recommends limiting sizes of openings permissible in the column strips and middle strips.

Reinforcement.—It is generally most convenient for the reinforcement to be arranged in bands in two directions, one parallel to each of the spans L_1 and L_2. Alternatively the bars should be arranged in two bands parallel to the spans and two diagonal bands, but this method produces congestion of reinforcement in relatively thin slabs.

With two-way reinforcement, 40 per cent. of the bars in the positive-moment reinforcement should remain in the bottom of the slab and extend at least over a length at the middle of the span equal to three-quarters of the span. No reduction of the positive-moment reinforcement should be made within a length of $0{\cdot}5L$ at the middle of the span. No reduction of the negative-moment reinforcement should be made within a distance of $0{\cdot}2L$ from the centre of the support. The negative-moment reinforcement should extend into the adjacent panel for an average distance of $0{\cdot}25L$; if the ends of the bars are staggered the shortest should extend for a distance of $0{\cdot}2L$. The amount of reinforcement in any section should not exceed 1 per cent. of the product of the width of the section and the effective depth.

Shearing Force.—The shearing stresses, which should not exceed the values given in *Table* 57 for conditions when no shearing reinforcement is provided, should be investigated at two sections, namely, around the edge of the drop and around the head of the column. For the latter the critical plane for shearing resistance is at a distance equal to half the total thickness of the slab, or drop, from the edge of the column-head as shown in the diagrams in *Table* 45.

Alternative Analysis.—A less empirical method of analysing flat slabs is described in B.S. Code No. 114, and is applicable to cases not covered by the foregoing rules. The bending moments and shearing forces are calculated by assuming the structure to comprise continuous frames, transversely and longitudinally.

Framed Structures.

A structure is statically determinate if the forces and bending moments can be determined by the direct application of the principles of statics. A cantilever, whether a simple bracket or the roof of a grandstand, a freely-supported beam, a truss with pin-joints, a three-hinged arch or frame, are some examples. A statically-indeterminate structure is one in which there is a redundancy of members or supports or both, and which can only be analysed by considering the elastic deformation under load. Restrained beams, continuous beams, portal frames, and other non-triangulated

structures with rigid joints, and two-hinged and fixed-end arches, are examples of statically-indeterminate structures, which can be analysed by the application of one or other of several methods, which include slope-deflection, column analogy, moment distribution, the theorem of least work, and, of more recent development, the displacement method and the ultimate-load method; reference should be made to books dealing fully with each of these methods. Most designers find that one method seems more simple to them than others, and it is therefore generally better for each designer to use the method he prefers; the same results should be obtained irrespective of the method used.

In this section the analysis of primary frames by the methods of slope-deflection and moment-distribution is described. Most analyses of complex rigid frames require an amount of calculation often out of proportion to the real accuracy of the results, and some approximate solutions are therefore given for common cases of building frames and similar structures.

Slope-deflection.—The principles of the slope-deflection method of analysing a restrained member are given in *Table* 46, and on the page facing the table, in which also are given the basic formulæ and the formulæ for the bending moments in special cases.

When there is no deflection of one end of the member relative to the other (for example, when supports which are not elastic are assumed), and when the ends of the member are either hinged or fixed, and when the load is symmetrically disposed, the general expressions are simplified and the resulting formulæ for the more common cases of restrained members are given in *Table* 46.

The bending moments on a framed structure are determined by applying the formulæ to each member successively. The algebraic sum of the bending moments at any joint equals zero. When it is assumed that there is no deflection (or settlement) d of one support relative to the other, there are as many formulæ for the restraint moments as there are unknowns, and therefore the restraint moments and the slopes at the ends of the members can be evaluated. For symmetrical frames on unyielding foundations and carrying symmetrical vertical loads it is common to neglect the change in the position of the joints due to the small elastic contractions of the members, and the assumption of $d = 0$ is reasonably accurate. If the foundations or other supports settle unequally under the load, this assumption is not justified and a value must be assigned to the term d for the members affected.

If a symmetrical or unsymmetrical frame is subjected to a horizontal force the sway produced involves lateral movement of the joints It is common in this case to assume that there is no elastic shortening of the member. Sufficient formulæ to enable the additional unknowns to be evaluated are obtained by equating the reaction normal to the member, that is the shearing force on the member, to the rate of change of bending moment. Sway cannot be neglected when considering unsymmetrical frames subject to vertical loads, or any frame on which the load is unsymmetrically disposed.

An example of the application of the slope-deflection formulæ to a simple problem is given on the page opposite *Table* 46.

Shearing Forces on Members of a Frame.—The shearing forces on any member forming part of a frame can be determined when the bending moments have been found by considering the rate of change of the bending moment. The uniform shearing force on a member AB due to end restraint only is $\frac{M_{AB} + M_{BA}}{L_{AB}}$, account

being taken of the signs of the bending moments. Thus if both restraint moments are clockwise, the shearing force is the numerical sum of the moments divided by the length of the member. If one restraint moment acts in a direction contrary to the other, the numerical difference is divided by the length to give the shearing force. For a member with end B hinged, the shearing force due to the restraint moment at A is $\frac{M_{AB}}{L_{AB}}$. The variable shearing forces due to the loads on the member should be algebraically added to the uniform shearing force due to the restraint moments, in a manner similar to that shown for continuous beams in *Table* 20.

Portal Frames —A common type of simple frame used in buildings is the portal frame with either a horizontal top member or two inclined top members meeting at the ridge. In *Table* 49, general formulæ for the bending moments at both ends of the columns, and at the ridge in the case of frames of that type, are given together with expressions for the forces at the bases of the columns. The formulæ relate to any vertical or horizontal load and to frames fixed or hinged at the base of the columns. In *Table* 50 are given the corresponding formulæ for special conditions of loading on frames of one bay.

Frames of the foregoing types are statically indeterminate, but a frame with a hinge at the base of each column and one at the ridge, that is a three-hinged frame, can be readily analysed. Formulæ for the forces and bending moments are given in *Table* 48 for three-hinged frames. Approximate expressions are also given for certain modified forms of these frames, such as when the ends of the columns are embedded in the foundations and when a tie-rod is provided at eaves level.

Method of Moment-distribution.—The analysis of frames by moment-distribution methods is described, with an example, in *Table* 47. Similar principles are applied to the solution of continuous beams in *Tables* 26 *et seq.*, but the different convention of signs adopted in the two cases should be noted.

Bending of Columns.

External Columns.—Provision should be made for the bending moments produced on the columns due to the rigidity of the joints in monolithic beam-and-column construction of buildings.

The external columns of a building are subjected to a greater bending moment than the internal columns (other conditions being equal), the magnitude of the bending moment depending on the relative stiffness of the column and beam and on the end conditions of the members. The two principal cases for exterior columns are when the beam is supported on the top of the column, as in a top story, and when the beam is fixed to the column at an intermediate point, as in intermediate stories. The second case is shown in the diagrams in *Table* 46. Since either end of the column or the end of the beam remote from the column can be hinged, fixed, or partially restrained, there are many possible combinations.

For the first case the maximum reverse moment at the junction of the beam and column occurs when the far end of the beam is hinged and the foot of the column is fixed. The minimum reverse moment at the junction occurs when the beam is rigidly fixed at the far end and the column is hinged at the foot. Conditions in practice generally lie between these extremes, and with any condition of fixity of the foot of the column the bending moment at the junction decreases as the degree of fixity at the far end of the beam increases. With any degree of fixity at the far end of the

beam the bending moment at the junction increases very slightly as the degree of fixity at the foot of the column increases.

The maximum reverse moment on the beam at the junction with the column in the second case occurs when the beam is hinged at the far end and the column is perfectly fixed at the top and the bottom as indicated in *Table* 46. With perfect fixity at the far end of the beam and hinges at the top and bottom of the column, as also shown in *Table* 46, the reverse moment on the beam at the junction is a minimum. Intermediate cases of fixity follow the following rules: increase in fixity at the end of the beam decreases the bending moment at the junction; decrease in fixity at either the top or the bottom of the column decreases the bending moment at the junction, and vice versa.

Formulæ for the maximum and minimum bending moments are given in *Table* 46 for a number of frames of a single bay. The bending moment on the beam at the junction is divided between the upper and lower columns in the ratio of their stiffness factors K when conditions at the ends of the two columns are identical. When the end of one column is hinged and the other fixed, the ratio of the bending moments allocated to each column is in accordance with the expression

$$\frac{\text{Bending moment on hinged portion}}{\text{Bending moment on fixed portion}} = \frac{0{\cdot}75K \text{ for hinged portion}}{K \text{ for fixed portion}}.$$

External Columns in Building Frames.—For the framework of ordinary buildings the bending moments on the external columns can be computed from the formulæ given in the lower part of *Table* 46. These expressions, which conform to those recommended in the B.S. Code, are only approximate as lack of uniformity in the end conditions of the beams or columns are neglected, but the formulæ are suitable for use in the design of ordinary buildings.

When there is no upper column (that is when the structure is only one story high or when the top story is being considered), the stiffness factor K_D is equal to zero.

Corner Columns.—A column at an external corner of a building is generally subjected to bending moments from beams in two directions at right-angles. These bending moments can be calculated by considering two frames (also at right-angles) independently, but practical methods of computing the stresses depend upon the relative magnitude of the bending moments and the direct load (see page 86).

Internal Columns.—There is less variation in the bending moments due to continuity between the beams and the internal columns of a building than is the case with external columns. End-fixing conditions of the various members do not affect the bending moments so seriously, and the bending moment on a beam at the junction with a column is less than the bending moment at the support when computed by, say, the Theorem of Three Moments neglecting fixity with the column. In *Table* 46 expressions are given for the bending moments in the upper and lower internal columns. In this case the value of $\frac{A}{L}$, when the spans are equal, should be for the live load only on one of the adjacent spans of the beam. When the spans are unequal, the greatest bending moments on the column occur when the value of M_{es} (see *Table* 46) is greatest, which is generally when the longer beam is loaded with live and dead load while the shorter beam is loaded with dead load only.

Approximate Methods.—The methods hitherto described for evaluating the bending moments in column-and-beam construction with rigid joints involve a fair amount of calculation, including that of the moments of inertia of the members. In

practice, and especially in the preparation of preliminary schemes, time is not always available to make these calculations, and therefore approximate methods are of value. Designs should be checked by more accurate methods.

For large columns and light beams, the effect on the column of the load on the beam is not great, and in such cases the difference between the permissible compressive stress for direct compression and for bending combined with direct compression (see *Table* 57) is generally sufficient to enable the preliminary design of the column to be based on the direct load only. Where the effect of the beam on the column is likely to be considerable, and in order to allow a margin for the bending stresses in the column, the column can be designed provisionally for the direct load increased, to allow for the effects of bending, by the amounts shown on the page facing *Table* 51 for the particular arrangement of beams supported by the column.

Bending Moments due to Wind.

In exposed structures such as water towers, bunkers and silos, and the frames of tall narrow buildings, the columns must be designed to resist the effects of wind. When conditions do not warrant a close analysis of the bending moments to which a frame is subjected due to wind or other horizontal forces, the methods described in the following and illustrated in *Table* 51 are sufficiently accurate.

Braced Columns.—For braced columns (of the same cross-section) forming an open tower such as that supporting an elevated water tower, the expressions at (*a*) in *Table* 51 give the bending moments and shearing forces on the columns and braces due to the effect of a horizontal force at the head of the columns. The increase or decrease of direct load on the columns is also given.

In general, the bending moment on the column is the shearing force on the column multiplied by half the distance between the braces. If a column is not continuous or is insufficiently braced at one end, as at unconnected foundations, the bending moment has twice this value.

The bending moment on the brace at an external column is the sum of the bending moments on the columns at the intersection with the brace. The shearing force on the brace is equal to the change of bending moment from one end of the brace to the other divided by the length of the brace. These shearing forces and bending moments are additional to those created by the dead weight of the brace and any external loads to which it may be subjected.

The overturning moment on the frame causes an additional direct load on the leeward column and a corresponding relief of load on the windward column, the maximum value of this direct load being approached at the foot of the column and being equal to the overturning moment divided by the distance between the centres of the columns.

The expressions in *Table* 51 for the effects on the columns and for the bending moments on the braces apply whether the columns are vertical or at a slight inclination. If the columns are inclined, the shearing force on a brace is $2M_B$ divided by the length of the brace being considered.

Columns supporting Massive Superstructures.—The case illustrated at (*b*) in *Table* 51 is common in bunkers and silos where a superstructure of considerable rigidity is carried on comparatively short columns. If the columns are fixed at the base, the bending moment on a single column is $\frac{Ph}{2N}$, where N is the number of columns if they are all of the same size; the signification of the other symbols is given in *Table* 51.

If the columns are of different sizes, since each column is deflected the same amount, the total shearing force should be divided among the columns in any one line in proportion to their separate moments of inertia. If N_1 is the number of columns with moment of inertia I_1, N_2 the number of columns with moment of inertia I_2, etc., the total moment of inertia is $N_1I_1 + N_2I_2 +$ etc. $= \Sigma I$. On any column having a moment of inertia I_n, the bending moment is $\frac{PhI_n}{2\Sigma I}$ as given in diagram (*b*) in *Table* 51. Alternatively, the total horizontal shearing force can be divided among the columns in the ratio of their cross-sectional areas (thus giving uniform shearing stress) and with this method the formula for the bending moment on any column with cross-sectional area A_n is $\frac{PhA_n}{2\Sigma A}$, where ΣA is the sum of the cross-sectional areas of all the columns resisting the total shearing force P.

Building Frames.—In the frame of a multiple-story building, the effect of the wind may be small compared with that of other loads, and in this case it is sufficiently accurate to divide the horizontal shearing force on the basis that an external column resists half the shearing force on an internal column. If N_T is the total number of columns in one frame, in the plane of the lateral force P, the effective number of columns is $N_T - 1$ for the purpose of calculating the bending moment on an interior column, the two external columns being equivalent to one internal column; see diagram (*c*) in *Table* 51. In a building frame subjected to wind pressure, the pressure on each panel (or story height) P_1, P_2, P_3, etc., is generally divided into equal shearing forces at the head and base of each story-height of columns. The shearing force at the base of any interior column, n stories from the top, is $\frac{\left(\Sigma P + \frac{P_n}{2}\right)}{(N_T - 1)}$, where

$$\Sigma P = P_1 + P_2 + P_3 + \ldots + P_{n-1}.$$

The bending moment is the shearing force multiplied by half the story-height.

A bending moment and a corresponding shearing force are caused on the floor beams in the same way as on the braces of an open tower. At an internal column the sum of the bending moments on the two beams meeting at the column is equal to the sum of the bending moments at the base of the upper column and at the head of the lower column.

Earthquake-resistant Structures.

Opinions may differ on whether structures to withstand the disruptive forces of earth tremors and quakes should be designed as rigid or flexible or semi-flexible, but, generally, a rigid construction seems to be favoured. The effect of an earth tremor is equivalent to a horizontal thrust additional to the loads and wind pressures for which the building is commonly designed. There are codes for earthquake-resistant construction in several countries, and recent codes seem more complex than earlier requirements. The most simple consideration is as follows.

The dead and live loads should be increased by 20 per cent. to allow for vertical movement. The magnitude of the horizontal thrust depends on the acceleration of the earth tremor, which may vary from less than 3 ft. per second per second in firm compact ground to over 12 ft. per second per second in alluvial soil and filling. A horizontal thrust equal to about one-tenth of the mass of the building seems to be sufficient for all but major shocks when the building does not exceed 20 ft. in height, and equal to one-eighth of the mass when the building is of greater height. The

horizontal shearing force on the building at any level is one-eighth (or one-tenth) of the total weight of the structure (including live loads) above this level. The analysis of the bending moments and shearing forces on the columns and floor beams is similar to that described for wind pressure on building frames in *Table* 51.

In order that the structure shall act as a unit, all parts should be effectively bonded together. Panel walls, finishes, and ornaments should be permanently attached to the frame, so that in the event of a shock they will not collapse independently of the main structure. Separate column footings should be connected by ties designed to take a thrust or a pull of one-tenth of the load on the heavier of the two footings connected.

Properties of Members of a Frame.

End Conditions.—Since the results given by the more precise methods of frame analysis vary considerably with different degrees of restraint at the ends of the members, it is essential that the end conditions assumed should be reasonably obtained in the actual construction. Absolute fixity is difficult to attain unless the beam or column is embedded monolithically in a comparatively large mass of concrete. Embedment in a brick or masonry wall represents more nearly the condition of a hinge, and should be considered as such. The ordinary type of separate foundation, designed only for the limiting uniform ground pressure under the direct load on a column, should also be considered as a hinge at the foot of the column. A continuous beam supported on a beam or column is only partly restrained, and where the outer end of an end span is supported on a beam a hinge should be assumed. A column built on a pile-cap supported by two, three, or four piles is not absolutely fixed but a bending moment can be developed if the resulting vertical reaction (upwards and downwards) and the horizontal thrust can be taken on the piles. A column can be considered as fixed if it is built on a thick raft and the necessary restraint can be developed in the raft.

In two-hinged and three-hinged arches, hinged frames, and some types of girder bridges, where the assumption of a hinged joint must be fully realised, it is necessary to form a definite hinge in the construction. This can be done by inserting a steel hinge (or similar), or by forming a hinge within the frame. Some types of hinged joints are illustrated in *Table* 94.

Moments of Inertia of Reinforced Concrete Members.—The moment of inertia of a reinforced concrete member is theoretically the equivalent moment of inertia of the stressed section about the neutral plane expressed in equivalent units of concrete or of steel. The concrete in tension is neglected (unless the tensile stress is limited as in the design of liquid containers); the reinforcement is considered as being equivalent in concrete units to its area multiplied by the modular ratio (see page 238). Until the cross-section of the member has been determined, or assumed, the calculation of the moment of inertia cannot be made with any precision. Moreover, the moment of inertia of an ordinary beam calculated on this basis changes considerably throughout its length, especially with a continuous or restrained beam in beam-and-slab construction which acts as a tee-beam at midspan but is designed as a rectangular beam towards the ends where reverse bending moments occur. Consideration should be given to whether the probable tensile stresses are sufficiently great to cause cracking, particularly with tee-beams and ell-beams if the flanges are in tension; although the beam may be designed on the assumption that the concrete

has cracked and that the reinforcement resists all the tension due to bending, cracking may not take place owing to the comparatively large area of concrete in the flange.

Since early comparisons of moments of inertia are required in the design of frames, the errors due to approximations are of little importance. It is, however, important that the method of assessing the moment of inertia should be the same for all members in a single calculation. It is generally sufficient to compare the moments of inertia of the whole concrete areas alone for members that have somewhat similar percentages of reinforcement. Thus the ratio of the moments of inertia of a rectangular column and a rectangular beam is $\frac{b_c d_c^3}{bd^3}$; b_c and d_c are the breadth and width of the column, and b and d are the breadth and depth of the beam.

In *Table* 64 are given values of the moments of inertia for square, rectangular, octagonal, and some other non-rectangular sections, together with factors and formulæ for determining the moment of inertia of tee-beams, for which the breadth of the flange assumed for the purpose of calculating the moment of inertia should not exceed the maximum permissible width given in *Table* 69. The particulars in *Table* 64 exclude the effect of the reinforcement, but the data in *Tables* 65 and 65A for some regular cross-sections take the reinforcement into account.

Alternative methods of assessing the ratio of the moments of inertia of two members are given in the examples on the page opposite *Table* 51, which show that approximate methods readily give comparative values that are accurate enough not only for trial calculations but for final designs. The two methods described are recommended in the B.S. Code.

Arches.

Arch construction in reinforced concrete occurs mainly in bridges and sometimes in roofs. The principal types of symmetrical concrete arch bridges are shown in *Table* 93. An arch may be either a three-hinge arch, a two-hinge arch or a fixed-end arch (see the diagrams in *Table* 52), and may be symmetrical or unsymmetrical, right or skew, or a single arch or one of a series of arches mutually dependent upon each other. The following consideration is restricted to symmetrical and unsymmetrical three-hinge arches and symmetrical two-hinge and fixed-end arches; reference should be made to other publications for information on more complex types. Arch construction may comprise an arch slab (or vault) or a series of parallel arch ribs. The deck of an arch bridge may be supported by columns or transverse walls carried on an arch slab or ribs, in which case the structure may have open spandrels; or the deck may be below the crown of the arch either at the level of the springings (as in a bow-string girder) or at some intermediate level. A bow-string girder is generally considered to be a two-hinge arch with the horizontal component of the thrusts resisted by a tie which generally forms part of the deck. If earth filling is provided to support the deck, an arch slab and spandrel walls are required and the bridge is a closed or solid spandrel structure.

Three-hinge Arch.—An arch with a hinge at each springing and with a hinge at the crown is statically determinate. The thrusts on the abutments, and therefore the bending moments and shearing forces, are not affected by a small movement of one abutment relative to the other, and this type of arch is therefore used when there is a possibility of unequal settlement of the abutments.

For any load in any position the thrust on the abutments can be determined

from the statical equations of equilibrium. For the general case of an unsymmetrical arch with a load acting vertically, horizontally or at an angle, the expressions for the horizontal and vertical components of the thrusts are given in the lower part of *Table* 52. For symmetrical arches the formulæ for the thrusts given for three-hinge frames in *Table* 48 are applicable or similar formulæ can be obtained from the general expressions in *Table* 52. The vertical component is the same as the vertical reaction for a freely-supported beam. The bending moment at any section of the arch is the algebraic summation of the moments of the loads and reactions to the thrusts on one side of the section. There is no bending moment at a hinge. The shearing force is likewise the algebraic sum of the reactions and loads, resolved at right-angles to the arch axis at the section considered, and acting on one side of the section. The thrust at any section is the sum of the reactions and loads, resolved parallel to the axis of the arch at the section, and acting on one side of the section.

The extent of the arch that should be loaded with live load to produce the maximum bending moment or shearing force or thrust at a given section is determined by drawing a series of influence lines. A typical influence line for a three-hinge arch and the formulæ necessary to construct an influence line for unit load in any position are given in the upper part of *Table* 52.

Two-hinge Arch.—The hinges of a two-hinge arch are placed at the abutments and thus, as in a three-hinge arch, only thrusts are transmitted to the abutments, there being no bending moment on the arch at the springings. The vertical component of the thrust from a symmetrical two-hinge arch is the same as for a freely-supported beam. Formulæ for the thrusts and bending moments are given in *Table* 52 and notes are given on the page facing the table.

Fixed Arch.—An arch with fixed ends exerts a bending moment on the abutments in addition to the vertical and horizontal thrusts. Like a two-hinge arch, and unlike a three-hinge arch, a fixed-end arch is statically indeterminate, and changes of temperature and shrinking of the concrete affect the stresses. As it is assumed in the general theory that the abutments are incapable of rotation or of translational movement, a fixed-end arch can only be used in such conditions.

Any section of a fixed arch rib or slab is subjected to a bending moment and a thrust, the magnitudes of which have to be determined. The design of an arch is a matter of trial and error since the dimensions and the shape of the arch affect the calculations, but it is possible to select preliminary sizes that reduce the repetition of arithmetical work to a minimum. The suggested method of determining the possible sections at the crown and springing as given in *Table* 53, and explained on the page facing *Table* 52, is based on treating the fixed arch as a hinged arch, and estimating the sizes of the cross-sections by reducing greatly the maximum stresses.

The general formulæ for thrusts and moments on a symmetrical fixed arch of any profile are given in *Table* 53 and notes on the application and modification of these formulæ are given on pages 230 and 232. The calculations involved in solving the general and modified formulæ are tedious, but some labour is saved by preparing the calculations in tabulated form. One such form is that recommended by the Ministry of Transport; this form is particularly suitable for open-spandrel arch bridges because the appropriate formulæ, which are those in *Table* 53, do not assume a constant value of s_1, the ratio of the length of a segment of the arch to the mean moment of inertia of the segment. A similar form is given in *Table* 55.

For an arch of large span the calculations are further rendered considerably

easier and more accurate by the preparation and use of influence lines for the bending moment and thrust at the crown, the springing, and the first quarter-point. Typical influence lines are given in *Table* 53, and such diagrams can be constructed by considering the passage over the arch of a single concentrated unit load and applying the formulæ for this condition. The effect of the dead load, and of the most adverse disposition of the live load, can be readily calculated therefrom. If the specified live load includes a moving concentrated load, such as the knife-edge load in the Ministry of Transport loading, influence lines are almost essential for the determination of the most adverse position, except in the case of the positive bending moment at the crown for which the most adverse position of the load is at the crown. The method of determining the data to establish the ordinates of the influence lines are given in the form in *Table* 55.

Fixed Parabolic Arches.—Consideration is given in *Table* 54 and on the facing page to symmetrical fixed arches that can have either open or solid spandrels and can be either arch ribs or arch slabs. The method is based on that of Dr. Strassner as developed by Mr. H. Carpenter, and the principal assumption is that the axis of the arch is made to coincide with the line of pressure due to the dead load. This results in an economical structure and a simple method of calculation. The shape of the axis of the arch is approximately a parabola and this method can therefore only be adopted when the designer is free to select the profile of the arch. The approximately parabolic form of arch may not be the most economical for large spans, although it is almost so, and a profile that produces an arch axis that coincides with the line of thrust for the dead load plus one-half of the live load may be more satisfactory. If the increase in thickness of the arch from the crown to the springing is a parabolic variation, only the bending moments and thrusts at the crown and springing need be investigated. The formulæ for the bending moments and forces are given on the page facing *Table* 54 and include a series of coefficients, values of which are given in *Table* 54; an example of the application of the method is given on the page facing *Table* 55. The component forces and moments are as in the following.

The thrusts due to the dead load are relieved somewhat by the effect of the compression causing elastic shortening of the arch. For arches with small ratios of rise to span, or for arches that are thick compared with the span, the stresses due to arch shortening may be excessive. This can be overcome by introducing temporary hinges at the crown and springings, which eliminate all bending stresses due to dead load. The hinges are filled with concrete after arch shortening and much of the shrinking of the concrete have taken place.

There are additional horizontal thrusts due to a rise of temperature or a corresponding counter-thrust due to a fall of temperature. A rise or fall of 30 deg. F. is often used for structures in Britain, but careful consideration should be given to those factors that may necessitate an increase, or may justify a decrease, in the temperature range. The shrinking that takes place when concrete hardens produces counter-thrusts, and can be considered as equivalent to a fall of temperature; with the common sectional method of constructing arches the effect of shrinking may be allowed for by assuming it to be equal to a fall of temperature of 15 deg. F.

The extent of the live load on an arch to produce the maximum stresses in the critical sections can be determined from influence lines, and the following are approximately correct for parabolic arches. The maximum positive bending moment at the crown occurs when the middle third of the arch is loaded; the maximum negative

bending moment at a springing occurs when four-tenths of the span adjacent to the springing is loaded; the maximum positive bending moment at a springing occurs when the whole span is loaded except the length of four-tenths of the span adjacent to the springing. In the expressions in *Table* 54 the live load is expressed in terms of an equivalent uniformly-distributed load. In *Table* 6 are given the values of the distributed load specified by the Ministry of Transport, but these loads have to be combined with the prescribed knife-edge load as shown in the example on the page facing *Table* 54.

When the corresponding normal thrusts and bending moments on a section have been determined, the area of reinforcement and the stresses at the crown and springing are calculated in accordance with the methods described on pages 76 *et seq.* There only remains to determine the intermediate sections and the profile of the axis of the arch. If the dead load is uniform throughout (or practically so) the axis will be a parabola, but if it is not uniform the axis must be shaped to coincide with the line of pressure for dead load. The latter can be plotted by force and link polygons (as in ordinary graphic statics), the necessary data being the magnitudes of the dead load, the horizontal thrust due to dead load, and the vertical reaction (which equals the dead load on half the span) of the springing. The line of pressure, and therefore the axis of the arch, having been established, and the thicknesses of the arch at the crown and springing determined, the lines of the extrados and intrados can be plotted to give a parabolic variation of thickness between the two extremes.

SECTION 3

MATERIALS AND STRESSES

THE properties of reinforcement and of the constituents of concrete are described in regulations, standards and codes of practice. Only those properties which concern the designer directly and working stresses are dealt with in this section; the specification in Section 6 embodies other requirements.

Concrete.

Cement.—Cements suitable for reinforced concrete are ordinary and rapid-hardening Portland cements, Portland-blastfurnace cement, low-heat Portland cement, sulphate-resistant cement, super-sulphate cement, and high-alumina cement. Quick-setting cements are not used in ordinary construction. Calcium chloride is sometimes added to ordinary and rapid-hardening Portland cement to accelerate the initial set, either for concreting in cold weather or to enable the moulds or shuttering to be removed earlier; cements with any such addition do not conform to British Standards. Cements of different types should not be used together.

Ordinary Portland Cement (B.S. No. 12, 1958).—The principal requirements are as follows. The initial setting-time must be not less than 45 minutes and the final setting-time not more than 10 hours. The specific surface area must be not less than 2250 sq. cm. per gramme. The minimum compressive strengths of 1 : 3 mortar cubes are 2200 lb. per square inch at three days and 3400 lb. at seven days. An alternative test on 4-in. concrete cubes with a cement aggregate ratio of about 1 : 6 (equivalent to 1 : 2 : 4), with aggregate from ¾ in. down, a water-cement ratio of 0·6, and a slump of ½ in. to 2 in., is included. The strength of such cubes must be not less than 1200 lb. per square inch at three days and 2000 lb. at seven days. According to the recommendations of B.S. Code No. 114, the crushing strength of 6-in. or 4-in. cubes of 1 : 2 : 4 ordinary concrete in preliminary tests should be not less than 2700 lb. per square inch at seven days. It is possible that this strength might not be obtained if cubes tested in accordance with B.S. No. 12 have only the minimum strength of 2000 lb. per square inch, but in this case the concrete would be acceptable so long as the strength at twenty-eight days be not less than 4000 lb. per square inch (see *Table* 57).

Rapid-Hardening Portland Cement (B.S. No. 12, 1958).—The principal physical difference between ordinary and rapid-hardening Portland cement is the greater fineness of the latter, which must have a specific surface area of not less than 3250 sq. cm. per gramme. The setting-times are the same, but the minimum compressive strengths of mortar cubes are 3000 lb. per square inch at three days and 4000 lb. at seven days. The minimum compressive strengths of concrete cubes are 1700 lb. per square inch at three days and 2500 lb. at seven days. An optional tensile test at one day (300 lb. per square inch) is included. The quicker hardening of this cement may enable shuttering to be removed earlier.

Portland-Blastfurnace Cement (B.S. No. 146, 1958).—The slag content must not exceed 65 per cent. The setting-times and the fineness are the same as for ordinary Portland cement. The minimum compressive strengths of mortar cubes are 1600 lb.

per square inch at three days, 3000 lb. at seven days, and 5000 lb. at twenty-eight days, and of concrete cubes 800 lb., 1600 lb., and 3200 lb. at these ages.

Sulphate-Resistant Cement.—There is no British Standard for this cement, which is used for concrete liable to chemical attack by sea-water, acid ground-waters, and other medium-sulphate liquids. It is a mixture of blastfurnace slag and Portland cement clinker, and has less free lime. Super-sulphate or high-alumina cements are required when sulphate concentrations are high.

High-Alumina Cement (B.S. No. 915, 1947).—This cement has extreme rapid-hardening properties due mainly to the proportion of alumina being up to 40 per cent. compared with about 5 per cent. in Portland cement; a minimum of 32 per cent. of alumina is required. The required fineness is between that of ordinary and rapid-hardening Portland cements. Initial setting must take place between two and six hours, and final setting within two hours after the initial set. The minimum compressive strengths of mortar cubes are 6000 lb. per square inch at one day and 7000 lb. at three days. High-alumina cement is more costly than Portland cement but it is immune from attack by sea-water and many corrosive liquids; because of its high early strength it is also used when saving of time is important.

Low-Heat Portland Cement (B.S. No. 1370, 1958).—Low-heat Portland cement generates less heat during setting and hardening than do other cements, and thus reduces the risk of cracks occurring in large masses of concrete due to a reduction of tensile stresses during cooling. The development of strength is slower than that of other Portland cements, but in course of time the strengths may be equal. The minimum compressive strengths of mortar cubes are 1100 lb. per square inch at three days, 2000 lb. at seven days, and 4000 lb. at twenty-eight days. The strength of concrete cubes must be 500 lb. per square inch at three days, 1000 lb. at seven days, and 2000 lb. at twenty-eight days. A high proportion of lime is not compatible with low heat of hydration, and therefore the permissible percentage of lime is less than for other Portland cements. The heat of hydration must not exceed 60 calories per gramme at seven days and 70 calories per gramme at twenty-eight days. The initial setting-time must be not less than one hour and the final setting-time not more than ten hours. The specific surface must be not less than 3200 sq. cm. per gramme.

Aggregates.—Fine aggregate (sand) and coarse aggregate (stone) must be clean, inert, and hard, non-porous and free from excessive quantities of dust, laminated particles, and splinters. Gravels and crushed hard stone are the commonly used materials for structural concrete. Broken brick is a cheap aggregate for plain concrete, generally of low strength. Clinker, foamed-slag, expanded shale and clay, pellets of pulverised-fuel ash, fire-brick, and pumice, are used as aggregates for lightweight and insulating concrete where great strength is not essential. In general, aggregates for reinforced concrete should comply with B S. No. 882, but air-cooled blastfurnace slag (B.S. No. 1047), foamed blastfurnace slag (B.S. No. 877), crushed dense clay brick and tile, some proprietary forms of expanded shale or clay, and clean pumice may be suitable for members such as wall panels and floor and roof slabs.

The size and grading of aggregates vary with the nature and source of the material, and the requirements in this respect depend upon the type of structure. For buildings and most reinforced concrete construction the fine aggregate should be graded from $\frac{3}{16}$ in. down to dust with not more than 3 per cent. passing a B.S. sieve No. 200. The coarse aggregate should be graded from $\frac{3}{16}$ in. up to $\frac{3}{4}$ in., and between these limits

the grading should be such as to produce a workable and dense concrete. The largest size of the coarse aggregate should be ¼ in. less than the cover of concrete over the reinforcement (except in slabs) or the space between the bars, and should not exceed a quarter of the smallest dimension of the concrete member. For the ribs and top slab of hollow clay-block slabs, and for shell roofs and similar thin members, the largest size of aggregate is generally ⅜ in. In non-reinforced concrete larger aggregate, say, 1½ in. to 3 in., is permissible and in concrete in large piers of bridges, massive foundations, or in concrete for filling large cavities or for kentledge, the use of hard stone "plums" is common.

For concrete subject to attrition, such as roads and the floors of garages, factories, and workshops, if a special finish is not applied, an angular aggregate and a coarse sand is preferable. For liquid containers the aggregates should give as dense a concrete as possible.

Concrete Mixtures.—The proportions in which the cement, fine aggregate, and coarse aggregate are mixed are usually expressed for convenience as volumetric ratios based on a unit volume of cement, for example, 1 : 2 : 4, meaning one part by volume of cement, two parts by volume of fine aggregate, and four parts by volume of coarse aggregate. Since it is important that the quantity of cement should be not less than that expected, the cement should be measured by weight. If Portland cement has a nominal weight of 90 lb. per cubic foot, 1 : 2 : 4 means 90 lb. of Portland cement to 2 cu. ft. of fine aggregate to 4 cu. ft. of coarse aggregate. Since the basis of a batch of concrete is generally a bag containing one hundredweight of cement, this mixture is equivalent to 112 lb. of cement to 2½ cu. ft. of fine aggregate to 5 cu. ft. of coarse aggregate, and in specifications and on working drawings the proportions of the concrete should be expressed in these terms.

So long as it is realised that the method of expressing proportions of concrete in nominal volumetric ratios is merely a convenience, its use can greatly facilitate reference to any concrete, and for this purpose may be preferable to expressing the proportions in terms of the volumes of fine and coarse aggregates to a specified weight of cement, which leads to cumbersome expressions, or to giving different proportions distinctive references, such as "Mixture A" or "Grade I". The latter reference is used in the London By-laws, but does not at sight give information regarding the mixture and there is also the difficulty of selecting a suitable reference for proportions that are between, or are variations of, the basic proportions. For these reasons nominal volumetric proportions are adopted in this book, for example, in *Tables* 56 and 57. The latter table relates to the B.S. Code No. 114; in *Table* 56 are given the corresponding references in accordance with the London By-laws (1952). The volumes of aggregate required with 1 cwt. of cement when using the more common proportions, which quantities also apply to the B.S. Codes, are given in *Table* 56. Recommendations for special mixtures to give a concrete of specified strength are given in the B.S. Codes, in which the limiting proportions of cement to aggregate are as in *Table* 57.

Proportions of Cement to Aggregate.—The proportion of cement to aggregate depends on the strength, impermeability, and durability required. Experience shows that a nominal 1 : 2 : 4 concrete is suitable for general construction in cost and strength. Leaner concrete is not generally recommended for reinforced concrete unless the mixture has been properly determined by trials with the actual materials it is proposed to use in the work. A nominal 1 : 3 : 6 concrete is suitable for non-reinforced construction or for concrete temporarily placed and which will be cut away

later. Workable mixtures richer in cement than 1 : 2 : 4, for example 1 : 1½ : 3 and 1 : 1 : 2, are stronger but more expensive owing to the higher proportion of cement, and they are not generally economical for beams. They are, however, often economical for heavily-loaded columns or for members subjected to combined bending and direct thrust when the direct thrust predominates. Mixtures richer than 1 : 1 : 2 contain so large a proportion of cement that, apart from the cost, shrinking upon hardening is excessive. Instead of using a rich mixture it is generally more economical to obtain the necessary compressive strength by careful proportioning of the aggregates and control of the amount of water. Suitable proportions for concrete for various parts and types of structures under ordinary conditions are given in *Table* 56.

In liquid-containing structures the nominal proportions should be not leaner than 1 :1⅔ : 3⅓, that is, 2 cu. ft. of fine aggregate and 4 cu. ft. of coarse aggregate to 1 cwt. of cement. If, however, the thickness of the concrete exceeds 18 in., nominal 1 : 2 : 4 concrete is permissible. Concrete having the proportions 1 : 1⅔ : 3⅓ is generally used for precast piles, unprotected roof slabs and for concrete deposited under water, and in other places where a concrete of a better quality than 1 : 2 : 4 is required. For hollow-block floors and similar narrow ribbed construction and for many precast products 1 : 1½ : 3 concrete is often specified, but with smaller aggregate than is used for ordinary construction. The blinding layer on the bottom of an excavation can consist of concrete having the proportions of 1 part of Portland cement to 8 parts of combined aggregate.

Proportions of Fine and Coarse Aggregates.—The ratio between the amounts of fine and coarse aggregate necessarily depends on the grading and other characteristics of the materials in order that the volume of sand shall be sufficient to fill the voids in the coarse aggregate to produce a dense concrete. Until the material for a particular structure has been delivered to the site it is not possible to say what will be the exact grading of the sand or stone. Therefore this information is not always available when the specification is written. Several courses are open to the engineer when specifying the proportions for the concrete. The proportions of a particular sand and a particular stone with the properties of which the engineer is acquainted can be specified. Two or more independent sources of supply should be available within reasonable distance of the site. If the material is specified in this way, the permissible variations of the essential properties should be given. Another method is to specify the proportions of coarse and fine aggregates having definite gradings and leave it to the contractor to supply a material conforming to these requirements. Probably a better method is to specify a provisional ratio of fine to coarse material, the maximum and minimum sizes (with such percentages of intermediate sizes as are necessary), and insert a provision to allow adjustment of the proportions after examination of the actual materials.

Generally the proportion of fine to coarse aggregate should be such that the volume of fine aggregate should be about 5 per cent. in excess of the voids in the coarse material. Since the volume of voids may be up to 45 per cent., a common ratio is one part of fine to two parts of coarse aggregate, as in mixtures such as 1 : 2 : 4 and 1 : 1½ : 3. Such proportions, however, relate to dry materials. Whereas the water in a damp coarse aggregate does not appreciably affect the volume, the water in a damp fine aggregate may increase the volume by 30 per cent. over the dry (or fully saturated) volume. The proportions specified should therefore apply to dry sand and must be adjusted on the site to allow for bulking due to dampness.

The ratio of 1 : 2 of fine (dry) to coarse aggregate should be altered if tests show

that a denser and more workable concrete can be obtained by using other proportions. Permissible lower and upper limits are generally 1 : 1½ and 1 : 2½ respectively; thus for a nominal 1 : 2 : 4 concrete, the variation of the proportions may result in the equivalent extreme proportions of approximately 1 : 2⅓ : 3⅔ and 1 : 1⅔ : 4⅓. For liquid-containers, in accordance with B.S. Code No. 2007, the ratio may vary between 1 : 1¼ and 1 : 3.

Standards of control to ensure that concrete of uniform quality and strength assumed in design is obtained at the site are described in " Quality of Concrete in the Field " (Inst. Civil Engrs.); the degrees of control and strength categories equivalent to each class of concrete are given in *Table* 56 and explained briefly on the page facing the table.

Quantity of Water.—The strength and workability of concrete depend to a great extent on the amount of water used in mixing. There is an amount of water for certain proportions of given materials that produces a concrete of greatest strength. A smaller amount of water decreases the strength, and about 10 per cent. less may be insufficient to ensure complete setting of the cement and may produce an unworkable concrete. More than the optimum amount increases workability but decreases strength; an increase of 10 per cent. may reduce the strength by approximately 15 per cent., while an increase of 50 per cent. may reduce the strength by one-half. With an excess of more than 50 per cent. the concrete becomes too wet and liable to separation. The use of an excessive amount of water not only produces low strength but increases shrinking, and decreases density and therefore durability.

Some practical values of the water–cement ratio for structural reinforced concrete are about 0·45 for 1 : 1 : 2 concrete, 0·50 for 1 : 1½ : 3 concrete, and 0·55 to 0·60 for 1 : 2 : 4 concrete.

Concrete placed by efficient mechanical vibrators may generally contain less water than concrete compacted by tamping or rodding, thereby obtaining greater strength. Increased workability can be obtained by incorporating a plasticising agent into the mixture; the consequent reduction in the amount of water required results in a gain in strength.

A practical method of assessing the amount of water required is to make trial mixtures and find the proportion of water which produces a concrete which is just plastic enough to be worked among and around the reinforcement bars These trial mixtures may also be used to determine the best ratio of fine to coarse aggregate. Several mixtures are made with the amounts of fine and coarse aggregates slightly different in each, but with the total volume of aggregate and weight of cement the same in each mixture. The amount of water is adjusted to give the required workability. The mixture that occupies the least volume, that is the densest, will produce the best concrete. When the best mixture has been determined, the slump may be determined and the slumps of subsequent batches checked. The slump test allows for the porosity and dampness of the aggregates but not for any variation in the grading, size, or shape of the aggregate. A maximum slump for reinforced concrete construction is about 6 in., but a stiffer mixture is often desirable and practicable; a slump of 1 in. may be suitable if the reinforcement is not intricate or congested. For plain concrete in massive foundations, roads and dams, and similar work the concrete may not contain enough water to produce any slump, but sufficient water must be present to hydrate the cement and to enable the concrete to be properly consolidated by vibration or ramming.

Properties of Concrete.

Weight and Pressure.—The weight of ordinary concrete is discussed on page 140, and the weights of ordinary reinforced concrete, lightweight concrete and heavy concrete are given in *Table* 1. A weight of 150 lb. per cubic foot is generally adopted in the structural design of reinforced concrete members.

In the design of shuttering a weight of not less than 150 lb. per cubic foot should be allowed for wet concrete. The horizontal pressure exerted by wet concrete is generally assumed to be 140 lb. per square foot of vertical surface per foot of depth placed at one time, but for narrow widths, for drier concretes, and where the reinforcement is intricate, the increase in pressure for each foot of depth is less.

Lightweight Concrete.—Concrete having a density less than that of concrete made with gravel or crushed stone is produced by using clinker, foamed-slag, expanded clay and shale, vermiculite, pumice, or other lightweight material. Such concretes have not generally a great strength, especially if they are specially low in weight, and therefore are not in general used alone for structural members; their low densities and high thermal insulation properties make them suitable for partitions and for lining walls and roofs. The use of concrete of medium weight with lightweight aggregates (such as sintered clay or shale) and having sufficient strength for some structural members, with or without reinforcement, is however extending. In some structural members the reinforcement is embedded in ordinary dense concrete and the remainder of the member is made of lightweight concrete.

Other methods of making lightweight concrete include mixing metallic powder or other materials with the concrete; the resulting reactions generate bubbles of gas or entrap air to form cellular concrete. Such concretes are not generally impermeable, and if reinforced to provide lightweight slabs suitable for roofs, wall panels, and the like, the reinforcement should be coated to protect it from corrosion.

"No-fines" concrete is a form of lightweight concrete suitable for cast-insitu, non-reinforced construction. It is generally ordinary gravel concrete with little or no aggregate less than $\frac{3}{8}$ in. in size, and has high thermal insulation properties.

Air-entrained Concrete.—Air is entrapped in Portland cement structural concrete with ordinary aggregates by adding resinous or fatty materials during mixing. Generally the amount of air is about 5 per cent. (by volume) and the results are decreases of about 3 per cent. in weight and up to 10 per cent. in strength, an increased resistance to frost and chemical attack and an improvement in workability.

Compressive Strength.—With given proportions of aggregates the compressive strength of concrete depends primarily upon age, cement content, and the cement-water ratio, an increase in any of these factors producing an increase in strength. A formula for determining the probable strength of concrete of known mixture is given on the page facing *Table* 56.

Compressive strengths vary from 1500 lb. per square inch for lean concretes to more than 8000 lb. per square inch for special concretes. The rate of increase of strength with age is almost independent of the cement content, and, with ordinary Portland cement concrete, about 60 per cent. of the strength attained in a year is reached at twenty-eight days; 70 per cent. of the strength at twelve months is reached in two months, and about 95 per cent. in six months. Working stresses are generally based on the strength at twenty-eight days. The strength at seven days is about two-thirds of that at twenty-eight days with ordinary Portland cement, and is a good indication of the strength likely to be attained.

The minimum crushing strengths of preliminary laboratory cubes and works cubes, at seven and twenty-eight days for Portland cement concrete of ordinary proportions, and at two days for high-alumina cement concrete, are given in *Tables* 56 and 57. The strengths of leaner concretes, which are generally used for non-reinforced work, are given in *Table* 56. The recommendations of B.S. Code No. 114 are given in *Table* 57. The requirements of the London By-laws for buildings and the recommendations of B.S. Code No. 2007 for liquid-containing structures are given in *Table* 56. The various grades of concrete included in the tables are the ordinary, lower, and higher qualities and the special grade of concrete described in B.S. Code No. 114, and the two qualities specified in the London By-laws.

Compression tests in Britain are made on 6-in. cubes, which should be made, stored, and tested in accordance with B.S. No. 1181. For cubes made on the site, three should be cast from one batch of concrete. Identification marks should be made on the cubes. Two sets of three cubes each are preferable, and one set should be tested at seven days and the other at twenty-eight days. If only one set of three cubes is made, they should be tested at twenty-eight days. The strengths of the cubes in any set should not vary by more than 15 per cent. of the average, unless the lowest strength exceeds the minimum required. The seven-days' tests are a guide to the rate of hardening; the strength at this age for Portland cement concrete should be not less than two-thirds of the strength required at twenty-eight days.

In some countries cylinders or prisms are used for compressive tests. For ordinary concrete the compressive strength as measured on 6-in. cylinders is about 85 per cent. of that as measured on 6-in. cubes of ordinary concrete, although the ratio may be two-thirds with high-strength concretes.

Tensile Strength.—The direct tensile strength of concrete is considered when calculating resistance to shearing force and in the design of cylindrical liquid-containers. The tensile strength does not bear a constant relation to the compressive strength, but is about one-tenth of the compressive strength.

The tensile resistance of concrete in bending is generally neglected in the design of ordinary structural members but is taken into account in the design of slabs and the like in liquid-containing structures. The tensile resistance in bending is measured by the bending moment at failure divided by the section modulus, the result being termed the *modulus of rupture*. The minimum moduli of structural concretes at three and twenty-eight days as given in B.S. Code No. 114 are given in *Table* 57. Formulæ for estimating the modulus of rupture and the direct tensile strength are given on the page facing *Table* 56.

Elastic Properties.—Notes on the elastic properties such as the modulus of elasticity, modular ratio, and Poisson's ratio for concrete and reinforced concrete are given on the page facing *Table* 56.

Thermal Properties.—The coefficient of thermal expansion is required in the design of chimneys, tanks containing hot liquids, and exposed or long lengths of construction, and provision must be made to resist the stresses due to changes of temperature or to limit the strains by providing joints. The thermal conductivity of concrete (k on page 107) varies with the density and porosity of the material. Some values of the coefficients of thermal expansion and conductivity are given on the page facing *Table* 57.

The nature of the aggregate is the principal factor in determining the resistance of concrete to fire, although the type of cement may affect this property to some

extent. The resistance to fire of a reinforced concrete structure is affected considerably by the thickness of cover of concrete over the bars, and for a high degree of resistance covers in excess of those ordinarily specified should be provided, especially for floor and roof slabs and walls; reference should be made to the table on the page facing *Table* 57, which gives the requirements of the London By-laws and B.S. Code No. 114. Aggregates that have been sintered are superior to other aggregates in their resistance to fire; also of high resistance, but less so than the foregoing, are limestone and artificial aggregates such as broken brick. The aggregate ordinarily used for structural concrete, such as crushed hard stone (excepting hard limestone, but including granite) and flint gravels, are inferior in resistance to fire although such aggregates produce the strongest concrete.

Shrinking.—Unrestrained concrete members exhibit progressive shrinking over a long period while they are hardening. For concrete that can dry completely and where the shrinking is unrestrained, the linear coefficient is approximately 0·00025 at twenty-eight days and 0·00035 at three months, after which period the increase is less rapid until at the end of twelve months it may approach a maximum of 0·0005. In reservoirs and other structures where the concrete does not become completely dry, a maximum value of 0·0002 is reasonable. A concrete rich in cement, or made with finely-ground cement or with a high water content, shrinks more than a lean concrete or one with a low water content.

If a concrete member is restrained so that reduction in length due to shrinking cannot take place, tensile stresses are caused. A coefficient of 0·0002 may correspond to a stress of 500 lb. per square inch when restrained; it is therefore important to reduce or neutralise these stresses by using a strong concrete, by proper curing and by the provision of joints. Shrinking is considered in the design of fixed arches on pages 44, 232 and 234.

Creep.—Creep is the slow deformation, additional to elastic contraction, exhibited by concrete under sustained stress, and proceeds at a decreasing rate over many years however small may be the stress. Characteristic values for creep, expressed in deformation per unit length, for 1 : 2 : 4 concrete loaded at 28 days with a sustained stress of 600 lb. per square inch, are 0·0003 at 28 days after loading, and 0·0006 at one year. Thus creep is of the same degree of magnitude as shrinking, and appears to be directly proportional to the stress. The earlier the age of the concrete at which the stress is applied the greater is the creep, which also appears to be affected by the same factors as affect the compressive strength of the concrete; generally the higher the strength the less is the creep. The effect of creep of concrete is not often considered in reinforced concrete design.

Reduction of Bulk upon Mixing.—When the constituents of concrete are mixed with water and tamped into position, a reduction in volume to about two-thirds of the total volume of the dry unmixed materials takes place. The actual amount of reduction depends on the nature, dampness, grading, and proportions of the aggregates, the amount of cement and water, the thoroughness of mixing, and the degree of consolidation. With so many variables it is impossible to assess exactly the amount of each material required to produce a cubic yard of wet concrete when deposited in place. The quantities given in *Table* 56 are ordinary values for gravel concrete but do not allow for waste. Alternative quantities are given for dry and damp fine aggregates. The quantities of fine and coarse aggregates for other proportions can be obtained from the ratios of the volumes of the materials.

Porosity and Permeability.—The porosity of concrete is the characteristic whereby liquids can penetrate the material by capillary action, and depends on the total volume of the spaces occupied by air or water between the solid matter in the hardened concrete. The more narrow and widely distributed these spaces are the more easily can liquids diffuse in the concrete.

Permeability is the property of the concrete that permits a liquid to pass through the concrete due to a difference in pressure on opposite faces. Permeability depends primarily on the size of the largest voids and on the size of the channels connecting the voids. Impermeability can only be approached by proportioning and grading to make the number and sizes of the voids the least possible, and by thorough consolidation to ensure that the concrete is the densest possible with the given proportions of the materials. Permeability seems to be a less determining factor than porosity when considering the effect on concrete of injurious liquids.

Fatigue.—The effect of repeatedly applied loads, either compressive or tensile, or a frequent reversal of load, is to reduce the strength of concrete; this phenomenon is called " fatigue ". If the resultant stress is less than half the strength, as is the case in compression on most concrete members, fatigue is not evident. When a stress exceeding half the strength of the unfatigued concrete is frequently caused, the strength of the concrete is progressively reduced, until it equals the stress due to the applied load, when the concrete fails. The number of repetitions of load to produce failure decreases the more nearly the stress due to the load equals the strength of the unfatigued concrete.

A relatively high frequency of repetition of stress would be ten times and upwards per minute. If intervals of time occur between successive applications of load, the effect of fatigue is delayed. The degree of fatigue differs for direct compression, direct tension, and bending. Since the tensile stress in concrete in bending more nearly approaches the strength than does the compressive stress, it is evident that fatigue due to tensile stress controls fatigue of concrete in bending.

Working Stresses in Concrete.

Compression due to Bending.—The permissible compressive stress p_{cb} in concrete due to bending is generally assumed to be one-third of the specified minimum crushing strength of cubes at twenty-eight days. Values of p_{cb} for concrete not leaner than 1 : 2 : 4 are given in *Tables* 56 and 57 and conform to general practice, and in particular to the London By-laws and the B.S. Codes. The London By-laws recognise two grades of concrete. The B S. Code considers four qualities of Portland cement concrete with aggregates complying with B.S. 882, namely, concrete of normal strength; concrete of lower strength, but not more than 25 per cent. less than ordinary concrete; concrete of higher strength, up to 25 per cent. in excess of ordinary concrete; and special concrete of higher strength. Two qualities of high-alumina cement concrete are considered. If the aggregates do not conform to B.S. 882, the B S. Code recommends working stresses based directly on the strength of site-made or laboratory-made cubes in accordance with the ratios given in *Table* 57.

For concrete in which the ratio of fine to coarse aggregate is not 1 : 2, the permissible stresses are equivalent to those for concrete having the same sum of the proportions of fine and coarse aggregate; thus mixtures of 1 : 2½ : 3½ and 1 : 1½ : 4½ are both equivalent to 1 : 2 : 4 as regards working stresses since the sum of the proportions of the fine and coarse aggregates is six in each case. For proportions between

those tabulated, the value of p_{cb} can be computed by proportion between the stresses given for the two nearest mixtures based on the sum of the separate volumes of the fine and coarse aggregate; an example is given on the page facing *Table* 62.

For liquid-containers made with 1 : 1⅗ : 3⅕ concrete, a maximum compressive stress of 1200 lb. per square inch should be used for slabs and beams subjected to bending. The 28-days' strength of concrete used for such structures should be not less than 5400 lb. per square inch for laboratory-made cubes and 3600 lb. per square inch for cubes made on the works. In thick construction 1 : 2 : 4 concrete is used having a 28-days' strength of 4500 lb. per square inch for laboratory-made cubes and 3000 lb. per square inch for site-made cubes; p_{cb} is 1000 lb. per square inch. These strengths and stresses are in accordance with B.S. Code No. 2007 and are given in *Table* 56.

Direct Compression.—For members in direct compression, such as concentrically-loaded columns, the permissible compressive stress is about 76 per cent. of the permissible compressive stress in bending as is given in *Tables* 56 and 57. This stress is assumed to occur over the whole cross-sectional area of an ordinary column and on the cross-sectional area of the core of a column with helical binding.

Combined Bending and Direct Force.—When a member is subject to bending moment combined with a direct thrust, as in an arch or a column forming part of a frame, or is subjected to a bending moment combined with a direct pull, as in the walls of rectangular bunkers and tanks, the same permissible compressive stress p_{cb} is used for the concrete as if the member were subjected to bending alone. Thus for a column built in 1 : 2 : 4 ordinary concrete (*Table* 57), the compressive stress should not exceed 760 lb. per square inch for the direct load considered alone, nor exceed 1000 lb. per square inch when the direct load is combined with a bending moment.

Tension.—In the design of members subjected to bending the strength of the concrete in tension is commonly neglected, but in certain cases, such as structures containing liquids and in the consideration of shearing resistance, the tensile strength of the concrete is important. For suspension members which are in direct tension and where cracking is not necessarily detrimental, the tensile strength of the concrete can be neglected and the reinforcement then resists the entire load. In a member that must be free from cracks, such as the wall of a cylindrical container of liquids, the tensile stress in 1 : 1⅗ : 3⅕ concrete should not exceed 190 lb. per square inch in accordance with B.S. Code No. 2007; a member in bending should be designed, as described on page 69, so that the tensile stress in the concrete does not exceed 270 lb. per square inch. The corresponding tensile stresses in 1 : 2 : 4 concrete are 175 and 245 lb. per square inch as given in *Table* 56. Overline railway bridges and similar structures where cracking may permit corrosive fumes to attack the reinforcement should also be designed with a limited tensile stress in the concrete.

Shearing Stresses.—The permissible shearing stress q in a beam is about 0·1 of the maximum permissible compressive stress in bending, but if the diagonal tension due to the shearing force is resisted entirely by reinforcement the shearing stress should not exceed $4q$ in accordance with B.S. Code No. 114 for buildings or $2{\cdot}5q$ in accordance with B.S. Code No. 2007 for liquid-containing structures; a maximum stress of less than $4q$ is advisable in all but primary beams in buildings. Values of the stresses q and $4q$, and $2{\cdot}5q$, are given in *Tables* 56 and 57. The relation between q and p_{cb} for concrete of quality other than those tabulated is given in *Table* 57.

Bond.—The permissible bond stress s_b between concrete and plain round bars

is slightly more than the shearing stress. Thus the shearing stress for 1 : 2 : 4 ordinary concrete is, according to the B.S. Code, 100 lb. per square inch and the average bond stress is 120 lb. per square inch. The local bond stress s_{b_1} (see page 62) is about 50 per cent. greater than the average bond stress, that is 180 lb. per square inch. Bond stresses corresponding to other proportions and qualities of concrete are given in *Tables* 56 and 57. For deformed bars the recommended stresses are 25 per cent. in excess of the stress for plain round bars.

Bearing on Plain Concrete.—Plain concrete mixed in leaner proportions, than 1 : 2 : 4 is used for filling under foundations and for massive piers, and thick retaining walls. Suitable working stresses in direct compression are given in *Table* 56 and these conform generally to the London By-laws. The bearing pressures on plain and reinforced concrete in piers subjected to a concentric load, or to an eccentric load, and permissible local pressures as under bearings are given in *Table* 104 in accordance with B.S. No. 111.

Modifications of Permissible Stresses.—The working stresses given in *Tables* 56 and 57 and specified in regulations should be related to the method of computing the bending moments and forces to which the member is subjected. These stresses must be reduced if departures are made from the ordinary methods of assessment of the load, the computation of the bending moment, or the calculation of the stresses.

The working stresses should be reduced when any of the following apply.

(*a*) If tests on works-made cubes made with the available aggregates give at twenty-eight days results lower than three times the permissible stress in bending, the working stresses should be reduced to one-third of the strength at twenty-eight days. The ratios recommended in B.S. Code No. 114 are given in *Table* 57.

(*b*) When the structure is constructed by inexperienced men and when competent whole-time supervision is not available.

(*c*) When the calculations are approximate or when the maximum load is uncertain. In preliminary designs such secondary effects as stresses due to changes in temperature or due to torsion may be omitted and a reduced working stress used. It is necessary, however, when preparing the calculations for the working drawings to include these factors and adopt the ordinary permissible stresses.

(*d*) Slender columns and piers must be designed for stresses less than the ordinary stress for direct load (see *Table* 83). Similarly, narrow beams and other members should be designed for lower compressive stresses (see page facing *Table* 62).

(*e*) When the structure is subject to vibration or impact the ordinary stresses should be reduced to allow for fatigue unless an allowance for these effects has been made in the calculation of the live load.

(*f*) If the greatest load can occur before the concrete has obtained a crushing strength equal to three times the compressive stress due to the combined dead and live loads.

(*g*) If all the load is dead load the working stresses should be reduced, unless means can be taken (for example, by introducing temporary supports) to prevent the full load acting on the structure until three months after completion.

(*h*) When the work is difficult to construct and when the concrete is deposited under water, in underpinning, or in similar difficult positions.

Under more favourable conditions reasons may exist for increasing the working compressive stresses, although the 28-days' strength divided by three should generally be adopted as a guide. Among such conditions are the following.

(*a*) When the concrete is specially proportioned and treated with the object of obtaining a high strength.

(*b*) If the structure, or particular members, will not be loaded for some time after completion of construction, the working stresses may be increased. The amounts of increase, in accordance with the B.S. Code, are given in *Table* 57.

(*c*) If the extreme live load occurs infrequently and for a short time, stresses for such a load can be increased at the designer's discretion, but the stresses due to the dead load and the ordinary live load should not exceed those generally permissible.

(*d*) When the calculated stresses include those due to the bending moments and forces caused by wind pressures on buildings, the stresses can be increased by $33\frac{1}{3}$ per cent. in accordance with the London By-laws and 25 per cent. in accordance with the B.S. Code. The stresses without wind should not exceed the ordinary permissible working stresses. This increase should not be made for structures in which the effects of wind are a predominating factor in the design.

In adopting the foregoing reductions or increases much has to be left to the judgment of the designer when considering the conditions of a particular structure and the requirements of any controlling authorities.

Properties of Reinforcement.

Strength.—Reinforcement is most commonly plain round mild steel bars. Less frequently, medium-tensile and high-tensile steels are used; these have higher yield-point stresses than mild steel because, in the case of high-tensile steel (which is obtainable as rolled deformed bars), of the higher carbon content. Medium-tensile steel is a superior type of mild steel. Other high-yield-stress bars are cold-worked bars, such as twisted ribbed bars, twisted square bars, and wire. Cold-drawn mild steel wire and small twisted square bars are generally in the form of oblong-mesh or square-mesh welded or woven fabrics and are used for the reinforcement of slabs and roads. Expanded metal is used for the same purpose. When available, and convenient to use, old rails, disused wire ropes, and light structural steel are occasionally embedded in concrete as reinforcement. The tensile strength, and the yield-point stress or equivalent yield stress, of hot-rolled mild steel, medium-tensile steel, high-tensile steel, hard cold-drawn wire, twisted square bars, and twin-twisted bars are given in *Table* 58; these data are in accordance with B.S. Nos. 785 and 1144. Twin-twisted bars are not now readily obtainable. The table also gives the equivalent yield stress and tensile strength of twisted ribbed bars, hard-drawn steel wire and expanded metal.

Cross-sectional Area, Perimeter, and Weight.—In *Table* 59 are given the cross-sectional areas and weights per foot of plain round bars and wire and twisted square bars of common sizes, including wire-gauge sizes. The cross-sectional area of any number up to twenty of plain round bars from $\frac{3}{16}$ in. to $1\frac{1}{2}$ in. diameter is given in *Table* 60, as well as the cross-sectional areas (per foot) of similar bars at various spacings from 3 in. to 24 in. The weights per foot and per square foot of most of the bars given in *Table* 60 are given in *Table* 59. The perimeters of plain round bars and the number of feet in a ton are given in *Table* 59. The cross-sectional areas and weights of twisted ribbed bars are the same as those of the same size of plain round bar since the nominal size of a ribbed bar is the diameter of the plain round bar having the same weight per foot. The overall diameter of a ribbed bar is 1·1 times the nominal diameter.

In Section 6 some methods of estimating and calculating the weight of mild steel reinforcement are described.

Table 61 gives data relating to bars of metric sizes, namely, the size of bars in millimetres and the equivalent in inches, the cross-sectional area of a specified number of bars up to ten (in square inches and square centimetres), and the cross-sectional area of bars at specified spacings from 8 cm. (about 3 in.) to 30 cm. (about 12 in.). The data relate to bars from 6 mm. (about ¼ in.) to 25 mm. (1 in.) and in some cases to 30 mm. (about 1¼ in.).

Working Stresses in Reinforcement.

Tension.—For rolled mild steel having a tensile strength of 28 tons to 33 tons per square inch the safe working stress in tension is generally 18,000 lb. per square inch for retaining walls, bridges, industrial and similar structures. For liquid-containers the maximum tensile stress should be 12,000 lb. per square inch when the tensile strength of the concrete is ignored (but see page 69). The permissible tensile stresses for reinforcement provided to resist shearing forces are discussed on page 72. A stress of 20,000 lb. is used for mild steel bars not larger than 1½ in. in buildings designed in accordance with the B.S. Code; for bars larger than 1½ in. the permissible stress is 18,000 lb. per square inch.

Since the yield-point stress of rolled mild steel is about 36,000 lb. per square inch, a working stress of 18,000 lb. per square inch represents a factor of safety of about two on the yield-point stress, and this factor is the true measure of the margin of security. For high-yield-stress steel the maximum working tensile stress permitted is generally 50 per cent. of the yield-point stress of the bar or wire in the form used in the concrete, with a limit of 30,000 lb. per square inch (or 20,000 lb. per square inch in shear reinforcement). The tensile stresses recommended in the B.S. Code and London By-laws are given in *Table* 58.

When deciding the working tensile stress suitable for the reinforcement in a part of a structure, the modifying factors given on page 57 should be considered, but the factors that represent a variation in the strength of the concrete only must be disregarded except where the bond stress is affected.

In liquid-containers in which cracking is to be avoided there is no advantage, so far as the tensile stress is concerned, in using a steel with a high yield-point stress if a working stress of 12,000 lb. per square inch is not exceeded although the tensile stress in mild steel reinforcement near the face of a member not in contact with liquid can, however, be 18,000 lb. or 20,000 lb. per square inch; these stresses are in accordance with B.S. Code No. 2007. The latter stresses might be adopted for a tank lined with an elastic impermeable membrane.

The working tensile stress in the reinforcement in buildings can be increased by one quarter when the increase is due solely to increased bending moments and forces caused by wind pressure. The B.S. Code No. 114 limits the increased stress in reinforcement to 30,000 lb. per square inch. The London By-laws permit an increase of one-third, with a limit of 27,000 lb. per square inch.

Compression.—The compressive stress in reinforcement depends on the compressive stress in the surrounding concrete if the modular-ratio theory of the action of reinforced concrete at working loads is applied. Since the strain of the two materials is equal so long as the bond is not destroyed, the stresses are proportional to the elastic moduli. Mild steel has a modulus of elasticity E_s of about 30,000,000 lb. per square

inch, and, if the modulus of elasticity of concrete E_c is assumed to be nominally 2,000,000 lb. per square inch, the compressive stress in the steel f_{sc} is fifteen times the compressive stress f_{cb} in the concrete, or generally $f_{sc} = mf_{cb}$ if m is the modular ratio $\frac{E_s}{E_c}$. The value of m is considered on the page facing *Table* 56. It is more convenient to calculate the compressive stress in the steel as additional to that in the concrete; this is $f_{cb}(m - 1)$. When this expression is used the resistance of the concrete can be calculated on the whole cross-sectional area, no deduction being necessary for the area of the bars.

When the moment of resistance of a beam is calculated in accordance with the steel-beam theory (see page 66), the stress in mild steel reinforcement in compression is generally considered to be 18,000 lb. per square inch. For high-yield-stress steel the permissible compressive stress is half the yield stress but not more than 23,000 lb. per square inch in accordance with the B.S. Code No. 114.

The compressive stresses in the main reinforcement in columns, and in other cases where the stress is independent of the stress in the concrete, in accordance with the London By-laws and B.S. Codes are given in *Table* 58.

Bond between Concrete and Steel.

Length for Bond of Tensile Reinforcement.—For a bar to resist tensile forces effectively there must be sufficient length of bar beyond any section to develop by bond between the steel and the concrete a force equal to the total tensile force in the bar at that section.

The minimum length for bond, or the minimum length of an overlap, can be expressed as N times the diameter D of the bar; for a plain round bar $N = \frac{f_{st}}{4s_b}$ where f_{st} is the tensile stress in the bar at the section considered and s_b is the permissible average bond stress. Values of s_b are given in *Tables* 56 and 57 for various qualities of concrete in buildings according to the London By-laws and the B.S. Code. Values of N for various combinations of common tensile stresses f_{st} and s_b are given in *Table* 62. The lengths of bond for stresses other than those tabulated are proportional to the tensile stresses, or inversely proportional to the permissible average bond stress. The lowest stress tabulated is 10,000 lb. per square inch and, although the working stress in the bar may be less, it is recommended that in no case should the length of bond provided be less than that required for 10,000 lb. per square inch. The B.S. Code recommends that the minimum length for bond should be $12D$ and the corresponding lengths for bars of various sizes are given in *Table* 62. If this recommendation is applied rigorously the length for small bars is only a few inches, and it is suggested that a minimum length of 6 in. (or preferably 12 in.) should be adopted, as tabulated. The minimum length of an overlap is $30D$; values are given in *Table* 62. For the common case of the permissible tensile stress p_{st} being 20,000 lb. per square inch (mild steel) and $s_b = 120$ lb. per square inch (1 : 2 : 4 concrete), N is $41\frac{2}{3}$; the length of bond for bars of various sizes for these stresses is given in *Table* 62.

If at a particular section of a beam a bar is no longer required to resist tension, it should not be stopped in the tensile zone but carried a short distance beyond the section (and preferably bent up at 45 deg. to extend into the compressive zone). Similar bars in slabs can remain in the tensile zone if continued a distance of $12D$ beyond the section. These requirements conform to the B.S. Code except the recommendations in parenthesis.

Anchorages.—If an anchorage is provided at the end of a bar in tension, the length for bond need not be so great as when no such anchorage is provided. An anchorage may be a semi-circular hook, a 45-deg. hook, a 90-deg. bend, or a mechanical anchorage. To obtain full advantage of the value in bond of an anchorage, the hook or bend must be properly formed. Suitable dimensions for common sizes of bars are given in *Table* 62, together with the length of bar required to form the anchorage additional to the overall length of the bent bar. The radius of the bend must be not less than $2D$. The length for bond must be measured from the point where the bar starts to deviate from the straight, that is, where the bend commences. The bond-resistance value of an anchorage of such dimensions is given in the B.S. Code as equivalent to a length of $4D$ for each 45 deg. through which the bar is bent; that is $8D$ for a right-angle bend, $12D$ for a 45-deg. hook, and $16D$ for a semi-circular hook. The bond values of each of these types of anchorage are given in *Table* 62 for bars of various sizes.

According to the B.S. Code No. 114, if an end anchorage is provided, the length required for bond (as represented by ND when N has the values in *Table* 62) can be reduced by the appropriate bond value (as also given in *Table* 62) of the anchorage, but the reduced length should be not less than $12D$. The bond-lengths required for bars with various forms of anchorages in accordance with B.S. Code No. 114 are given in greater detail in *Table* 63. For more conservative design, the length given by ND can be considered to be the minimum if a semi-circular hook of value $16D$ is provided; if other forms of anchorage are provided, the length for bond should be ND plus $16D$ minus the value of the anchorage as given on the page facing *Table* 62.

A mechanical anchorage can be either a hook embracing an anchor bar (the internal diameter of the hook being equal to the diameter of the anchor bar) or the end of the bar can be threaded and provided with a plate and nut. The size of the plate should be such that the compression on the concrete at, say, 1000 lb. per square inch of net area of contact (that is the gross area of the plate less the area of the bar) should be equal to the tensile resistance required.

Length of Bond for Compression Reinforcement.—The length of bond required to resist a compressive force in a plain round bar is 80 per cent. of that required for a tensile force of the same magnitude. Therefore the values of N in *Table* 62, multiplied by 0·8, can be used for compression reinforcement subjected to a stress of the same numerical value. For example, for a bar subjected to a compressive stress of 20,000 lb. per square inch in concrete the permissible average bond stress for which is 120 lb. per square inch, the value of N is 0·8 times 42 approximately, say, 34, and the bond-length is $34D$. The same rules apply to the length of overlap of bars forming compression reinforcement. The minimum length of bond for bars in compression is $12D$, but the minimum length of an overlap is $24D$ (as in *Table* 62). The end of a bar in compression need not be provided with an anchorage.

Bars in Liquid-containers.—For liquid-containers, using 1 : 1$\frac{2}{3}$: 3$\frac{1}{3}$ concrete, the length required for bond or for overlap of bars in tension, based on $s_b = 130$ lb. per square inch and $p_{st} = 12{,}000$ lb. per square inch, as in B.S. Code No. 2007, is about $24D$ (see *Table* 62); it is recommended (but it is not required by the Code) that a semi-circular hook should be provided in addition to this length, since it is preferable to provide hooks on all plain bars in direct tension in liquid-containers. Hooks can generally be omitted from reinforcement in planar slabs. Greater lengths must be provided where the permissible tensile stress of 18,000 lb. per square inch is adopted.

For bars in compression, end anchorages are not necessary but a minimum length of $20D$ should be provided for bond and for overlaps. For liquid-containing structures of 1 : 2 : 4 concrete, the lengths should be as for ordinary construction.

Deformed Bars.—Twisted-ribbed, twisted-square, and similar bars provide greater mechanical bond resistance than plain round bars. The B.S. Code defines a deformed bar as one having a bond resistance (determined by test) 25 per cent. greater than that of a plain round bar, and for such a bar the bond stresses permissible are 25 per cent. higher than those given in *Table* 57. Therefore the length provided for bond can be based on a value of N equal to 0·8 of that given in *Table* 62; if such a length is provided, no end anchorage is required for a deformed bar. Bond-lengths for deformed bars in tension or compression are given on the page facing *Table* 63.

Bond of Beam Reinforcement.—The local bond stress s_{b1} due to variation in the tensile stress in reinforcement in beams should be investigated by application of the formulæ given on pages 246 and 248. The permissible stress s_{b1} is about 50 per cent. greater than the ordinary bond stress. The permissible local bond stresses in accordance with the B S. Code No. 114 are given in *Table* 57.

Details of Reinforcement.

Length and Size of Bars.—If attention is given to a number of points regarding the length and size of reinforcement bars, fixing the bars is facilitated and the construction is more efficient. As few different sizes of bars as possible should be used, and the largest size of bar consistent with good design should be used, thus reducing the number of bars to be bent and placed. Large bars are cheaper than small bars. The basic price is that of ⅜-in. bars, all larger bars being supplied at this rate; smaller bars cost more for each ⅛ in. in diameter below ⅜ in.

Generally the longest bar economically obtainable should be used, but regard should be paid to the facility with which a long bar can be transported and placed in position. Consideration should also be given to the greatest length that can be handled without being too whippy; these lengths are about 20 ft. for bars of $\frac{5}{16}$ in. diameter and less, 25 ft. from $\frac{5}{16}$ in. to ½ in., 40 ft. for ⅝ in., 60 ft. for 1 in., and 75 ft. for bars over 1¼ in. The basic price only applies to bars up to 40 ft. long, and extra for every foot in length over 40 ft. is charged. Bars up to ⅜ in. can be obtained in long lengths in coils at ordinary prices and sometimes at lower prices. Over certain lengths it is more economical to lap two bars than to buy long bars, the extra cost of the increase in total length of bar due to overlapping being more than offset by the increased charge for long lengths. Long bars cannot always be avoided in long piles, but bars over 40 ft. require special road or rail vehicles which may result in delay and extra cost.

The total length of each bar should, where possible, be given to a multiple of 3 in. and as many bars as possible should be of one length, thus keeping the number of different lengths of bars as small as practicable.

Bar-bending Schedule.—The method of giving bending dimensions and marking the bars should be uniform throughout the bar-bending schedules for any one structure. A system of bending dimensions is illustrated in *Table* 62 and conforms to B.S. No. 1478 in which bars of other shapes also are given. Most designers have their own form of bending schedule to suit the work in hand. Generally, however, it is necessary to give the following information thereon: Position of the bar in the structure, the diameter and total length of the bar, the number of bars of one type

in a single member, the number of identical members and the total number of bars of one type, the shape and bending dimensions of the bar, and perhaps a reference mark for each bar or bundle of bars.

Cover of Concrete.—For the proper protection of the reinforcement and in order to ensure that the thickness of concrete around a bar is adequate to develop the bond resistance, it is necessary to provide sufficient concrete over the bars and sufficient space between adjacent bars. The minimum cover of concrete to a reinforcement bar should be as given on the page facing *Table* 63 and in no case should it be less than the diameter of the bar.

It should be noted that much of the deterioration of reinforced concrete structures is due to insufficient cover of concrete to the bars; therefore, the designer should not hesitate to increase the minimum covers if it is thought to be desirable to do so for a particular structure.

Spacing of Bars.—The minimum clear distance between two bars in any one layer in a beam should be not less than the diameter of the bar, or 1 in., or the largest size of the aggregate plus $\frac{1}{4}$ in., whichever is the greatest. The minimum clear distance between successive layers of bars in a beam should be $\frac{1}{2}$ in. and this distance should be maintained by the provision of $\frac{1}{2}$-in. diameter spacer bars placed every 3 ft. or 4 ft. throughout the length of the beam wherever two or more layers of reinforcement occur. Where the bars from transverse beams pass between the layers, spacer bars are unnecessary. If the bars in a beam exceed 1 in. in diameter, it is preferable to increase to $\frac{3}{4}$ in. or 1 in. the clear space between the layers. If concrete is to be compacted by vibration, a space of at least 3 in. should be provided between groups of bars for the insertion of the vibrator if of the internal or poker type.

SECTION 4

RESISTANCE OF STRUCTURAL MEMBERS

Properties of Cross-sections of Members.

THE geometrical properties of plane figures, the shapes of which conform to those of the cross-sections of common reinforced concrete members, are given in *Table* 64. The data include areas, section moduli, moments of inertia, and radii of gyration. In particular are given data for readily calculating the moment of inertia of tee-sections which are also applicable to other flanged sections such as ell-sections and inverted channels. These data are suitable for cases when the amount of reinforcement need not be taken into account as in the case of comparative moments of inertia (see page facing *Table* 51).

The data in *Tables* 65A and 65B apply to reinforced concrete members of rectilinear and polygonal cross-sections when the reinforcement is taken into account on the basis of the modular ratio. Two conditions are considered, namely, when the entire section is subjected to stress, and when the concrete in tension in members subjected to bending is not taken into account. The data given for the former condition includes the effective area, the position of the centroid, the moment of inertia, the section modulus, and the radius of gyration. For the condition when a member is subjected to bending and the concrete is ineffective in tension, the data include the position of the neutral plane, the lever-arm, and the moment of resistance. The corresponding general formulæ for irregular and regular sections are given on the page facing *Table* 65B.

Design of Beams: Modular-ratio Method.

There are two fundamental methods of analysing the stresses in, and calculating the resistance of, a reinforced concrete member in bending: the ordinary elastic or modular-ratio method, and the ultimate-load or load-factor method; both methods are included in B.S. Code No. 114. The load-factor method is dealt with on page 70.

The modular-ratio method applies to conditions at working stresses only. The basis of design of a structural member subjected to bending is that the internal resisting couple is equal to the bending moment produced by the external load. In a reinforced concrete member, the equal forces forming the couple are the compressive resistance of the concrete and the tensile resistance of the tensile reinforcement; the arm of the couple, that is the distance between the lines of action of the resultant forces, is called the lever-arm. It is assumed that the strain in any plane of the member is proportional to the distance of the plane from the neutral plane, that the tensile resistance of the concrete is neglected, that the reinforcement resists all the tensile forces (except in certain cases of liquid-containers), and that the concrete and steel are elastic within the range of the permissible working stresses. The formulæ for the position of the neutral plane, the lever-arm, the moments of resistance, and the maximum stresses in rectangular beams and flanged beams (that is tee-beams and ell-beams), based on the foregoing principles are given in *Table* 66. The notation is summarised on the page facing the table. For beams of other regular cross-sections, the expressions for the lever-arm and moments of resistance given in *Tables* 65A

and 65B are applicable. For a beam of any general or irregular cross-section the method described on the page facing *Table* 68 can be used.

Formulæ (1) and (1*a*) in *Table* 66 apply to members of any cross-section subjected to bending and give the neutral-plane factor n_1 expressed in terms of the permissible and actual stresses respectively; n_1 is dependent only on the modular ratio m and the ratio of the maximum stresses in the tensile reinforcement and in the concrete, that is p_{st} and p_{cb} or f_{st} and f_{cb} respectively. Values of n_1 for $m = 15$ and for various ratios r of the maximum stresses are given in *Table* 68 or can be obtained from the curves in *Table* 67.

The formulæ (14) for the maximum stresses and formulæ (15) for the ratio of the maximum stresses (in *Table* 66) involve n_1 and m only, and are therefore applicable to a member of any cross-section.

Rectangular Beams.—Formulæ (2) and (2*a*) in *Table* 66 apply to rectangular beams whether reinforced to resist tension only or to resist tension and compression and give the neutral-plane factor in terms of the proportions of tensile and compression reinforcement, that is r_t and r_c respectively; values of n_1 for various values of r_t, or conversely, are given in *Tables* 67 and 68 for $m = 15$.

The formulæ for the lever-arm l_a and the lever-arm factor, a_1 in general, a_c for the concrete in compression and a_s for the compression reinforcement, are given by the formulæ in the series (5) in *Table* 66. For values of $a_1 = a_c$ see *Tables* 67 and 68.

The moment of resistance of a rectangular beam reinforced in tension only is given by formulæ (8) and (8*a*), depending on whether the resistance to compression or tension (M_{rc} and M_{rt} respectively) determines the strength. If n_1 is based on the maximum permissible stresses, that is formula (1*a*), the values of M_{rc} and M_{rt} may differ; if n_1 is based more properly on formula (2), M_{rc} and M_{rt} will be equal.

The moment of resistance in compression can be expressed conveniently in terms of a factor Q_c such that $M_{rc} = Q_c b d_1^2$. Values of bd_1^2 for various values of b and d_1 are given in *Table* 69, and values of Q_c for various stresses with $m = 15$ are given in *Table* 68.

The maximum stresses produced by a given bending moment, with n_1 based on formula (2), are calculated from formulæ (15).

Tables 70A, 70B and 70C give the moments of resistance and areas of reinforcement for rectangular beams of various depths and of unit width reinforced in tension only for tensile stresses of 18,000 lb., 20,000 lb., and 27,000 lb. per square inch in the reinforcement and compressive stresses of 750 lb., 1000 lb., and 1100 lb. per square inch in the concrete ($m = 15$); *Table* 70D gives similar data for a tensile stress of 30,000 lb. per square inch and compressive stresses of 750 lb., 1000 lb., and 1250 lb. per square inch.

When sufficient depth or breadth of beam cannot be obtained to provide enough compressive resistance from the concrete alone compression reinforcement is provided. This extra reinforcement is not generally economical, although some concrete is saved thereby, but in some cases, such as the support sections of continuous beams, the ordinary arrangement of the reinforcement provides compression reinforcement conveniently; the amount of compression reinforcement should not exceed 4 per cent. in accordance with B.S. Code No. 114, and compression reinforcement in excess of this amount should be neglected in the calculation of the resistance of the beam.

If the compressive resistance of the concrete is not neglected, the moment of

resistance of a beam with compression reinforcement is the sum of the moments of resistance of the concrete and compression reinforcement. The moment of resistance of the concrete is calculated as for a beam with tensile reinforcement only, and the additional moment of resistance due to the compressive reinforcement is as given by formula (8*b*) in *Table* 66, in which n_1 is based on formulæ (2*a*). The maximum stresses due to a given bending moment are derived from formulæ (14) in which l_a is based on a_1 calculated from formula (5*a*), or approximately from (5*b*), and n_1 from formula (2*a*); note that formula (5*b*) does not apply if r_c is small compared with r_t.

The rational limit of application of the formulæ for rectangular beams with compression reinforcement is when $A_{sc} = A_{st}$, and for this condition the moment of resistance if given by

$$M_r = \left[\tfrac{1}{2}n_1 a_c + r_c(m - 1)\left(\frac{n_1 - f_2}{n_1}\right)(1 - f_2)\right]p_{cb}bd_1^2,$$

and the proportion of tensile reinforcement, which is equal to the proportion of compression reinforcement, is given by

$$r_t = \frac{0{\cdot}5n_1}{r - (m - 1)\left(\dfrac{n_1 - f_2}{n_1}\right)},$$

in which r is the ratio of the permissible stresses $\left(= \dfrac{p_{st}}{p_{cb}}\right)$. Values of M_r and r_t are given at the foot of *Tables* 70A, 70B, 70C, and 70D for common permissible stresses and $m = 15$. The moments of resistance and areas of reinforcement for rectangular beams of various depths and of unit width reinforced in tension and compression are also given in these tables, from which a beam to resist a specified bending moment can be selected since the moment of resistance for the limits of $A_{sc} = 0$ and $A_{sc} = A_{st}$ are given for stresses of 18,000 lb., 20,000 lb., 27,000 lb., and 30,000 lb. per square inch in the tensile reinforcement and stresses of 750 lb., 1000 lb., and 1100 lb. or 1250 lb. per square inch in the concrete. The compressive resistance of the concrete is not neglected; the stress in the compression reinforcement is therefore m-times the stress in the surrounding concrete.

To prevent the compression reinforcement from buckling, binders should be provided at a pitch not exceeding twelve times the diameter of the smallest bar in the compression reinforcement. The binders should be so arranged that each bar is effectively tied.

Balanced Design.—In the design of a beam it is of course necessary that the permissible stresses in the steel and concrete are not exceeded, but it is also desirable generally for the greatest stresses to be equal to the permissible stresses. When this condition is obtained, the design is considered to be " balanced ". There is, for each ratio of permissible stresses, a proportion of tensile and compression (if any) reinforcement which gives balanced design, and expressions for this amount are given in *Table* 66. The percentage of reinforcement corresponding to the proportion for a given ratio of stresses is sometimes called the " economic percentage ", but this may be a misnomer because the relative amounts of steel and concrete in the most economical beam or slab are dependent not only on the permissible stresses but also on the costs of the materials and shuttering.

Steel-beam Theory.—If the amount of compression reinforcement required equals or exceeds the amount of tensile reinforcement when using the formulæ in

Table 66, a beam may be designed by the "steel-beam theory" in which the compressive resistance of the concrete is neglected and

$$A_{st} = A_{sc} = \frac{M}{(1 - f_2)d_1 p_{st}}.$$

When this method is adopted the spacing of the binders should not exceed eight times the diameter of the bars forming the compression reinforcement, and p_{st} should not exceed 18,000 lb. per square inch. The indiscriminate application of the steel-beam theory is not recommended. At first sight it seems that a beam of any size can be designed to resist almost any bending moment irrespective of the compressive stress in the concrete. In fact with a theoretical compressive stress of 18,000 lb. per square inch in the reinforcement, the theoretical compressive stress in the concrete may exceed 1200 lb. per square inch, which for ordinary concrete leaves little margin for accidental overloading, differences between theoretical and actual bending moments and stresses, poor workmanship, and other factors. Partial safeguards against unreasonable use of the steel-beam theory include the provision of sufficient area of concrete to resist the shearing forces, the space required for the bars in the top and bottom of the beam, and the reduction in the lever-arm consequent upon large numbers of bars requiring more than one layer of reinforcement in the top and bottom. When A_{sc} approaches equality with A_{st} it is preferable to design by the load-factor method described on page 70.

Flanged Beams: Tee-beams, Ell-beams and I-beams.—If a beam is constructed monolithically with a slab, the slab forms the compression flange of the beam if the bending moment is such that compression is induced in the top of the beam. If the slab extends an equal distance on either side of the rib, that is the beam is a tee-beam, or if the slab extends on one side only of the rib, as in the case of an ell-beam (or inverted ell), the breadth of slab assumed to form the effective compression flange should not exceed the least of the dimensions given in *Table* 69.

There are two cases to consider, namely, where the neutral plane is within the thickness of the slab and where the neutral plane is below the slab. In the former case, a flanged beam is dealt with in the same way as a rectangular beam the breadth b of which is the effective width of the flange. If the neutral plane is below the slab, the small compressive resistance afforded by the concrete between the neutral plane and the underside of the slab is often neglected, and the corresponding formulæ in *Table* 68 apply. Note the approximate expression for the lever-arm in formulæ (6*b*) and (6*c*); this value is sufficiently accurate for most tee-beams and ell-beams.

It is not common for beams with compression flanges to require compression reinforcement, but if this is unavoidable the same principles apply as for rectangular beams. The theoretical formulæ in this case are too complex to be of practical value, although they may be of some use for I-beams, the design of which is dealt with on page 258.

Beams with Concrete effective in Tension.—In the design of liquid-containers, and some other structures, resistance to cracking of the concrete in the tension zone is important; therefore the members concerned are calculated taking the concrete as effective in tension. The corresponding formulæ for rectangular and flanged beams are given in *Table* 69.

Proportions and Details of Beams.—The dimensions of beams are primarily determined from considerations of the moment of resistance and resistance to shearing force, but beams having various ratios of depth to breadth may give the resistances

required. There are in practice several other factors that may affect the relative dimensions.

To limit the deflection, the depth of a rectangular beam should be in accordance with the limiting ratios given in *Table* 15. A rule for determining a trial section for a rectangular beam or tee-beam is that the total depth in inches should be equal to the clear span in feet. The breadth of a rectangular beam or the breadth of the rib of a tee-beam generally varies from one-third to one times the total depth; for rectangular beams in buildings a reasonable breadth is one-half to two-thirds of the total depth; in industrial structures beams having proportions of breadth to depth of one-half to one-third are often convenient. The lower ratio in each case applies principally to tee-beams. Much, however, depends upon the conditions controlling a structure, especially such factors as clearances below beams and the cross-sectional area required for resistance to shearing. The breadth of beams should also conform with widths of timber as commercially supplied. The cross-sectional dimensions of beams and columns and similar members should be multiples of 1½ in. where practicable (see B.S. No. 2539). In buildings the breadth of beams may have to comply with the nominal thicknesses of brick walls, that is 4½ in., 9 in., 13½ in., or 18 in. If the ratio of the span to the breadth of a beam exceeds 30, the permissible compressive stress must be reduced in accordance with the rules given on the page facing *Table* 62.

The breadth of the rib of a flanged beam is generally determined by the cross-sectional area required for resistance to the shearing force, but consideration must also be given to the accommodation of the reinforcement; a preliminary rule for determining the minimum breadth of rib in inches is 2½ times the number of bars in one layer. When the calculations have been completed the breadth should be checked to ensure that sufficient space is provided for the bars.

Methods of designing beams or determining the stresses in beams either by the use of the tables or by application of the formulæ, are given on the page facing *Table* 67. Examples of the use of the tables are given on the page facing *Tables* 68 and 69. Typical details and calculations for freely-supported and continuous beams are given in Appendix II.

Design of Solid Slabs: Modular-ratio Method.

A slab is generally calculated for a strip 1 ft. wide; hence a slab is equivalent to a rectangular beam with $b = 12$ in., and the moment of resistance and the bending moment are expressed per foot of width. The formulæ in *Table* 66 for rectangular beams apply to slabs, but as b has a constant value, the expressions can be modified to facilitate computation; for example, the effective depth and the area of reinforcement required can both be expressed as simple functions of the bending moment as given in *Table* 71. Notes and examples on the use of *Table* 71 are given on the page facing the table, which can be used for almost any permissible stresses.

When the tensile stresses are 18,000 lb., 20,000 lb., 27,000 lb., or 30,000 lb. per square inch and the compressive stresses are 750 lb., 1000 lb., or 1250 lb. per square inch, or in the same ratio as pairs of these stresses, suitable slabs can be selected from *Table* 72 in which are given the moments of resistance of, and area of reinforcement for, slabs 12 in. wide without compression reinforcement; examples are given on the page facing the table.

Common arrangements of, and notes on, the reinforcement in solid slabs are given on pages 266 and 268. It is not usual to provide compression reinforcement in slabs unless a convenient arrangement of reinforcement is obtained thereby, but

when it is necessary to do so the method of calculation is the same as that for rectangular beams. Links or other means to prevent the bars from buckling should be provided at centres not exceeding twelve times the diameter of the bars in compression; otherwise the bars in compression should be neglected when computing the resistance.

Reinforcement for resistance to shearing force is not generally necessary in slabs. Shearing stresses need not be considered unless the span is small.

Although slabs 2 in. thick can be constructed with fine concrete, for ordinary cast-insitu slabs 3 in. should be the minimum thickness. The minimum thickness of the slab should be not less than the limiting values given in *Table* 62.

Typical details and calculations for solid slabs are given in Appendix II.

Slabs in Liquid-containing Structures.

Tables 73 and 74 give data for the design of walls and other slabs in a structure containing water or other aqueous liquid in accordance with the recommendations of British Standard Code No. 2007 (1960). There are three principal cases to consider, namely, I. Slabs subjected to bending only; II. Slabs subjected to direct tension only; III. Slabs subjected to bending and direct tension. *Table* 73 deals with case I and *Table* 74 with cases II and III. The following conditions are applicable to all cases.

The tensile stress p_{st} in the reinforcement is not to exceed 12,000 lb. per square inch (except in tensile reinforcement near the face remote from the liquid for slabs not less than 9 in. thick). The nominal volumetric proportions of the concrete are 1 : 1·6 : 3·2. The compressive stress in the concrete is not to exceed 1200 lb. per square inch; this requirement is seldom critical. The tensile stress in the concrete is not to exceed 190 lb. per square inch in slabs in direct tension and 270 lb. per square inch in slabs in bending; this requirement applies when designing for resistance to cracking. The cover of concrete over the reinforcement is to be not less than $1\frac{1}{2}$ in. or the diameter of the bar if exceeding $1\frac{1}{2}$ in.

The basic formulæ and conditions from which the expressions and data in *Tables* 73 and 74 are derived and examples are given on pages 270, 272 and 274.

Case I. Slabs subjected to Bending Only.—Slabs subjected to bending have to be designed to resist cracking and, should cracking occur, they must be reinforced sufficiently and be sufficiently thick to avoid overstressing the reinforcement in tension or the concrete in compression. This basis of design applies to all slabs when the strain at the face in contact with the liquid is tensile, and to slabs less than 9 in. thick whether tensile strain occurs at the face in contact with, or remote from, the liquid. If in a slab not less than 9 in. thick, the tensile strain is at the face remote from the liquid, the method of design is as for an ordinary slab with the stress in the reinforcement not greater than 18,000 lb. per square inch in plain bars or 20,000 lb. per square inch in deformed bars, and the compressive stress in the concrete not greater than 1200 lb. per square inch.

Case II. Slabs subjected to Direct Tension Only.—Planar slabs subjected to direct tension only without bending are not common; a curved slab, such as the wall of a cylindrical tank, is the more common case. The two conditions of design are that the tensile stress p_{ct} in the concrete should not be so great as to cause cracking, and, should cracking occur, the reinforcement should be able to resist the whole of the tensile force without exceeding the permissible tensile stress. The first of these conditions determines the thickness of the slab. The second condition determines the amount of reinforcement.

Case III. Slabs subjected to Direct Tension combined with Bending.—The two cases are when the tensile strain is at the face in contact with the liquid, and when this face is subjected to compressive strain and the opposite face to tensile strain.

There are two conditions for the case of tensile strain at the face in contact with the liquid, namely, when tensile stresses only are produced, and when tensile and compressive stresses are produced. When there are *tensile stresses only*, which condition occurs when e does not exceed $d_1 - d\frac{1}{2}$, the case is like a slab subjected to direct tension only and therefore the tensile stress in the concrete must not exceed the stress permissible in direct tension if no cracking is to occur; the thickness of the slab is determined from this requirement. Should cracking occur, the permissible tensile stress in the reinforcement must not be exceeded; this condition determines the amount of reinforcement.

If there are *tensile and compressive stresses*, which condition occurs when e exceeds $d_1 - \frac{1}{2}d$, the case is analogous to a slab subjected to bending only, and therefore the limiting stresses for that condition apply. Since the effect of reinforcement in the compression zone of a slab is insignificant, such reinforcement, if any, can be neglected.

When the tensile strain is at the face remote from the liquid, there are two conditions to be considered, namely, when the slab is less than and not less than 9 in. thick. For the former case the design is exactly as in the immediately preceding case, taking into account whether there are tensile stresses only or tensile and compressive stresses.

Design of Beams and Slabs: Load-factor Method.

The load-factor, or ultimate-load, method is based on the behaviour of the member when near failure. The safe resistance under working conditions is a fraction of the ultimate resistance, say, one-third if the crushing strength of the concrete determines the ultimate resistance, or, say, one-half if the yield stress of the reinforcement is decisive. Formulæ and factors for these conditions for beams and solid slabs subjected to bending are given in *Tables* 75 to 79, and are based on the recommendations in B.S. Code No. 114. The notation is given on the page facing *Table* 75, and the basis of the tables and examples on their use are given on the pages facing *Tables* 75 to 79. The basic formulæ for the moment of resistance and basic design formulæ for beams with or without compression reinforcement are given in *Table* 75, together with the corresponding formulæ simplified for cases in which ordinary 1 : 2 : 4 concrete is used and mild steel bars or high-yield-stress bars are provided in beams and solid slabs. Values of some of the factors used in these formulæ are given in *Table* 76. Numerical values of moments of resistance and areas of reinforcement for common conditions for rectangular beams with mild steel reinforcement are given in *Table* 77A, and with high-yield-stress bars in *Table* 77B; similar data for solid slabs are given in *Table* 78, and for flanged beams in *Table* 79.

Deflection of Beams.

The deflection of reinforced concrete members cannot be calculated with precision, but this is not of serious consequence since in most cases comparative deflections only are required, and the indefinite numerical values offset each other to a large extent. If the amount of the actual maximum deflection of a beam is required to be known,

it can be found approximately from $\frac{WL^3K}{E_cI_c}$, where W is the total load on the beam, L is the span, E_c is the modulus of elasticity of concrete in compression, I_c is the equivalent moment of inertia, and K is the coefficient of deflection depending on the type of load and the conditions at the supports of the beam; values of K for some common loads are given in *Tables* 16, 17 and 17A. If all the terms are in inches and pounds, the deflection will be in inches.

An appropriate value for E_c is 3,750,000 lb. per square inch, corresponding to a modular ratio of 8, but if a more accurate value can be obtained from tests it should be used. The moment of inertia should be expressed in concrete units and should be that at the centre of the span, that is, at the position of the maximum positive bending moment. The moment of inertia in this instance should be computed for the whole area of the concrete within the effective depth, that is, the area of concrete between the neutral plane and the tensile reinforcement should be included as well as the area of the concrete above the neutral plane. The area of the tensile and compression reinforcement should be allowed for by transforming to the equivalent additional area of concrete, that is, the effective modular ratio [say $(8 - 1) = 7$] multiplied by the area of the reinforcement. The moment of inertia should be taken about the centroid of the transformed area, and is approximately $\frac{(1 + 4mr_t)bd_1^3}{12(1 + mr_t)}$ for a rectangular beam reinforced in tension only, the proportion of tensile reinforcement being r_t. The corresponding expressions for rectangular beams with compression reinforcement, and for tee-beams, are so complex that it is easier to calculate I_c with actual numerical values from first principles, or use the data in *Tables* 65 and 65A or the formulæ on page 352.

Data for limiting the deflection of beams and solid slabs are given in *Table* 15.

Resistance to Shearing Force.

Shearing Stresses.—Shearing force produces diagonal tensile stresses in the concrete and, if the stresses exceed the safe tensile stress in the concrete, reinforcement should be provided in the form of either inclined bars or binders, or both, to provide the shearing resistance. The shearing stress q in a member subjected to a bending moment or a shearing force Q acting alone is calculated from the formulæ given on the page facing *Table* 80. If the beam is of uniform cross-section, the shearing force, and therefore the shearing stress, is independent of the bending moment, except in so far that shearing force is the rate of change of bending moment. Values of the safe shearing stresses with or without reinforcement are given in *Tables* 56 and 57.

The distribution of shearing stress is such that at any point in a beam there exists a horizontal shearing stress equal in intensity to the vertical shearing stress at the point. In a plain rectangular concrete beam of uniform cross-section, the mean intensity of the shearing stress varies parabolically from nothing at the top and bottom edges to a maximum of $1{\cdot}5\frac{Q}{bd}$ at the mid-depth of the beam. In a reinforced concrete beam in which the concrete in tension below the neutral plane is ignored, there is a parabolic increase from nothing at the top edge to $\frac{Q}{l_ab}$ at the neutral plane, and this stress q is constant from the neutral plane to the centre of the tensile reinforcement. Shearing stresses due to torsion on a rectangular section attain maximum values at the middle of the longer sides.

Without assistance from reinforcement the concrete alone is sometimes considered as capable of resisting the total shearing force when the shearing stress q is not greater than the safe shearing stress. The Ministry of Transport permits this rule to be applied to slabs, but for beams in bridges only one-third of the shearing force may be resisted by the concrete, the remainder being resisted by reinforcement. In industrial structures it is preferable to resist the entire shearing force by reinforcement in the form of inclined bars, and to provide resistance to shearing forces even when the calculations for ordinary loading indicate that the shearing stresses are small; by so doing, some provision is made for abnormal loads to which beams in such structures may be subjected. The B.S. Code recommends that in narrow beams (as defined on page 246), the entire shearing force should be resisted by reinforcement whatever the shearing stress.

If reinforcement is provided to resist the whole of the shearing force the stress q should not exceed four times the safe shearing stress; this is in accordance with the B.S. Code and London By-laws. Generally, however, this extreme value of q should not be used and a limit of 2½ times the safe shearing stress is preferable for secondary beams which may be subjected to greater incidental loads, although the higher limit might be used for main beams in buildings (not warehouses) if the full design load is not likely to occur.

Reinforcement to resist Shearing Forces.—The reinforcement provided to resist shearing forces may be in the form of vertical binders or inclined bars. The shearing resistance of such reinforcement is given in *Table* 81 for tensile stresses of 18,000 lb. and 20,000 lb. per square inch; the resistances at other stresses are proportional. In some cases, such as beams subjected to vibration and impact, the stress in the reinforcement provided to resist shearing forces should be less than 18,000 lb. per square inch, say, 12,000 lb. per square inch, and binders of small diameter closely spaced are preferable. In liquid-containing structures (B.S. Code No. 2007) the stress should not exceed 12,000 lb. per square inch, that is the resistances are two-thirds of those for 18,000 lb. per square inch in *Table* 81. B.S. Code No. 114 recommends a limiting tensile stress of 20,000 lb. per square inch in shearing reinforcement, comprising mild steel or high-yield-stress bars, in buildings and not more than 18,000 lb. per square inch in mild steel bars greater than 1½ in. in diameter.

Notes on the spacing, diameter, and shape of binders are given in *Table* 80 and on the page facing the table.

The principle assumed in evaluating the shearing resistance of inclined bars is that the bars form the tension members of a lattice girder, the inclined compression in the concrete providing the corresponding compression member. The shearing resistance at any section is the sum of the vertical components of all the inclined bars and compression "struts" cut by the section. Notes on the arrangement of inclined bars as affecting the stresses therein are given on pages 288 and 289. The resistances in *Table* 81 are for bars from ½ in. to 1½ in. in diameter inclined at 45 deg. and 30 deg.

Inclined bars are commonly provided by bending up the main tensile reinforcement, but in so doing inspection must be made to ensure that the bar is not required to assist in providing the moment of resistance beyond the point at which it is bent. The points at which bars can be dispensed with as reinforcement against bending are given in *Table* 69, which applies to beams having up to eight bars as the principal tensile reinforcement. Although a bar can be bent up at the points indicated, it is

not implied that if it is not bent up it can stop at these points, since it may not have sufficient length of bond from the point of critical stress. This length depends on the rate of change of bending moment, and should be investigated in any particular beam. Examples of the calculation of the shearing resistance at any section of a beam and the design of a member to resist shearing force are given on the page facing *Table* 81.

Torsion.

The design of reinforced concrete members subject to twisting moments is based largely on tests. The data in *Table* 82 are derived from the results of tests made by Mr. Leslie Turner and Mr. V. C. Davies.

Members subject to twisting moments should not have re-entrant angles. In designing for ordinary shearing force and bending moment, lower stresses than usual should be adopted, thereby leaving a margin for the secondary torsional stresses. The moment of resistance to torsion of a reinforced member can be expressed as RD^3q, where D is the equivalent diameter (in.), and q is the safe shearing stress (lb. per square inch). R is a section-coefficient the value of which depends upon the percentage of reinforcement and the shape of the member; values are given in *Table* 82 for the shapes most commonly used. The equivalent diameter D is the diameter of the largest circle that can be inscribed within the solid section. If the member is subjected to a bending moment or a shearing force that causes the concrete below the neutral plane to crack, the area of the cracked concrete should be ignored when assessing the shape of the cross-section and the value of D. The amount of reinforcement is equal to the sum of the amounts of longitudinal reinforcement and binding inserted solely to resist torsion. Only longitudinal bars near the outer faces of the section can be considered as effective. It is common to provide, for this purpose, equal amounts of binders and longitudinal bars, since lateral binding is of considerable value for resisting torsion especially if arranged diagonally.

The method of designing a member to resist torsion is to design first for the ordinary bending moments and shearing forces (using reduced stresses) and to determine D for this section. The value of q given in *Table* 56 or 57 is then used to determine the section-coefficient from $R = \frac{T}{D^3q}$, where T is the applied twisting moment. The amount of reinforcement required for the appropriate shape can be determined from *Table* 82 and should be divided equally between lateral binders and longitudinal bars. The combination of torsional shearing stress with the shearing stress due to the ordinary shearing force is complex and, failing the adoption of a more accurate analysis, the designer should assure himself that, at any section reinforced for shearing force, the numerical sum of the two stresses does not exceed a reasonable value, say, not more than twice the shearing stress commonly allowed. Consider the numerical example given on the page facing *Table* 82, in which at the point of contraflexure (approximately at the fifth-point of span) the shearing stress due to the ordinary shearing force is 54 lb. per square inch and the torsional shearing stress does not exceed 100 lb. per square inch; thus the sum of the two shearing stresses does not exceed 154 lb. per square inch which is less than twice 100 lb. per square inch.

Arcate Beams (Bow Girders).

Bow girders and beams that are not rectilinear in plan are subjected to twisting moments in addition to the normal bending moments and shearing forces. Beams forming a circular arc in plan may form part of a complete circular system supported

on columns equally spaced and each span equally loaded such as occurs in water towers, silos, and similar cylindrical structures. The equivalent of these conditions also occurs if the circle is incomplete so long as the appropriate negative bending moment can be developed at the end supports. This type of circular beam occurs in structures such as balconies.

The method of designing for twisting moments as given in the foregoing and in *Table* 82 applies. The maximum twisting moment occurs at the point of contraflexure; therefore the design of the section in torsion can be based on the entire section, assuming no cracking of the concrete due to tensile stresses caused by bending or shearing force. The position of the point of contraflexure expressed in terms of the angular distance from the support is also given in *Table* 82. The maximum shearing force, equal in value to half the total load on the beam, occurs at the edge of the support. The maximum positive bending moment occurs midway between the supports; it is preferable to design for a positive bending moment in excess of the theoretical value, say, half that of the negative bending moment at the support.

The critical design sections are therefore: (*a*) Midspan—maximum positive bending moment and zero shearing force and zero twisting moment; (*b*) Point of contraflexure—zero bending moment, maximum negative twisting moment combined with shearing force; (*c*) Support—maximum negative bending moment, maximum shearing force, and maximum positive twisting moment.

An analysis of curved beams allows for the torsional rigidity of the section and gives the following expressions for the twisting and bending moment at any point X in a beam carrying a uniformly-distributed load.

$$M = M_S \cos\theta_X + T_S \sin\theta_X - F_S R \sin\theta_X + wR^2(1 - \cos\theta_X)$$
$$T = -M_S \sin\theta_X + T_S \cos\theta_X + F_S R(1 - \cos\theta_X) - wR^2(1 - \sin\theta_X)$$

where M_S, T_S, and F_S are respectively the bending moment, twisting moment, and shearing force at support S, R is the radius of the beam, and θ_X is the angle defining the position of the point X, as shown in the diagram on *Table* 82.

If a curved beam is subjected to a concentrated load P such that the angle between the radii through the point of application of P and support S is α, the following expressions apply at any point X if $\theta_X > \alpha$.

$$M = M_S \cos\theta_X + T_S \sin\theta_X - F_S R \sin\theta_X + PR \sin(\theta_X - \alpha)$$
$$T = -M_S \sin\theta_X + T_S \cos\theta_X + F_S R(1 - \cos\theta_X) - PR[1 - \cos(\theta_X - \alpha)].$$

If $\theta_X < \alpha$, the terms containing PR equal zero.

For a number of common cases of circular beams, formulæ and coefficients for evaluating M_S, T_S, and F_S are given in *Table* 82, together with the corresponding factors for the bending moment, twisting moment, and shearing force at mid-span and at the point of contraflexure. The general formulæ given in the foregoing are derived from the analyses given by Professor A. J. S. Pippard and Professor J. F. Baker, and the coefficients and expressions in *Table* 82 are based on the development (as subsequently modified) by Professor W. T. Marshall and Dr. G. G. Meyerhof.

Safe Loads on Short Columns.

The imposed loads for which columns in buildings should be designed are the same as those for beams as given in *Table* 3, except that the minimum load does not apply. The imposed load on the floors supported by the columns may be reduced (see *Table* 3) when calculating the load on the column in accordance with the scale

given for multiple-story buildings. External columns in buildings, and internal columns under certain conditions, should be designed to resist the bending moments due to the restraint at the ends of beams framing into the columns, and due to wind (see *Tables* 46 and 51). An approximate method of allowing for the bending moment on a column forming part of a building frame is to design for a concentric load F times the actual load, where F is as given on page 228 for different arrangements of beams framing into the column; so many factors affect the value of F that the tabulated values can be only approximate and the final design must be checked by more accurate calculation.

The working stress for which a column should be designed is the direct compressive stress p_{cc} (*Tables* 56 and 57) when the column is subjected to direct load only. When the column is subjected to bending in addition to the load, the stresses should be determined in accordance with the methods described in *Tables* 85 to 88, and the compressive stress in the concrete should not exceed the permissible compressive stress p_{cb} in bending (*Tables* 56 and 57). Columns subject to bending should, however, be checked to ensure that under the load assumed to be acting alone, the permissible direct compressive stress is not exceeded. The stresses in the foregoing may be exceeded by not more than one-third (London By-laws), or one-quarter (B.S. Code), if the effects of wind are included in the calculation of the load and the bending moment, but the stress p_{cc} or p_{cb} must not be exceeded when these effects are excluded. The modifications to specified working stresses as given on page 57 should be considered when assessing the safe working stress in a column. When the load on the column is ascertained approximately the stress in the concrete should be well below the permissible stress unless it is known for certain that the approximate load has been over-estimated. When the column loads are carefully calculated and the elastic reactions from the beams are taken into account, the calculated stress can be increased to the permissible stress. The foregoing rules apply to " short " columns. The reduction of the safe load on slender columns is discussed below.

Reinforced concrete columns are generally either rectangular in cross-section with separate binders, or circular or octagonal with helical binding. In some multiple-story residential buildings columns of ell-shape or tee-shape are formed at the intersection of reinforced concrete walls. In most reinforced concrete columns the main vertical bars are secured together by means of separate links or binders. Rules for the arrangement of such binding, the limiting amount of main reinforcement and formulæ for the safe load on such columns are given in *Table* 83; safe loads on square columns with separate binders are given in *Table* 84.

By forming the binding in a column in the form of a continuous helix instead of separate binders, the safe load on the column is increased. The form of the binding must be circular, or nearly so on plan. The conditions given in *Table* 82 and on the page facing the table conform to the B.S. Code.

Columns with helically-bound cores are either square, octagonal, or circular; generally an octagonal section is the most economical, since the shuttering is less costly than for a circular column and there is less ineffective concrete in the corners than in a square column. The minimum outside size of the column is about 2 in. more than the diameter of the bound core. Although helically-bound columns are not necessarily the most economical form of column construction, the extra cost is mostly offset by the advantages arising from the extra available floor space and reduced dead weight.

So many variants enter into the design of a column that it is not easy readily to decide which combination gives the most economical member. For a short column carrying a load exceeding 100 tons, the following may apply.

Other things being equal, the richer the concrete the more economical is the column. For a square column, the minimum amount of longitudinal reinforcement produces the cheapest member for a specified quality of concrete. Also, for any concrete a square column is generally less costly than an octagonal column with helical binding. Taking eight designs of columns to carry loads from 100 tons to 500 tons, the order of economy is generally as follows, the most economical design being the first: 1 : 1 : 2 concrete, square column with minimum vertical steel; 1 : 1 : 2 concrete, octagonal column with maximum volume of helical binders and minimum area of vertical reinforcement; 1 : 1½ : 3 concrete, square column with minimum area of vertical reinforcement; 1 : 1½ : 3 concrete, octagonal column with maximum volume of helical binders and minimum area of vertical reinforcement; 1 : 2 : 4 concrete, square column with minimum area of vertical reinforcement; 1 : 2 : 4 concrete, octagonal column with maximum volume of helical binders and minimum area of vertical reinforcement; 1 : 2 : 4 concrete, octagonal column with maximum volume of helical binders and maximum area of vertical reinforcement; 1 : 2 : 4 concrete, square column with maximum area of vertical reinforcement.

Long Columns.

If the ratio of the effective length of a column to the least radius of gyration exceeds about 50, the column is a "long" or slender column and the safe load on the column is less than that on a "short" column. For square and rectangular columns it is more convenient to calculate the slenderness ratio on the least lateral dimension (provided that the section has no re-entrant angles) instead of on the radius of gyration. Formulæ and data relating to both bases of determining the reduction of the concentric safe load on "long" columns are given in *Table* 83 and on the page facing *Table* 84, on which page also the effective length compared with the actual length of the column is considered.

Combined Stresses.

The stresses in structural members such as arches, walls of rectangular containers, columns subject to eccentric load, and chimneys, are due to the combined effect of a bending moment and a direct force, which may be either a pull or a thrust. The method of determining the magnitude and distribution of the stress depends on the nature of the direct force and the relative magnitudes of the bending moment and the force. There are three principal cases: (i) When the direct force is a thrust and the resultant stresses are wholly compressive; (ii) When the direct force is a pull and the resultant stresses are wholly tensile; (iii) When the direct force is either a pull or a thrust and both tensile and compressive forces are produced.

The analysis of the stresses for these cases is generally based on a method analogous to the elastic modular-ratio methods of analysing beams and columns subjected to concentric loads; this method is accepted by the B.S. Code and is the basis of *Tables* 85, 86, 87 and 89. The Code also describes a load-factor method of analysis, which is given in *Table* 88 and described on page 83.

The permissible stresses in members subjected to bending and compression are those permitted for bending only. In long columns subjected to bending, the per-

missible stresses must be reduced by the factor R_L given in *Table* 83, but no reduction need be made for stresses within one-eighth of the length of the column from either end. In slender beams subjected to axial thrust, the reduction factors should be in accordance with the rules in *Table* 57 and on page 246.

The effect of a bending moment M and a direct force N acting simultaneously is equivalent to the direct force N acting at a distance e from the centroid of the stressed area where $e = \frac{M}{N}$. The eccentricity e is sometimes measured from the centroid of the concrete section and, except in case (iii) if the eccentricity is small, the error involved by this approximation is small. In some problems the eccentricity of the load about one face of the member is known, and before the stresses can be determined this eccentricity must be converted to that about the centroid of the stressed area (or of the concrete section).

The value of e relative to the dimensions of the member determines into which of the three cases the problem falls. For problems in case (i) the maximum and minimum compressive stresses are calculated by adding and subtracting respectively the stresses due to the direct force alone and to the bending moment alone. The limit of this case is reached when the tensile stress produced by the bending moment alone (assuming that the whole of the concrete and the reinforcement are fully effective) is equal to the compressive stress due to a concentric load N. For a rectangular section this limiting condition is reached when $\frac{e}{d}$, d being the total depth of the member, is 0·167 for a section with no reinforcement and up to about 0·3 with large percentages of reinforcement. As a small tensile stress may be permitted in the concrete in some cases, an upper limit for $\frac{e}{d}$ may be about 0·5. If no tensile stress is permitted in the concrete the limiting value of e is $\frac{Z}{A}$, where A is the effective area of the section expressed in concrete units and Z is the modulus of the effective section (also expressed in concrete units) measured about the axis passing through the centroid of the equivalent section. Expressions for the effective area and section modulus of reinforced concrete sections subjected to stress over the entire section are given in *Tables* 65A and 65B; these expressions take into account the reinforcement; for preliminary approximate calculations it may not always be necessary to allow for the reinforcement, in which case the expressions in *Table* 64 apply.

When N is a pull and the stresses are entirely tensile, the problem is in case (ii) when $\frac{e}{l_{as}}$ is less than 0·5, where l_{as} is the distance between the centroids of the reinforcement near opposite faces of a reinforced section, the tensile resistance of the concrete being entirely neglected.

When case (i) is applied to a problem in which N is a thrust and an excessive tensile stress is produced in the concrete, or when case (ii) is applied to a problem with N a pull and compressive stresses are produced, the problem is in case (iii). Various methods of calculating the stresses for this case have been devised. Any direct method is complicated since an exact analysis involves the solution of a cubic equation, and rapid computation necessitates an impracticably large number of graphs or tables if provision is to be made for all the probable variations of the terms in the equation. In the method given in *Tables* 85 and 86 the depth to the neutral plane is assumed first; the assumed depth is later checked and adjusted.

For rectangular sections, or sections capable of being reduced to equivalent

rectangles, the notation is as indicated in *Table* 86, and for an irregular section the notation is as shown in *Table* 85. When there is no compression reinforcement the term A_{sc} in the formulæ is zero and simplifications consequently follow. An abstract of the methods of determining the stresses in a rectangular member subject to a bending combined with a direct thrust is given in *Table* 86, together with the values of some of the terms involved in the calculation. For values of $\frac{e}{d}$ exceeding, say, 1·5, an approximate method can be used that gives the stresses with sufficient accuracy.

Rectangular Section subjected to Bending and Thrust (Modular-ratio Method).

When e does not exceed $\frac{1}{6}d$.—In this case, with any amount of reinforcement, compressive stresses only are developed and the maximum and minimum values are given by the formula in *Table* 86. The expression for the section modulus is correct if A_{sc} equals A_{st} and is approximately correct if A_{sc} differs from A_{st}. For more accurate expressions, see *Table* 65A. The design of a section for this case involves the assumption of trial dimensions and reinforcement. For the special case of $m = 15$ and $A_{sc} = A_{st}$ the factors given in *Table* 87 can be used, the method being described on the page facing the table.

When e is greater than $\frac{1}{6}d$ *and less than* $\frac{1}{2}d$.—With no reinforcement, tension is developed in one face of the member when e exceeds $\frac{1}{6}d$, but as the proportion of reinforcement increases the ratio of e to d also increases before tensile stresses are developed. The limiting value of $\frac{e}{d}$ depends on the amount of A_{sc} and A_{st} and the relative values of d_2, d_1, and d. Cases where $\frac{e}{d}$ lies between $\frac{1}{6}$ and $\frac{1}{2}$ should be first calculated as if $\frac{e}{d}$ did not exceed $\frac{1}{6}$, and if no tensile stress is shown to be developed the stresses calculated by this method are the theoretical stresses. Even if a small tensile stress is developed, treatment as in the preceding paragraph is generally justified so long as the tensile stress in the concrete for the worst combination of M and N does not exceed about one-tenth of the allowable compressive stress. If the tensile stress exceeds this amount the tensile resistance of the concrete should be ignored, and the stresses calculated as in the next paragraph.

When e is greater than $\frac{1}{2}d_1$ *and less than* $1\frac{1}{2}d_1$.—This is the general case, when tension in the concrete is ignored, and the method given in *Table* 86 is applicable to members with or without compression reinforcement and with any value of d_2 and any modular ratio.

The first step is to select a trial position for the neutral plane by assuming n_1, the neutral-plane factor, and then calculating the maximum stresses f_{cb} and f_{st} in the concrete and reinforcement respectively from the formulæ given in *Table* 86, in which also are given the expressions for the factors and some numerical values of the factors. The term $\bar{x}$ is the distance from the compressive edge of the section to the centroid of the stressed area. Since $\bar{x}$ may be very nearly equal to $\frac{1}{2}d$ it is sufficiently accurate in the first trial calculation to assume this value, but for the second or final calculation, $\bar{x}$ should be determined from the appropriate expression.

The value of n_1 obtained by substituting the calculated values of f_{cb} and f_{st} in $n_1 = \dfrac{1}{1 + \dfrac{f_{st}}{mf_{cb}}}$, should coincide with or be very nearly equal to the trial value of n_1.

If in the first trial there is a difference between the two values of n_1, the factors F, G, H and J should be recalculated with a second trial value of n_1 and the recalculated values of f_{cb} and f_{st} should give a satisfactory value of n_1. Values of n_1 for various values of $\frac{f_{st}}{f_{cb}}$ for different modular ratios and values of G and H are given in *Table* 86, and in *Table* 68 also values of n_1 for $m = 15$ are given; see also *Table* 67.

When the member is reinforced in tension only, $H = 0$ and the formulæ for the stresses are

$$f_{cb} = \frac{NF}{Gbd_1} \quad \text{and} \quad f_{st} = \frac{f_{cb}J - N}{A_{st}}.$$

For the special case of $m = 15$ and $A_{sc} = A_{st}$, the stresses can be obtained approximately from the factors given in *Table* 87.

A member which does not generally require compression reinforcement can be designed by first assuming a value for d_1 (and therefore d) and calculating the breadth required from $b = \frac{NF}{p_{cb}d_1G}$, in which G is calculated from the value of n_1 corresponding to the permissible stresses p_{cb} and p_{st} or taken from *Table* 86. The area of tensile reinforcement required is given by $\frac{p_{cb}J - N}{p_{st}}$. If the value of b thus obtained is unsuitable, another value of d_1 may give suitable proportions. (For a slab, $b = 12$ in. if N and M are expressed in terms of one foot of width.) If suitable proportions cannot be obtained in this way, a convenient section may be found by reducing the stress in the tensile reinforcement, thereby increasing the area of concrete in compression, or by adding compression reinforcement, or by combining both methods.

If reinforcement is added to increase the compressive resistance, or if the member is such that ordinary design or other considerations require the provision of compression reinforcement (for example, columns, piles, the support section of beams, and members subject to reversal of flexure), it is necessary to assume (or determine from other considerations) suitable values of b as well as d_1. With these values, and with the ratio of the allowable stresses in the tensile reinforcement and the concrete, the factors F, H, J and G can be calculated or read from *Table* 86. The amount of compression reinforcement required is given by

$$A_{sc} = \frac{1}{H(1 - f_2)}\left(\frac{NF}{p_{cb}} - bd_1 G\right).$$

The area of tensile reinforcement required is given by

$$A_{st} = \frac{p_{cb}(J + HA_{sc}) - N}{p_{st}}.$$

In evaluating F, the value of $\bar{x}$ may be assumed to be $\frac{1}{2}d$, but in important members the stresses should be checked using the calculated value of $\bar{x}$.

If the calculated value of A_{sc} exceeds A_{st} both values should be adjusted by reducing the tensile stress or by modifying the dimensions of the concrete.

When e is greater than $1\frac{1}{2}d_1$.—When the eccentricity of the thrust is large compared with the dimensions of the member the stresses are primarily determined by the bending moment, the thrust producing only a secondary modification. In this case the stresses should first be calculated for the bending moment acting alone as described on the pages facing *Tables* 66 and 67. The combined stresses can then be determined approximately by adding a stress f_c to the maximum compressive stress

in the concrete, and by deducting mf_c from the tensile stress in the reinforcement where f_c is given by the formula at the bottom of *Table* 86.

Examples of the use of *Table* 86 are given on the page facing the table.

Any Section subjected to Bending and Thrust (Modular-ratio Method).

Compressive Stresses Only.—The first step in the determination of the stresses when the value of $\frac{M}{N}$ is small is to determine the equivalent area A_e and the moment of inertia I_X of the section about an axis passing through the centroid as given by the expressions at the top of *Table* 85; an irregular section is divided into a number of narrow strips as in the diagram. The maximum and minimum compressive stresses are obtained from the appropriate formulæ in the table. The limit of this case is when $f_{cb(min.)} = 0$. A small negative value of $f_{cb(min.)}$ may be permissible if this tensile stress does not exceed, say, one-tenth of the permissible compressive stress.

If the section is symmetrically reinforced and is rectangular (bending about a diagonal), circular, octagonal, or has any of the symmetrical shapes given in *Table* 64, the area of A and the modulus Z_c of the concrete section can be obtained from the data given in the table. The additional area A_A and the additional modulus Z_A due to the reinforcement are given by $A_A = (m - 1)\Sigma\delta A_c$ and $Z_A = \frac{2(m - 1)}{D}\Sigma h^2.\delta A_c$, where δA_c is the area of a bar or group of bars placed at a distance h from the centroid of the section. Thus $A_e = A_c + A_A$ and $Z = Z_c + Z_A$; the maximum and minimum compressive stresses in the concrete are given by $\frac{N}{A_e} \pm \frac{M}{Z}$. The limit for this case is when $\frac{M}{N} = \frac{Z}{A_e}$. For other common sections the expressions for the effective area and modulus in *Tables* 65A and 65B can be used.

Compressive and Tensile Stresses.—When the stress $f_{cb(min.)}$ determined in the preceding paragraph is negative or exceeds the permissible tensile stress, or when e is so large compared with D that the simultaneous production of compressive and tensile stresses can be assumed at the outset, the total tension should be resisted by the reinforcement only. In this case it is necessary to select a trial position of the neutral plane, either after consideration of the maximum permissible stresses or otherwise, and to plot the plane on a diagram of the section drawn to scale, as indicated in the diagram in *Table* 85. Next find the position of the centre of tension below the top edge of the section. The next stage is to divide the compression area above the neutral plane into a number of narrow horizontal strips. The depth δx of each of these strips need not be the same, as any regularity in the conformation of the section may suggest convenient subdivisions. When the strips are of equal depth, or when the section is symmetrical or hollow, simplifications should be readily perceived. For each strip determine the factors a and x_n. The position of the centre of compression below the top edge can then be found. The distance x of the centroid of the stressed area below the compressed edge of the section can now be evaluated, and the maximum compressive and tensile stresses can be calculated from the formulæ in *Table* 85. The value of n_1 corresponding to these stresses should be compared with the assumed value, and if necessary a second trial should be made. The values of p_a and a for individual bars or groups of bars and for individual compression strips are not affected by the value of n_1. An example of the application of this method is given at the bottom of *Table* 85.

Bending combined with Tension (Modular-ratio Method).

When e is less than $l_{as} - \bar{x}'$.—If the distance between the centroids of the reinforcement on opposite faces of any member is l_{as}, and if e is measured about the centroid of the combined reinforcement, as shown in the diagram at the top of *Table* 89, then if e does not exceed $l_{as} - \bar{x}'$, the stresses are wholly tensile. The average stresses in the group of bars near the face nearer the line of action of N and in the group of bars near the face remote from the line of action of N are given by the formulæ for f_{st1} and f_{st2} respectively. The maximum stress in a bar depends upon the distance of the farthest bar in any group from the centroid of that group, and is given by the formula for $f_{st1(max.)}$ in the table. The expressions for f_{st1} and f_{st2} can be re-arranged to give the areas of reinforcement required for a specified permissible stress.

Simplified formulæ are given in *Table* 89 for this case for regular sections, such as rectangular sections in which the bars are in two rows only. Further simplification obtains if the area of the bars in each row are equal, as also given in *Table* 89.

Rectangular Section: When e is greater than $l_{as} - \bar{x}'$ and less than $1\frac{1}{2}d_1$.—This is the general case, and the method of treatment is similar to that given previously for combined bending moment and direct thrust, modifications being introduced to allow for the difference between a direct thrust and a direct pull as given in the lower part of *Table* 89; the factors J, G, and H can be obtained from *Table* 86.

When the section is reinforced in tension only, $H = 0$ and the formulæ for the maximum stresses are

$$f_{cb} = \frac{NL}{bd_1G} \quad \text{and} \quad f_{st} = \frac{f_{cb}J + N}{A_{st}}.$$

When designing a member to resist a bending moment and a direct tension an approximate method is as follows. If compression reinforcement is not likely to be required, assume d_1 (and d) and determine the minimum breadth from $b = \frac{NL}{p_{cb}d_1G}$, where G is calculated (or read from *Table* 86) from the value of n_1 corresponding to the permissible stresses. If the value of b is unsatisfactory, d_1 should be adjusted or compression reinforcement added. The area of the tension reinforcement required is given by $A_{st} = \frac{p_{cb}J + N}{p_{st}}$.

For a singly-reinforced slab subject to a bending moment and a direct tension, such as the wall or bottom of a tank or bunker, a simple approximate procedure is given at the bottom of *Table* 89. Determine the eccentricity e_s of the line of action of the direct tension from the centre of the tensile reinforcement. The total tensile reinforcement required is given by $\frac{N}{p_{st}}\left(\frac{e_s}{l_a} + 1\right)$. The value of d_1 (and d) is that required to resist the bending moment acting alone, and the value of the lever arm l_a is that corresponding to the permissible stresses.

In designing a member in which compression reinforcement is required, first assume or otherwise determine suitable values of b and d_1, and with these values and the allowable maximum stresses determine the area of compression reinforcement from $A_{sc} = \frac{1}{Hl_{as}}\left(\frac{NL}{p_{cb}} - bd_1G\right)$. The area of tensile reinforcement is found from $A_{st} = \frac{p_{cb}(J + HA_{sc}) + N}{p_{st}}$. For this method L can be based on $\bar{x} = \frac{1}{2}d$, but in important members the stresses should be checked by using the calculated value of $\bar{x}$.

If it is necessary to reduce the amount of compression reinforcement, this can

often be effected by reducing f_{st} and thus increasing n_1. Generally in problems of bending and direct tension the tension is the deciding factor, and a more economical member can be obtained by decreasing the stress in the concrete.

Rectangular Section: When e is greater than $1\frac{1}{2}d_1$.—The determination of the approximate stresses in this case is similar to that described for the corresponding case for combined bending moment and direct thrust. The stresses are first computed for the bending moment acting alone. The next step is to evaluate f_c and deduct f_c from the stress in the concrete and add mf_c to the tensile stress in the reinforcement to obtain the maximum stresses in the concrete and steel, f_c being obtained from the expression given for this case near the bottom of *Table* 89.

To design a member, such as a slab with tensile reinforcement only, the following approximate method is applicable. The depth or thickness d, and the breadth b in the case of a beam, are determined for the bending moment acting alone. Evaluate the eccentricity e_s about the tensile reinforcement. The area of tensile reinforcement required is given by substitution in the formula given at the bottom of *Table* 89 in which l_a is the lever-arm of the section designed for bending only.

Any Section: Compressive and Tensile Stresses.—With the modifications necessary to allow for N being a pull instead of a thrust, the method given for the corresponding case for combined bending moment and direct thrust can be applied to the determination of the stresses on any section that cannot be treated as rectangular. A trial position for the neutral plane is taken and the part of the section above the neutral plane is divided into a number of narrow horizontal strips as in the diagram in *Table* 89. The values of R, p_t, $\bar{x}$, p_c, x_n and a are determined and substituted in the formulæ for the maximum stresses given in the table. If the value of d_n corresponding to these stresses is approximately equal to that assumed, the stresses are approximately the maximum stresses produced by the applied bending moment and direct tension. If the difference between the calculated and trial values of d_n is great, a second trial value must be selected and the summations revised by taking in a greater or smaller number of strips to accord with the different value of d_n.

Position of the Neutral Plane.

The accuracy of the results of some of the methods described in the foregoing and the labour entailed in arriving at these results depend upon the accuracy with which the position of the neutral plane is selected. From a consideration of the member and the forces acting upon it, it is possible to assume a value for d_n very close to that corresponding to the calculated stresses. The maximum stresses for which the section has been designed may indicate a reasonable value of d_n for the first trial, or consideration can be given to the ratio of stresses for bending only as determined by the proportion of tensile reinforcement. The selected value of d_n should differ from the value for bending alone in accordance with the following rules. For bending and compression, the selected value of d_n should be greater than the value for bending alone, the difference increasing as $\frac{e}{D}$ or $\frac{e}{d}$ or $\frac{e}{d_1}$ decreases. For bending and tension, the selected value of d_n should be less than the value for bending alone, the difference decreasing as $\frac{e}{D}$ or $\frac{e}{d}$ or $\frac{e}{d_1}$ increases. If the difference between the first assumed value, say d_{n1}, and the value corresponding to the calculated stresses, say d_{ns}, is such that it is necessary to select another value, say d_{n2}, intermediate between d_{n1} and d_{ns}, the following considerations apply. For bending and com-

pression, the value of d_{n2} should be nearer d_{ns} than it is to d_{n1}. For bending and tension, the value of d_{n2} should be nearer d_{n1} than it is to d_{ns}.

Columns subjected to Bending: Load-factor Method.

A load-factor method of designing columns subjected to bending, which is applicable to all similar members, is recommended in B.S Code No. 114, and formulæ and procedures based on this method are given in *Table* 88 and examples are given on the page facing the table. Since columns subjected to concentric load (no bending) are in general designed by a load-factor method, it is reasonable to apply a similar method to columns subjected to bending instead of changing to the modular-ratio method when bending occurs. The load-factor method in the Code applies directly only to rectangular columns with symmetrical reinforcement. It can be used for checking designs of columns, but it cannot be readily applied to the direct design of columns subjected to bending because many of the factors are related to the dimensions and other properties of the cross-section of the column.

Notation.

A_c = cross-sectional area of concrete (excluding reinforcement). A_{sc1} = cross-sectional area of reinforcement at each of the two opposite faces normal to the plane of bending, $A_{sc} = 2A_{sc1}$.

b = breadth of column.

d = overall "depth" of column. d_1 = "effective depth" of column $(d - d_2)$.

d_2 = distance from face of concrete to centre of adjacent reinforcement $(f_1 d)$.

E_s = secant modulus of elasticity of the steel at a tensile stress of $2p_{st}$ (30×10^6 lb. per sq. in. for mild steel).

e = eccentricity of applied load N $\left(=\frac{M}{N}\right.$ if M is the applied bending moment$\left.\right)$; this eccentricity and e_b are measured about the centroid of the gross section.

e_b = limiting eccentricity of limiting eccentric load P_b.

$K_I = \left(\frac{P_I}{bd}\right)$, the intensity of the safe load at eccentricity e for Case I. $K_{II} = \left(\frac{P_{II}}{bd}\right)$, the intensity of the safe load at eccentricity e for Case II. $K_b = \left(\frac{P_b}{bd}\right)$, the intensity of the limiting eccentric load. $K_0 = \left(\frac{P_0}{bd}\right)$, the intensity of the safe eccentric load.

N = working load applied at eccentricity e.

P = maximum safe eccentric load on a short column at eccentricity e; $= P_I$ when $N < P_b$; $= P_{II}$ when $N > P_b$. P_b = limiting eccentric load on a short column at limiting eccentricity e_b. P_0 = maximum safe concentric load on a short column.

p_{cc} = direct compressive stress permissible in concrete. p_{sc} = compressive stress permissible in reinforcement. p_{st} = tensile stress permissible in reinforcement.

r = ratio of cross-sectional area of reinforcement (at the two opposite faces normal to the plane of bending) to gross cross-sectional area of column $\left(= \frac{2A_{sc1}}{bd}\right)$.

Limiting Load and Limiting Eccentricity.—The preliminary stage in the calculation of the safe eccentric load on a column of given size and with specified

reinforcement is the determination of the limiting eccentric load P_b on the column, and the corresponding limiting eccentricity e_b. Then, if the load N applied at eccentricity e is less than the limiting load P_b, the safe load P at eccentricity e is determined by the tensile resistance of the column. If the load N applied at eccentricity e is not less than the limiting load P_b, the safe load P at eccentricity e is determined by the compressive resistance of the column and is related to the maximum safe concentric load P_0 on a short column of the same dimensions and reinforcement. N must not exceed the safe eccentric load P calculated for the condition obtaining.

The following formulæ apply.

$$P_b = p_{cc}bd_1X - A_{sc1}(p_{st} - p_{sc}) \text{ in which } X = \frac{85{,}000}{100{,}000 + \dfrac{60p_{st} \times 10^6}{E_s}};$$

or $$P_b = K_b bd \text{ in which } K_b\left(= \frac{P_b}{bd}\right) = p_{cc}X(1 - f_1) - (p_{st} - p_{sc})\frac{r}{2} \qquad \text{(A)}$$

and $$X = \frac{0{\cdot}85}{1 + \dfrac{600}{E_s}p_{st}}.$$

$$e_b = \frac{1}{P_b}[p_{cc}bd_1^2X(1 - 0{\cdot}5X) + A_{sc1}p_{sc}(d_1 - d_2)] - \frac{d_1 - d_2}{2},$$

or $$\frac{e_b}{d} = \frac{1}{2}\left[\frac{(1 - f_1)^2(2 - X)Xp_{cc} + rp_{sc}(1 - 2f_1)}{K_b} - (1 - 2f_1)\right] \qquad \text{(B)}$$

Case I.—Applied Load less than P_b.—If the applied load N acting at eccentricity e is less than P_b, the tensile resistance determines the strength of the member, and the maximum safe load that can be applied at eccentricity e is given by

$$P = p_{cc}bd\left[\left(\frac{1}{2} - \frac{e}{d} - Y\right) + \sqrt{\left(\frac{1}{2} - \frac{e}{d} - Y\right)^2 + r\frac{d_1 - d_2}{d}\cdot\frac{p_{sc}}{p_{cc}} + Y\left(2\frac{d_1}{d} - Y\right)}\right]$$

in which $Y = \dfrac{r(p_{st} - p_{sc})}{2p_{cc}}$. These formulæ can be rewritten as follows.

Maximum safe load (at eccentricity e) $= P_{\mathrm{I}} = K_{\mathrm{I}}bd$ in which K_{I} is the intensity of the maximum safe load (at eccentricity e) and is given by

$$K_{\mathrm{I}}\left(= \frac{P_{\mathrm{I}}}{bd}\right) = p_{cc}\left\{U + \sqrt{U^2 + (1 - 2f_1)\frac{p_{sc}}{p_{cc}}r + Y[2(1 - f_1) - Y]}\right\} \qquad \text{(C)}$$

where $U = \dfrac{1}{2} - \dfrac{e}{d} - Y$. The applied load N must not exceed P_1.

Case II.—Applied Load greater than P_b.—When the applied load N is greater than P_b, the compressive resistance determines the strength of the member, and the maximum safe load P which can be applied at eccentricity e is $P = \dfrac{P_0}{1 + \left(\dfrac{P_0}{P_b} - 1\right)\dfrac{e}{e_b}}$, in which p_0 is the safe concentric load on a short column, that is

$$P_0 = p_{cc}A_c + p_{sc}A_{sc}.$$

These formulæ can be rewritten as follows.

Maximum safe concentric load on a short column $= P_0 = K_0bd$, in which K_0 is the intensity of the maximum safe concentric load on a short column and is given by

$$K_0\left(= \frac{P_0}{bd}\right) = p_{cc} + r(p_{sc} - p_{cc}).$$

Maximum safe load (at eccentricity e) $= P_{II} = K_{II}bd$, in which K_{II} is the intensity of the maximum safe load (at eccentricity e) and is given by

$$K_{II}\left(= \frac{P_{II}}{bd}\right) = \frac{K_0}{1 + \left(\frac{K_0}{K_b} - 1\right)\frac{e}{e_b}} \quad . \quad . \quad . \quad . \quad \text{(D)}$$

The applied load N must not exceed P_{II}.

Special Formulæ and Factors.—Under conditions excluding the effects of wind, the permissible stresses in mild steel bars not exceeding $1\frac{1}{2}$ in. diameter are 20,000 lb. in tension and 18,000 lb. per square inch in compression. For bars exceeding $1\frac{1}{2}$ in. diameter the corresponding stresses are 18,000 lb and 16,000 lb. per square inch. If the effects of wind are included in the calculations of the loads and bending moments (and therefore the eccentricities) these stresses may be increased by 25 per cent. The elastic modulus for mild steel is 30,000,000 lb. per square inch. The stress coefficients X and Y are non-dimensional. Therefore for mild steel bars not exceeding $1\frac{1}{2}$ in. diameter and excluding the effects of wind, $X = 0{\cdot}607$ and $Y = \frac{1000r}{p_{cc}}$. If the stresses and coefficients are substituted in the general formulæ (A) for K_b, (B) for $\frac{e_b}{d}$ and (C) for $\frac{K_I}{p_{cc}}$, the formulæ (A), (B) and (C) series (1) to (4) in the upper part of *Table* 88 are obtained; series (1) and (3) apply when the effects of wind are not included, and series (2) and (4) apply when these effects are included. Similarly series (1) and (2) apply to bars not exceeding $1\frac{1}{2}$ in. diameter, and series (3) and (4) to larger bars. Series (1) apply to the conditions most common for the columns of a building frame, and for these conditions the numerical factors are as given in the lower part of *Table* 88 and include the preliminary factor K_I and the short-column concentric-load factor K_0 for various proportions of mild steel reinforcement with the common range of cover ratios, and for ordinary-quality 1 : 2 : 4 concrete. This part of *Table* 88 also gives values of the safe eccentric load factors K_I for Case I when the applied load is less than the limiting eccentric load P_b. The safe eccentric load for Case II, when the applied load is greater than the limiting eccentric load, is calculated readily by direct substitution in formula (D) which is applicable to all conditions of design.

The permissible stress in high-yield-stress bars is half the yield stress, but with limiting stresses of 30,000 lb. per square inch in tension and 23,000 lb. per square inch in compression. If the effects of wind are included the stresses can be increased by 25 per cent. but must not exceed 30,000 lb. per square inch; therefore the maximum permissible stresses when effects of wind are included are 30,000 lb. per square inch in tension and 28,750 lb. per square inch in compression. These stresses apply to bars having a yield stress of not less than 60,000 lb. per square inch. Formulæ (A), (B) and (C) series (5) excluding wind, and series (6) including wind, in *Table* 88, are derived by substituting the foregoing stresses in formulæ (A), (B) and (C). The elastic modulus of cold-worked bars depends on the method of manufacture and other conditions; a suitable value should be determined by tests of bars of the type to be used. For preliminary designs a modulus of 15×10^6 lb. per square inch is reasonable.

Design Procedure.—The design procedure in accordance with the load-factor method is as follows. (i) Select a reasonable trial section. (ii) Compare the calculated safe eccentric load with the applied eccentric load. (iii) Make adjustments to the trial section if it is inadequate or too large or contains too much reinforcement. The examples on the page facing *Table* 88 show the application of this procedure.

Bending Moments about Two Axes.

Some methods of calculating the stresses when a section is subjected to bending moments M and M_1 acting about each of the two axes mutually at right-angles simultaneously with a concentric compressive load N are given in *Table* 90. The two cases of when the stresses are entirely compressive, and when tensile and compressive stresses are produced, are considered. The method in the former case is accurate, but the method in the latter case is approximate and applies only if M is much greater than M_1. If M and M_1 are more nearly equal, a semi-graphical method, which is only worth while for important members, can be applied by combining vectorially M and M_1 to obtain the resultant moment M_R. Assume a position of the neutral plane, which will be at right-angles to the plane of action of M_R and proceed as described for the irregular section in *Table* 85. The foregoing procedures are based on the modular-ratio method.

Combination of Stresses acting in Different Directions.

If three stresses act on a square element of uncracked concrete the principal tensile and compressive stresses, mutually at right-angles, are given by substitution in the general formulæ in *Table* 90; the plane in which the principal tensile stress acts can also be established. The general formulæ apply if a tensile stress acts normal to one face of the element, a tensile stress acts normal to an adjacent face, and a shearing stress acts in the plane of the element. If either of the direct stresses are compressive, the sign of the appropriate term in the formula is changed. Formulæ are also given for cases in which one or both of the direct stresses are compressive or do not act.

SECTION 5

STRUCTURES AND FOUNDATIONS

THE loads and consequent bending moments and forces on the principal types of structural components, and the stresses in, and resistances of, these components are dealt with in the preceding sections. In this section some complete structures, which are mainly assemblies or special cases of such components, and their foundations are considered.

Buildings.

A building may be constructed entirely of reinforced concrete or the roof, floors, walls, stairs and foundations, or one or more of these parts, may be of reinforced concrete in conjunction with a steel frame. Alternatively, the interior and exterior walls may be of cast-insitu reinforced concrete and support the floors and roof, the columns and beams being formed in the thickness of the walls. In the following are given some notes relating to the design of building components.

Floors.—Concrete floors may be of monolithic beam-and-slab (with the slabs spanning in one or two directions), flat-slab, or hollow-slab construction, or may be of precast slabs supported on cast-insitu or precast concrete beams. Some typical details of beams and solid slabs, and hollow-block slabs are given in Appendix II. (B.S. Code No. 114 gives recommendations for the design and construction of floors and flat roofs comprising hollow blocks, ribbed slabs, and precast slabs.)

Openings in Slabs.—The slabs around openings in floors or roofs should be strengthened with extra reinforcement, unless the opening is large compared with the span of the slab (for example, stair-wells or lift-wells) in which case beams should be provided around the opening. For small openings in solid slabs the cross-sectional area of the extra bars placed parallel to the principal reinforcement should be at least equal to the area of principal reinforcement interrupted by the opening. A bar should be placed diagonally across each corner of an opening.

Holes for pipes, ducts, and other services should be formed when the floor is constructed and it should not be permitted to cut such holes afterwards, unless done under the supervision of a competent engineer. It is therefore an advantage to provide, at the time of construction, a number of holes that can be used for electric conduits and small pipes, even when they are not required for the known services. Suitable positions are through floor slabs in the corners of rooms or corridors, and through the ribs of beams immediately below the slab.

Hollow-block Slabs.—If the span of a floor or roof slab exceeds 10 ft. it is often economical to provide a hollow-block slab, which is light in weight and requires less concrete than a solid slab. Such a floor consists of a thin top slab ($1\frac{1}{4}$ in. to $3\frac{1}{2}$ in. thick) overlying concrete ribs. The ribs may be at 6-in. to 36-in. centres and may be from $2\frac{1}{2}$ to 5 in. wide. The spaces between the ribs may be left open, but in order to simplify the shuttering they are frequently filled with hollow blocks of burnt clay or lightweight concrete. The combined depth of the rib and slab is determined in the same way as the depth of a solid slab, and the thickness of the top slab is made sufficient

to provide adequate compressive area. The width of the rib is primarily determined by the shearing force. Alternative designs of hollow-block slabs are given in Appendix II. Weights of solid and hollow-block slabs are in *Table* 1. The principal requirements of B.S. Code No. 114 are that the thickness of the top slab be not less than one-twelfth of the distance between ribs, with a minimum of 2 in. but, if the blocks are assumed to add to the strength of the construction and the clear distance between the ribs does not exceed 18 in., the top slab should be not less than 1 in. thick. The distance between the centres of the ribs should not exceed 3 ft. The width of a rib should be not less than 2½ in. or one-third of its depth; for resistance to shearing, the effective width of the rib is assumed to be the actual width plus the thickness of one wall of the block.

Stairs.—Stairs can be designed to span transversely (that is, across the flight) or longitudinally (that is, in the direction of the flight). When spanning transversely supports must be provided on both sides of the flight, either by walls or stringer beams. In this case the waist or thinnest part of the stair construction need be only, say, 2 in. thick, the effective lever-arm for resisting the bending moment being about one-half the maximum thickness from the nose to the soffit measured normal to the soffit. When the slab spans longitudinally the thickness required to resist bending determines the thickness of the waist. The loads for which a flight of stairs should be designed are described on page 9. The bending moments should be calculated from the total weight of the stairs and the total imposed load combined with the horizontal span. The stresses produced by the longitudinal thrust are small and are generally neglected. Unless circumstances otherwise dictate, a suitable shape for a step is 7-in. rise with 10-in going, which with 1-in. nosing or undercut gives a tread of 11 in.; stairs in industrial buildings may be steeper.

Recommendations for the design of stairs and landings are given in the B.S. Code.

Reference should be made to other publications for the design of helical stairs.

Flat Roofs.—A flat reinforced concrete roof is designed similarly to a floor and may be a simple solid slab, or beam-and-slab construction, or a flat slab. In beam-and-slab construction the slab may be a solid cast-insitu slab, a hollow-block slab, or a precast concrete slab. A watertight covering, such as asphalt or bituminous felt, is generally necessary, and with a solid slab some form of thermal insulation may be required. The watertight covering is sometimes omitted from a flat solid slab forming the roof of an industrial building, but in such a case the concrete should be particularly dense, and the slab should be not less than 4 in. thick and should be laid to a slope of at least 3 in. in 10 ft. to expedite the discharge of rainwater. Sodium silicate or tar, well brushed into the surface of the concrete, will improve the watertightness if there are no cracks in the slab.

For ordinary buildings the slab of a flat roof is generally built level and the slope for draining, often about 1 in. in 10 ft., is formed by a mortar topping. The topping is laid directly on the concrete and below the asphalt or other watertight covering, and may form the thermal insulation if it is made of sufficient thickness and of light-weight concrete of low thermal conductivity.

Sloping Roofs.—Planar slabs with a continuous steep slope are not common in reinforced concrete, except for mansard roofs, the covering of pitched roofs being generally metal or asbestos-cement sheeting, glass, wood-wool slabs, or other light-weight material. Such coverings and roof glazing require purlins for their support and, although the purlins are frequently of steel, reinforced concrete purlins, which

may be either cast-insitu or more commonly precast, are provided especially if the roof structure is of reinforced concrete.

Precast Concrete Purlins.—The size of a precast concrete purlin depends not so much on the stresses due to bending as on the deflection. Excessive deflection, although not necessarily a sign of structural weakness, may lead to defects in the roof covering. The shape of the purlin should be such that lightness is combined with resistance to bending, not only in a vertical plane but also in a direction parallel to the slope of the roof. An ell-shape, which is often used, is efficient in these respects, but a wedge-shape is often less costly to make for small spans. The weight of a precast concrete purlin may be excessive for spans over 15 ft. The dimensions depend on the span and the load, and for purlins spaced at 4 ft. 6 in. centres and carrying ordinary roof sheeting and spanning 15 ft. suitable sizes are 5 in. for the width across the top flange and 8 in. for the overall depth. For a span of 10 ft. the corresponding dimensions are 4½ in. and 6 in. For purlins on sloping roofs, the vertical weights and the wind pressure normal to the slope of the roof should be combined vectorially before computing the bending moment. The stresses should then be calculated with the neutral plane normal to the line of action of the resultant load. A semi-graphical method, as described in *Table* 85, is most suitable for the calculation of the stresses.

The purlins may be supported on cast-insitu or precast concrete frames or rafters. If the rafters are cast insitu the ends of the purlins can often be embedded in the rafters so as to obtain some fixity, which increases the stiffness of the purlin. If the rafters are of precast concrete, the type of fixing of the purlin is generally such that the purlin should be designed as freely supported.

Non-planar Roofs.—Roofs which are not planar, other than the simple pitched roofs considered in the foregoing, may be constructed in the form of a series of planar slabs (prismatic or hipped-plate construction), or as domes or vaults (segmental or cylindrical shells). The elementary analysis of such structural forms is given in *Table* 91, but reference should be made to other publications for more comprehensive analyses and more complex designs. Notes on the matter given in *Table* 91 are given on the page facing the table.

In *Table* 97 expressions are given for the forces in domed slabs such as are used for the bottoms and roofs of cylindrical tanks. In a building a domed roof has generally a much larger ratio of rise to span and, when the dome is part of a spherical surface and has an approximately uniform thickness throughout, the analysis in *Table* 91 applies. Shallow segmental domes and truncated conical "domes" are also dealt with in *Table* 91.

Segmental or semi-cylindrical roofs are generally designed as shell structures. A thin curved slab acting as a shell is assumed to offer no resistance to bending and not to deform under distributed loads. Except near the edge and end stiffeners, it is subjected only to direct "membrane" forces, namely, a direct force acting longitudinally in the plane of the slab, a direct force acting tangentially to the curve of the slab, and a shearing force. The membrane forces per unit length of the slab of a segmental shell roof, supported at the ends only, are given by the formulæ in *Table* 91.

Panel Walls.—Panel walls filling in the structural frame and not designed to carry loads (other than ordinary wind pressures) should be not less than 4 in. thick (for constructional reasons), and should be reinforced with not less than ¼-in. mild steel bars spaced at 6-in. centres or the equivalent in bars of other sizes but at not more than 12-in. centres (or an equivalent fabric); this reinforcement should be provided

in one layer in the middle of the wall. Bars ½ in. or more in diameter should be placed above and at the sides of openings, and ½-in. bars 4 ft. long should be placed diagonally across the corners of openings. The slab must be strong enough to resist the bending moments due to spanning between the members of the frame. The connections to the frame must be strong enough to transfer the pressures on the panel to the frame either by bearing, if the panel is set in rebates in the members of the frame, or by the resistance to shearing of reinforcement projecting from the frame into the panel. A bearing is preferable since the panel can be completely free from the frame and therefore not subjected to secondary stresses due to deformation of the frame nor is the connection between the panel and the frame subjected to tensile stresses due to contraction of the panel caused by shrinkage of the concrete or thermal changes. By setting the panel in a chase the connection is also lightproof. The wind pressure which a panel wall should be designed to resist is a pressure or suction of $0{\cdot}8p$ as given in *Table* 8. If not rigidly connected to the frame the panel of slab should be designated as a slab spanning in two directions without the corners being held down (see *Table* 38).

Load-bearing Walls.—B.S. Codes Nos. 114, 111, and 123—101 give recommendations for load-bearing walls which are summarised in the following. The requirements of the London By-laws are less conservative and should be consulted where applicable.

Walls exposed to the weather should be not less than 6 in. thick, but in other cases not less than 4 in. thick. Party walls should, for acoustical reasons, be cavity walls. A load-bearing reinforced concrete wall should be designed as a column, but the minimum cross-sectional area of the vertical reinforcement need not be less than 0·2 per cent. of the area of the horizontal cross-sections of the wall, and horizontal reinforcement should be provided, the cross-sectional area of which should be not less than 0·2 per cent. of the area of the vertical cross-section. If the concrete alone can carry the imposed load, without assistance from the vertical bars, lateral reinforcement tying in the vertical bars is not required, and the minimum amount of vertical reinforcement is 0·2 per cent. and of horizontal reinforcement 0·1 per cent. If the

REINFORCEMENT IN LOAD-BEARING WALLS.

Thickness of wall (in.)	Minimum reinforcement (Area)				Applicability	
	0·1 per cent.		0·2 per cent.		0·1 per cent.	0·2 per cent.
	Total sq. in.	On each face	Total sq in.	On each face		
4	0·048	$\frac{3}{16}$ in. at 12 in.	0·096	¼ in. at 12 in.	Horizontal bars if vertical bars are not taken into account (Code No. 114)	Horizontal bars if vertical bars are taken into account (Code No. 114) Vertical bars for all cases. (Codes Nos. 114 and 111–201) Horizontal bars (Code No. 111–201)
5	0·060	¼ in. at 18 in.	0·120	$\frac{5}{16}$ in. at 15 in.		
6	0·072	¼ in. at 15 in.	0·144	$\frac{5}{16}$ in. at 12 in.		
7	0·084	¼ in. at 13½ in.	0·168	⅜ in. at 15 in.		
8	0·096	¼ in. at 12 in.	0·192	⅜ in. at 13½ in.		
9	0·108	$\frac{5}{16}$ in. at 16½ in.	0·216	⅜ in. at 12 in.		

NOTES. (1)—Area of reinforcement is percentage of cross-sectional area of wall. (2)—" Total " is cross-sectional area of reinforcement in sq. in. per foot length or height of wall taking into account bars on both faces of wall. (3)—If wall is reinforced by a single layer of bars, the spacings given in the table should be halved.

load on the wall is such that the resistance of the vertical bars is required, it is necessary to provide transverse ties or crimped horizontal bars additional to the horizontal reinforcement; the spacing of the ties should be such that the vertical bars are restrained at points not more than twelve times their diameter apart. The minimum reinforcement in walls of various thicknesses is given in the table on page 90. The minimum amount of reinforcement recommended in B.S. Code No. 114 may be insufficient to resist the effects of temperature and shrinkage, to withstand which the minimum reinforcement recommended in B.S. Code No. 111—201 should be provided; the effect of the method of construction on the shrinkage stresses and the degree of exposure as it affects the probable thermal changes should be considered.

The load-carrying capacity of a reinforced concrete wall depends on the ratio of the effective height H_e of the wall to the effective thickness D_e which is the actual thickness D of a solid wall or, if the wall is stiffened by pilasters, D_e may exceed D; if P_1 is the ratio of the distance between the centres of the pilasters to the width of the pilasters, and P_2 is the ratio of the overall thickness of the pilaster to D, empirical values of D_e from which intermediate values can be interpolated are as follows.

P_1	6	8	10	15
$P_2 = 2$	$1{\cdot}4D$	$1{\cdot}3D$	$1{\cdot}2D$	$1{\cdot}1D$
$P_2 = 3$	$2{\cdot}0D$	$1{\cdot}7D$	$1{\cdot}4D$	$1{\cdot}2D$

If $P_2 = 1$ or is 20 or more, $D = D_e$. The ratio $\frac{H_e}{D_e}$ should not exceed 24. If this ratio exceeds 15, the permissible compressive stress in the concrete (and also in the vertical reinforcement if this is taken into account) should be reduced in accordance with the rules given in *Table* 83. The effective height H_e of a wall is calculated in the same way as the effective length L_E of a column as described on page 296, but if the wall is stiffened by cross walls or pilasters and the distance between the cross walls or pilasters is less than H_e, this distance may be substituted for H_e. The safe load calculated for a column may be increased for walls that are long compared with their height, that is if the length L is two or more times the story height H, the safe load (or the permissible stresses) may be increased by 20 per cent., and by 10 per cent. if $L = H$; there is no increase if the length is two-thirds or less of the story height. In this consideration L is the overall length or, if there are openings in the wall, the length between adjacent openings.

Walls subjected to Bending.—A reinforced concrete wall subjected to bending should be designed in accordance with the requirements for columns under similar conditions. Where floor beams are monolithic with walls, with or without pilasters, the bending moments on the wall (and at the ends of the beams) can be calculated by the formulæ for columns. The bending moment and load from the beam may be concentrated immediately around the junction of the beam and wall (or pilaster), but will be dispersed through the wall. The angle of dispersion in a concrete wall, as specified in B S. Code No. 111, is 45 deg.; it is reasonable to distribute the bending moments to the same extent.

Bridges.

Types of Bridges.—A bridge may be one of two principal types, namely, an arch bridge or a girder bridge, and either of these types may be statically determinate

or statically indeterminate. The basic types of bridges are illustrated in *Table* 93. A bowstring girder is a special type of arch, and a rigid-frame bridge can be considered as a type of girder or arch bridge.

The selection of the type in any particular instance depends principally on the span, the nature of the foundation, and the clearance required. It may be that more than one type is suitable, in which case the economy of one over the others may be the deciding factor. If a bridge is fairly high above the railway, road, or waterway, an arch is generally the most suitable if the ratio of the span to the rise does not greatly exceed ten and if the foundation is able to resist the inclined forces from the arch. If settlement of the foundation is probable, an arch provided with hinges can be used but the ratio of the span to the rise should not greatly exceed five. For other conditions a girder bridge is more suitable; although some girder bridges have the appearance of solid-spandrel arches, they are not arches as they are designed so as not to impose any horizontal thrust on the abutments or piers. In a bowstring-girder bridge, which has the advantages of an arch but does not require the same rigidity in the foundation, the deck is at the level of the springing of the arch and is therefore suitable when the level of the deck is not much above the level of the waterway, road, or railway that is spanned. "Memorandum on Bridge Design and Construction" issued by the Ministry of Transport gives guidance on most aspects of the subject of the general design of bridges and the selection of type.

Loads.—The live loads on road and railway bridges are described on pages 11 and 12. Particulars of the Ministry of Transport loading on road bridges, the weights of typical road and rail vehicles, and the loading requirements of B S. No. 153 (Part 3A) for reinforced concrete decks of steel bridges are given in *Table* 6. Notes on the foregoing are on the page facing the table.

Deck.—The design of the deck of a reinforced concrete bridge is almost independent of the type of the bridge. Some typical cross-sections are given in *Table* 93. In the simplest case the deck is a reinforced concrete slab spanning between the abutments and bearing freely thereon, as in a freely-supported type of bridge, or built monolithically therewith as in a rigid-frame bridge. This type of deck is suitable only for small spans, say, up to 15 ft. The more common case is for the main arches or girders to support a reinforced concrete slab that spans transversely between these principal members. If the latter are spaced at more than, say, 7 ft., an economical deck is provided by inserting transverse beams and designing the slab to span in two directions. In a solid-spandrel arch bridge with earth filling, the deck may be a concrete slab laid on the filling. A deck common in bridges of large span is one of cellular construction in which the road forms the top slab and a soffit slab forms the bottom of a series of boxes, the sides of which are the longitudinal arch ribs or girders.

The underside of the deck of a bridge over a railway should have a flat soffit, thereby avoiding pockets in which smoke may collect. For such bridges the corrosion of the concrete by the smoke from steam locomotives has to be prevented. Smoke-guards may not entirely protect the structure. A dense concrete, free from cracks through which the fumes can reach and attack the steel reinforcement, is necessary. The cover of concrete should be greater than that provided in buildings, and the tensile stresses in the concrete should be calculated and limited in value, as in liquid-containing structures.

A bridge less than 15 ft. wide is often economical if the deck slab spans transversely between two outer longitudinal girders. These girders may be the parapets of the

bridge, but for major structures the parapets should not be used as principal structural members. If the width of the bridge exceeds 15 ft., an economical design is produced by providing several longitudinal girders or arch ribs spaced at about 7-ft. centres.

Footpaths are sometimes cantilevered off the principal part of the bridge. Water, gas, electrical and other services are generally installed in a duct under the footpaths.

Clearances.—The approximate clearances required for a bridge over a railway or road and for subways are given in *Table* 92. Other clearances may be required for railways and roads abroad, and in some instances in Britain. As each railway has its own structure-gauge such requirements are generally specified, as are also the requirements of the navigation authorities for bridges over rivers and canals.

The diagram for a double line of railway in *Table* 92 illustrates the minimum desirable clearances required by the Ministry of Transport, but for curved lines these clearances may have to be increased to allow for curvature and superelevation of the line and for the length of the rolling stock. For a new bridge or a bridge being rebuilt, the lateral clearance, after making these allowances, should be not less than 2 ft. 4 in. from the widest part of the broadest vehicle likely to use the line. Except where platforms occur, the clearance of 2 ft. 4 in. must extend for the full height from rail level to the top of the highest carriage-door. The overhead clearance (except for underground railways or for railways having electric traction with overhead equipment) should be 12 in. above the loading gauge, allowance being made for superelevation.

Parapets.—The height of the parapets on an underline railway bridge should be 4 ft. 6 in. above the level of the rail, and should be solid for the whole height. If a check rail is provided in the track the parapet need be solid for a height of only 1 ft. above rail level, a guard rail being provided 4 ft. 6 in. above rail level. The height of the parapets of road bridges or foot-bridges over railways must not be less than 4 ft.

Vertical Curves.—If a road bridge is constructed at a slightly higher level than the adjacent land, the approach roads often rise towards the centre of the bridge, where a vertical curve is formed to ease the change of gradient and to provide an adequate view of on-coming traffic. Proportions for such vertical parabolic curves in conformity with British practice are given on *Table* 92 The formulæ for the sighting distance over the hump of the bridge is based on the level of the eye being 3 ft. 9 in. above the crown of the road.

Girder Bridges.—The types of girder bridges shown in *Table* 92 are classified on a structural basis. Some of the types shown are statically determinate, the exceptions being the restrained and continuous beams, and the Vierendeel girder. The statically-determinate girders of two or more spans are made up of a series of elements each of which consists of a beam of one span with a cantilever at one or both ends. The junction between two adjacent elements is made by means of a freely-suspended span. To preserve freedom of movement and the condition of statical determinacy it is necessary that such girders should not be rigidly connected to the supporting piers or abutments; the beams should be free to rotate and, at all but one support for each element, the bearing should be free to slide. The formulæ in *Table* 92 indicate some suitable lengths for the cantilevers in a bridge consisting of a series of equal spans. The bending moments and shearing forces are calculated by determining those for a freely-supported beam, and superimposing those due to the cantilever. For bridges designed for Ministry of Transport loading, the shearing force (lb) at a point at a distance aL from the centre of the span of a freely-supported girder of span L ft. is $0{\cdot}5w_a\ LB(a + 0{\cdot}5)^2 + 2700B(0{\cdot}5 + a)$, where w_a is the uniformly-distributed load

(lb. per square foot) for a loaded span of $(0{\cdot}5 + a)L$, and B ft. is the spacing of the longitudinal girders.

The foregoing articulated types of girder bridges are suitable when there is a probability of unequal settlement of the foundations. When unequal settlement of the foundations is not likely, a girder designed as a continuous beam is permissible. This type of girder may have a considerable variation of moment of inertia throughout each span, and it is necessary to adopt the formulæ and methods described for this case in *Table* 19. If the girder is constructed monolithically with the supports, the effect of the fixity at the supports should be allowed for and the structure should be designed as a frame.

An open-type bridge girder may be a lattice girder similar to the truss of a steel bridge and may have a single, double, or treble lattice. These trusses are generally analysed in the same way as a steel truss, assuming pin joints, but the rigidity of the joints in monolithic reinforced concrete construction largely invalidates this method of calculation. The Vierendeel truss, in which the panels are rectangular and the joints rigid, is more appropriate to construction in reinforced concrete. The accurate analysis of such a truss is complex, but a practical approximation is to calculate the shearing forces and consequent bending moments in each member and combine therewith the primary forces due to the general bending action.

Frame Bridges.—Bridges incorporating portal frames are statically indeterminate and three principal types are shown in *Table* 92. The frames, either slabs or ribs, may be hinged at the base or fixed. The simplest form of framed bridge is a rigid frame of one span; for small spans, say, up to 20 ft, the frame may be a slab forming the deck and the abutment wall without beams or counterforts. For larger spans the slab would span horizontally between beams and counterforts which together form the frame. The frame would be analysed as described in *Tables* 49 and 50.

Arch Bridges.—The principal types of arch bridges are illustrated in *Table* 92 and include arch ribs or arch slabs. In the case of the former type the deck is supported on columns carried on the ribs or, in the case of a bridge with the deck below the level of the crown of the arch, the deck is partly supported on columns and partly suspended from the ribs. In the case of an arch slab, the deck is supported on earth filling deposited on the slab and retained by spandrel walls. Arch ribs or slabs may be three-hinged arches which are statically determinate, or two-hinge or fixed arches which are statically-indeterminate The design of hinged and fixed arches is dealt with in *Tables* 52 to 55, and is described on pages 43 *et seq.*

Precast Concrete Bridges.—Where the erection of temporary falsework is prohibited and the working periods are limited, precast concrete can be used with advantage. Some types of bridges incorporating precast construction are given in *Table* 93. The piers and abutments are constructed first and precast concrete beams (or in some cases precast concrete arch ribs), which are generally I-beams, are erected thereon, being either spaced apart or laid closely together. A cast-insitu slab is generally laid on the beams. If the beams are spaced apart the slab forms the flange of a tee-beam of which the precast beam forms the rib, the two parts being bonded together by vertical bars projecting from the precast beam. The reinforcement in the precast concrete beam is calculated as the reinforcement required for a tee-beam carrying the total load, but the stresses induced while it is an independent member must be allowed for and must not be excessive. The stresses induced during erection can be kept low by selecting suitable positions for the attachment of the crane slings.

When precast concrete beams are laid together an interlocking device should be provided to distribute the live load over two or more beams to prevent one beam deflecting considerably more than the others. A thin cast-insitu reinforced concrete slab bonded to the top of the precast beams serves this purpose, and the combined construction is a single structural member.

It should be noted that, due to its own weight, the weight of the shuttering, and the weight of the wet concrete in the superimposed slab, the precast beam will be strained before it can act as a beam of the full cross-section of the final structural member. The initial stresses so caused must be included in the calculation of the stresses caused by any further dead load and by the live load, although the additional stresses may be computed for a beam of increased size.

Piers and Abutments.—The piers for girder bridges are generally subjected only to the vertical load due to the total loads on the girders, the abutments of girder bridges have to resist the vertical loads from the girders and the horizontal earth pressure on the back of the abutment. There may also be a horizontal force due to friction on bearings (see *Table* 91) and, in the case of railway bridges, a horizontal force due to braking, acceleration, etc. (see page 146) Continuity between the girders and the abutments is assumed in rigid-frame bridges, and consequently the foregoing forces on the abutment must be combined with the bending moments and horizontal thrusts due to action as a frame.

The abutments of an arch bridge have to resist the vertical loads and the horizontal thrusts from the arch. Stability is obtained by constructing massive piers in plain concrete or masonry, or by providing tension and compression piles, or by a cellular reinforced concrete box filled with earth. Part of the horizontal thrust on the abutments will be resisted by the active earth pressure on the abutment, but in the case of fixed arches this pressure should be assumed to relieve the thrust from the arch only when complete assurance is possible that this pressure will always be effective. Adequate resistance to sliding should also be assured, and the buoyancy effect of foundations below water should be investigated.

Mid-river piers, if not protected by independent fenders, should be designed to withstand blows from passing vessels or floating débris, and should be provided with cutwaters.

Culverts and Subways.

Pipe Culverts.—For conducting small streams or ditches under embankments, culverts can be constructed with precast reinforced concrete pipes, which must be strong enough to resist the vertical and horizontal pressures from the earth and other superimposed loads. The pipes should be laid on a bed of concrete, and where they pass under a road they should be surrounded with reinforced concrete at least 6 in. thick. The culvert should also be reinforced longitudinally to resist bending due to unequal vertical earth pressure or unequal settlement. Owing to the uncertainty of the magnitude and disposition of pressures on circular pipes embedded in the ground, accurate analysis of the bending moments is impracticable. A basic guide is that the positive bending moments at the top and bottom of a circular pipe of diameter D and the negative bending moments at the ends of a horizontal diameter are $0{\cdot}0625pD^2$, where p is intensity of downward pressure on the top and of upward pressure on the bottom, assuming the pressures to be uniformly distributed on a horizontal plane.

Loads on Culverts.—The load on a pipe culvert, or on the top slab, includes

the weight of the earth, the imposed load (if any), and the weight of the top slab. When a trench has been excavated in consolidated ground for the construction of the culvert and the depth from the surface of the ground to the roof of the culvert exceeds, say, three times the width of the culvert, it can be assumed that the maximum earth pressure on the culvert is that due to a depth of earth equal to three times the width of the culvert. Although a culvert passing under a newly-filled embankment may be subjected to more than the full weight of the earth above, there is little reliable information concerning the actual load carried, and therefore any reduction in load due to arching of the ground should be made with discretion. If there is no filling and wheels or other concentrated loads can bear directly on the culvert, the load should be considered as carried on a certain length of the culvert. In the case of a box culvert, the length of the culvert supporting the load should be determined by the methods illustrated in *Table* 6. The concentration is modified if there is any filling above the culvert and, if the depth of filling is D^1, a concentrated load W can be considered as spread over an area of $4D_1^2$. When D_1 equals or slightly exceeds half the width of the culvert, the concentrated load is equivalent to a uniformly-distributed load of $\frac{W}{4D_1^2}$ lb. per square foot over a length of culvert equal to $2D_1$. For values of D_1 less than half the width of the culvert, the bending moments will be between those due to a uniformly-distributed load and a central concentrated load.

The weights of the walls of the culvert can be assumed to produce a uniform pressure on the ground. The weights of the bottom slab and the water in the culvert are carried directly on the ground below the slab and thus do not produce bending moments, although these weights must be taken into account when calculating the maximum pressure on the ground. The horizontal pressure due to the water in the culvert produces an internal triangular load or a trapezoidal load if the surface of the water outside the culvert is above the top, when there will also be an upward pressure on the underside of the top slab. The magnitude and distribution of the horizontal pressure due to the earth against the sides of the culvert can be calculated in accordance with the formulæ given in *Tables* 10, 11 and 12, consideration being given to the possibility of the ground becoming waterlogged with consequent increased pressures and the possibility of flotation.

Bending Moments on Culverts.—The maximum bending moments can be calculated by considering the possible incidence of the loads and pressures. Generally there are only two conditions to consider: (*a*) Culvert empty: full load and surcharge on the top slab, the weight of the walls, and maximum earth pressure on the walls; (*b*) Culvert full: minimum load on the top slab, minimum earth pressure on the walls, weight of walls, maximum horizontal pressure from water in the culvert, and possible upward pressure on the top slab. In some circumstances these conditions may not produce the maximum positive or negative bending moments at any particular section, and the effect of every probable combination should be considered. The direct thrusts and tensions due to various loads should be combined with the bending moments to determine the maximum stresses.

The bending moments produced in monolithic rectangular culverts are generally determined by considering the four slabs as a continuous beam of four spans with equal bending moments at the end supports. The load can be conveniently divided as follows: (*a*) A uniformly-distributed load on the top slab and an equal reaction from the ground below the bottom slab; (*b*) An imposed load on the top slab and an equally-

distributed reaction from the ground below the bottom slab; (*c*) Upward pressure on the bottom slab due to the weight of the walls; (*d*) A triangularly-distributed horizontal pressure on each wall due to the increase in earth pressure in the height of the culvert; (*e*) A uniformly-distributed horizontal pressure on each wall due to pressure from the earth and any surcharge above the level of the roof of the culvert; (*f*) The internal horizontal pressure from the contents of the culvert. These loads are indicated in *Table* 99 and the bending moments at the corners due to the various types of loading can be found from the formulæ given in the table. These expressions are applicable when the thicknesses of the top slab and the bottom slab are about equal, but may be equal to or different from the thicknesses of the walls.

For the bending moments due to the outward pressures from the water in the culvert, which are greatest when the culvert is entirely submerged, the formulæ for the corresponding inward pressures can be used but with the sign changed.

Subways.—A subway of rectangular cross-section is subjected to external earth pressures similar to those on a culvert and the formulæ in *Table* 99 can be used for the purpose of calculating the bending moments. Internal pressures do not generally have to be considered. The minimum sizes for subways are given in *Table* 92.

Bearings, Hinges, and Joints.

In the construction of frames and arches, hinges are necessary at points where it is assumed there is no bending moment. Sliding and roller bearings are necessary in some types of bridges to ensure statical determinacy. Some types of bearings and hinges are illustrated in *Table* 94, and notes on these designs are given on the page facing the table.

Joints in monolithic concrete construction are required to allow free expansion and contraction due to changes of temperature and shrinking in such structures as retaining walls, reservoirs, roads, and long buildings, and to allow unrestrained deformation of the walls of cylindrical containers when it is undesirable to transfer any bending moment or force from the walls to the bottom slab. Some designs of joints for various purposes are illustrated in *Table* 95 and notes on these designs are given on the page facing *Table* 94. Joints in road slabs are illustrated in *Table* 96.

Concrete Roads.

A concrete road may be a concrete slab forming the complete road or may be a slab underlying bituminous macadam, granite setts, asphalt, wooden blocks, or other surfacing. On the site of extensive works it is sometimes convenient to lay concrete roads before constructional work begins, these roads being the bases of permanent roads. A type of concrete road much used for motorways and similar main roads comprises a layer of plain cement-bound granular material (called "dry-lean" concrete), the mixture being about 1 : 18, with a bituminous surfacing. In this section, the design of all-concrete reinforced concrete roads only is dealt with. For details of the preparation of the foundation (a very important aspect of the construction of a road) and methods of construction, reference should be made to other publications.

The design of concrete roads is based as much on experience as on calculation, since the combined effects of the expansion and contraction of the concrete due to moisture and temperature changes, of the weather, of foundation friction, of spanning over weak places in the foundation, of fatigue, and of carrying the loads imposed by

traffic are difficult to assess. The provision of joints assists in controlling some of these stresses. The notes in the following give the basic principles only.

Stresses due to Traffic.—The stresses in a concrete road slab due to vehicles are greatest when a wheel is at the edge or near a corner of the slab, but considerably less when it is remote from an edge or corner; therefore, from the point of view of stresses due to traffic, it is desirable to reduce the number of joints across which loads cannot be transferred and to prevent cracks, thereby reducing the number of effective edges and corners. The empirical formulæ derived by Westergaard for the calculation of the stresses are the basis of many subsequent attempts to reconcile the stresses derived theoretically with measured stresses; the formulæ (published in 1933) are given in a modified form in *Table* 96, and have since been modified to apply to aircraft runways. See page 150 and *Table* 6 for weights of vehicles and aircraft.

Thickness of Slab.—For all-concrete roads the slab may be from 5 in. to more than 12 in. thick, depending on the weight of traffic and the type of soil. Some common thicknesses are given in *Table* 96 for various weights of traffic and types of soil, which are defined approximately in the table. The thicknesses should be increased for particularly adverse conditions, such as for very heavy traffic on dockside roads on very poor soil, for which upwards of 12 in. may be necessary. The concrete should not be leaner than 1 : 1⅔ : 3⅓ unless special mixtures are "designed" to give a dense strong concrete with a lower cement content. For the wearing surface, rounded aggregates are not recommended and a hard crushed stone should be used. In districts where suitable crushed stone is costly an economical and durable slab can be formed by making the lower part of the slab of 1 : 2 : 4 concrete made with uncrushed gravel aggregate, and the upper part, to a depth of about 1½ in., with 1 : 1½ : 3 concrete made with crushed stone graded from ½ in. to $\frac{3}{16}$ in. Exposure to weather and abrasion from traffic subject all-concrete roads to severe conditions, and all reasonable means of attaining a concrete of high quality should be taken.

Reinforcement.—When a concrete road is laid on a firm and stable foundation, experience shows that reinforcement is not always necessary, but some engineers take the view that the provision of reinforcement is a precaution that justifies the cost. When mild steel reinforcement is used the amount provided is generally between 6 lb. and 10 lb. per square yard provided in a single layer near the bottom or top of the slab, but for roads subject to heavy traffic reinforcement is provided near the top and bottom to give a total weight of 10 lb. to 20 lb. per square yard. If high-yield-stress steel is used two-thirds of the foregoing weights may be sufficient. Typical weights of mild steel reinforcement, which should preferably be placed near the top of the slab, are given in *Table* 96 for various types of traffic and soil. The arrangement of the reinforcement depends on the width of the road and the spacing of the transverse joints. If the joints are at distances apart equal to about the width of the slab, the reinforcement should be arranged to give equal strength in both directions, but if the transverse joints are provided at long intervals to form panels of length, say, three or more times their width, nine-tenths of the reinforcement should be parallel to the length of the road; for panels of intermediate proportions, ratios of between 0·5 and 0·9 of the total reinforcement should be placed longitudinally. Additional bars, say, ½ in. diameter, are required in the top at the corners of the panels.

Joints.—Although some concrete roads have no transverse joints, the provision of such joints and, in wide roads, the provision of longitudinal joints, may assist in reducing cracking. In Britain the common spacing of expansion and contraction

joints is from 30 ft. to 100 ft. The end of each day's concreting should coincide with a joint. One form of transverse expansion and contraction joint is illustrated in *Table* 96; a clear gap of about ½ in. is left between the faces of adjacent panels and the space is almost filled with a resilient material and sealed with a bituminous compound. In the centre of the slab, and at intervals of about 1 ft. to 2 ft. 6 in. across the width of the road, mild steel dowel bars 2 ft. or 3 ft. long and from ½ in. to 1 in. in diameter project horizontally from one panel to the next. One half of each bar is greased or otherwise treated to allow freedom of movement and is fitted with a ferrule; the other half is embedded in the concrete. Dowel bars prevent one panel rising relatively to its neighbour, partially prevent warping and curling, and transfer a part of the load on one panel to the other, thereby reducing the stresses.

Simple contraction joints (or dummy joints) are provided at intervals of about 15 ft. between transverse expansion and contraction joints; such a joint is illustrated in *Table* 96 and is merely a slot ½ in. wide and 2 in. deep, filled with bituminous compound, formed in the top only (or in some cases in the bottom only) of the slab. The provision of dummy joints enables the slab to crack at intervals without being unsightly, irregular or injurious.

Longitudinal joints are generally provided in roads more than 15 ft. wide so as to divide the road into strips about 10 ft. wide. A longitudinal joint may be a simple butt-joint, but some form of interlock is desirable to avoid one slab rising relatively to the adjacent slab and to enable transfer of load to take place. Thus a dowelled joint is sometimes provided, or a tongued-and-grooved joint as illustrated in *Table* 96.

The joints shown in *Table* 96 are typical and are suitable for a reinforced concrete road slab 7 in. thick on a medium or good soil carrying medium traffic. Similar and other designs are given in the publications of the Road Research Board.

Containers.

The weights of materials and the calculation of the horizontal pressure due to dry materials and liquids contained in tanks, reservoirs, bunkers, silos and other containers are given in *Tables* 4, 10, 11, 12 and 14. This section deals with the design of containers, and with the calculation of the forces and bending moments produced by the pressure of the contained materials. Where containers are required to be watertight, the recommendations given in B.S. Code No. 2007 (1960) for reinforced concrete structures for the storage of water have been adopted. Containers are conveniently classified as tanks containing liquids, and bunkers and silos containing dry materials, each class being sub-divided into cylindrical and rectangular structures. Some typical details of walls of containers are given in Appendix II.

Tanks.

Direct Tension in the Wall of a Cylindrical Tank.—The wall of a cylindrical tank is primarily designed to resist direct tension due to the horizontal pressures of the contained materials, and, if p lb. per square foot is the pressure at any depth, the direct tension T in a horizontal ring 1 ft. in depth is $0 \cdot 5pD$ lb., where D is the internal diameter of the tank (ft.). Sufficient circumferential reinforcement must be provided to resist this tension; appropriate formulæ are given in *Table* 97. For tanks containing liquids, the tensile stress in the circumferential reinforcement should not exceed 12,000 lb. per square inch, but for a container of dry materials a stress of 18,000 lb. per square inch can be allowed. The length of any overlaps in the circumferential reinforcement must be sufficient to enable the maximum tensile stress to be developed,

and for this reinforcement hooks at the ends of the plain round bars should be invariably provided in liquid-containers. For tanks containing liquids the thickness of the wall should be determined in relation to the total tension, as described on page 69 and in *Tables* 73 and 74, to reduce the risk of cracking. The minimum thickness of the wall is given by the appropriate general expression in *Table* 97, or the special expressions in *Tables* 73 and 74. It is sometimes recommended that the thickness of the wall of a tank containing liquid should not be less than 4 in. (or 5 in.) and not less than $2\frac{1}{2}$ per cent. of the depth of liquid plus 1 in. For cylindrical vessels containing dry materials or for lined tanks containing liquids, it is not so essential to design for no cracking, and the thickness of the wall is determined from practical considerations.

Bending Moments on the Walls of Cylindrical Tanks.—In addition to the horizontal tension in the wall of a cylindrical container, bending moments are produced by the restraint at the base of the wall. Unless a joint is made at the foot of the wall, as illustrated at (*b*) in *Table* 95, there is some continuity between the wall and the base slab which causes vertical deformation of the wall and reduces the circumferential tension. There are three principal factors, namely, the magnitude of the bending moment at the base of the wall, the point at which the maximum circumferential tension occurs, and the magnitude of the maximum circumferential tension. Coefficients and formulæ for determining these factors are given in *Table* 97 and are derived from Mr. H. Carpenter's translation of Dr. Reissner's analysis. The shape of the wall has some effect on the value of the coefficients, but the difference between the bending moments at the bases of walls of triangular or rectangular vertical section is so small that the common intermediate case of a trapezoidal section can be considered to be the same as a rectangular wall. The small error involved partly offsets the error of assuming perfect fixity at the junction of the wall and the base.

The procedure is first to determine the maximum vertical bending moment and provide a wall having an equal moment of resistance at the bottom. The maximum circumferential tension and the height up the wall at which this occurs are next determined; sufficient area of steel and thickness of concrete must be provided at this height to resist the maximum tension. Above this height the area of reinforcement can be uniformly decreased to a nominal amount, and below it the area of reinforcement can be maintained equal to that required for the maximum circumferential tension, although some reduction towards the bottom may be justified. Some typical details of walls of tanks with and without restraint at the bottom are given in Appendix II.

Octagonal Tanks.—If the wall of a tank is in plan, a series of straight sides instead of circular, the shuttering may be less costly but extra reinforcement or increased thickness of concrete or both is necessary to resist the horizontal bending moments which are produced in addition to the direct tension. If the tank is a regular octagon the bending moment at the junction of adjacent sides is $\frac{pL^2}{12}$, where L is the length of side of the octagon. If the distance across the flats is D, the direct tension in each side is $\frac{1}{2}pD$, and at the centre of each side the bending moment is $\frac{pL^2}{24}$. If the shape of the tank is not a regular octagon, but the lengths of the sides are alternately L_1 and L_2, and the corresponding thicknesses are D_1 and D_2, the bending moment at the junction of any two sides is $\dfrac{p\left[L_1^3 + L_2^3\left(\frac{D_1}{D_2}\right)^3\right]}{12\left[L_1 + L_2\left(\frac{D_1}{D_2}\right)^3\right]}$.

Walls of Rectangular Tanks.—The bending moments on, and direct tensions in, the walls of rectangular tanks are calculated in the same manner as described on page 102 for bunkers. For impermeable construction, however, the maximum tensile stress in the reinforcement and, to reduce the risk of cracking, the tensile stress in the concrete should not exceed the values given on page 69; the consequent design formulæ are given in *Table* 74; this data applies to suspended bottoms of tanks as well as to walls.

The walls of large rectangular reservoirs generally span vertically and are monolithically with the roof and floor slab, the floor being generally laid directly on the ground. If the wall is considered as freely supported at the top and bottom, and if P is the total water pressure on the wall, the force at the top is $0{\cdot}33P$ and at the bottom $0{\cdot}67P$. If the wall is assumed to be freely supported at the top and fixed at the bottom, the forces are $0{\cdot}2P$ and $0{\cdot}8P$ at the top and bottom respectively. As neither of these conditions is likely to be obtained, a practical assumption is that the forces at the top and bottom are $0{\cdot}25P$ and $0{\cdot}75P$ respectively; the positive bending moment at about the midpoint of a wall of height H ft. and the negative bending moment at the bottom are each $0{\cdot}083\ PH$. If the walls span vertically and horizontally *Table* 40 applies. Some typical details are given in Appendix II.

Bottoms of Elevated Tanks.—The type of bottom provided for an elevated cylindrical tank depends on the diameter of the tank and the depth of water. For small tanks a flat beamless slab is satisfactory, but beams are necessary for tanks from 10 ft. up to, say, 25 ft. in diameter. Some designs are indicated on the page facing *Table* 97; notes on the designs which include bottoms with beams and domed bottoms, and examples, are given on the pages facing *Tables* 97 and 98.

The weight of liquid in a tank having only one compartment should be considered as a dead load when calculating the bending moments on the slabs and beams of the bottom since all spans will be loaded simultaneously. Tee-beams in the bottom of a tank should be designed so that the tensile stress at the face of the concrete in contact with water, due to negative bending moments, does not exceed the safe tensile stress in the concrete (see page 69); for this purpose the maximum width of slab considered as the flange should be as given in *Table* 69.

Columns supporting Elevated Tanks.—It is important that there should not be unequal settlement of the foundations of the columns supporting an elevated tank and a raft should be provided if the nature of the ground is such that unequal settlement is likely. In addition to the bending moments and shearing forces due to the pressure of the wind on the tank, as described in *Table* 8, the wind force causes a thrust on the columns on the leeward side and a tension in the columns on the windward side; the values of the thrusts and tensions can be calculated for a group of columns from the expressions given on the page facing *Table* 98.

Effects of Temperature.—For a tank containing a hot liquid, the working stresses should be lower than for other tanks, or the probable increase in stress due to the higher temperature should be calculated as described on page 107.

In Britain the effects of temperature due to weather variations are seldom sufficiently great to be considered in the design of the tank, but elsewhere it may be necessary to protect the tank of a water tower from extreme exposure to the sun. External linings of timber, brick, or other material may be provided or the tank should be designed for the effects of the differences of temperature on opposite faces of the wall.

Joints.—Permanent joints are provided in large tanks, reservoirs, and similar

containers to allow for expansion and contraction due to changes of temperature or to shrinkage of the concrete, or to relieve parts of the structure from stresses due to indeterminate restraints that would otherwise be imposed by adjacent parts. Details of joints suitable for reservoirs, swimming pools, and tanks are given in *Table* 95.

Pipes.—Pipes built into concrete tanks are sometimes made of non-ferrous alloy, since deterioration due to corrosion is much less than for ferrous metals and replacements that may affect the water-tightness of the structure are obviated. Pipes built into the wall of a tank should have an additional intermediate flange cast in such a position that it will be buried in the thickness of the wall and thus form a water-bar.

Underground Tanks.—Underground or submerged tanks are subjected to external pressures due to the surrounding earth or water, which produce direct compression in the walls. The stress produced by this compression in the wall of a cylindrical tank is a maximum when the tank is empty, and is given by the expression in *Table* 97. Unless conditions are such that the permanence of the external pressure is assured, the relief to the tension provided by the compression should be disregarded in the calculation of the stresses in the tank when full. When empty, the structure should be investigated for flotation if it is submerged in a liquid or is in waterlogged ground.

Reservoirs with earth or other material banked up against the walls should be designed for earth pressure from outside with the tank empty. When the reservoir is full no reduction should be made to the internal pressure by reason of the external pressure, but in cases where the designer considers such reduction justified the amount of the reduction should be considerably less than the theoretical pressure calculated by the formulæ for active pressures in *Tables* 10, 11 and 12.

The earth on the roof of a reservoir should be considered as a live load, although it is ultimately a uniformly-distributed load acting on all spans simultaneously. When the earth is being placed in position, conditions may occur whereby some spans are loaded and others are unloaded. Often, however, the designer can ensure that the earth is deposited in such a manner as to keep the bending moments at a minimum.

Bunkers and Silos.

Properties of Contained Materials.—The weights of materials commonly stored in bunkers and silos are given in *Table* 4, and the pressures of these materials are dealt with in *Tables* 10 and 11. When calculating the size of a structure of a specified capacity, the weight of the material should not be overrated nor too small a value assumed for the angle of repose. When calculating the weight to be carried on the bottom and the pressures to which the sides will be subjected, the weight should not be under-estimated nor the angle of internal friction over-estimated. Generally two assumptions are therefore necessary in designing a container; examples of these assumptions are given in *Table* 11.

Walls.—The walls of bunkers and silos are designed to resist bending moments and tensions caused by pressure of the contained material. If the wall spans horizontally, it is designed for the bending moments and direct tension combined as in *Table* 89. If the wall spans vertically, horizontal reinforcement is provided to resist the direct tension and vertical reinforcement to resist the bending moments. In this case the horizontal bending moments due to continuity at corners should be considered, and it is generally sufficient if as much horizontal reinforcement is provided at any

level at the corners as is required for vertical bending at this level; the amount of reinforcement provided for this purpose, however, need not exceed the amount of vertical reinforcement required at one-third of the height of the wall. The principal bending moment on walls spanning vertically is due to the triangularly-distributed pressure from the contained material. Bending-moment coefficients for this distribution of load are given in *Tables* 16 and 17 for conditions of fixity or free-support at one or both ends of the span. The practical assumption described for the walls of rectangular reservoirs should be observed in this connection (see page 101).

For walls spanning horizontally the bending moments and forces depend upon the number and arrangement of the compartments. For structures with several compartments, the intermediate walls act as ties between the outer walls, and in *Table* 98 expressions are given for the negative bending moments on the outer walls of rectangular bunkers with various arrangements of intermediate walls or ties. The corresponding expressions for the reactions, which are a measure of the direct tensions in the walls, are also given. The positive bending moments can be readily calculated when the negative bending moments at the corners are known. An external wall is subject to maximum stresses when the adjacent compartment is filled, since it is then subjected simultaneously to the maximum bending moment and the maximum direct tension. An internal cross wall is subjected to maximum bending moment when the compartment on one side of it is filled, and to maximum direct tension (but no bending moment) when the compartments on both sides of the wall are filled.

In small bunkers the panels of wall may be of such proportions that they span both horizontally and vertically, in which case *Table* 40 should be used to calculate the bending moments since the pressure along the horizontal span will be uniformly distributed, while along the vertical span triangular distribution will occur. An example is given on page 320.

In the case of an elevated bunker the whole load is generally transferred to the columns by the walls, and when the span exceeds twice the depth of the wall the wall can be designed as a beam. Owing to the large moment of inertia of the wall (as a beam bending in a vertical plane) compared with that of the columns, the beam can be assumed to be freely supported but the heads of the columns under the corners of the bunker should be designed to resist a bending moment equal to, say, one-third of the maximum positive bending moment on the beam. If the provision of sufficient moment of resistance so requires, a compression head can conveniently be constructed at the top of the wall, but there is generally ample space to accommodate the tensile steel in the base of the wall. When the distance between the columns is less than twice the height of the wall the reinforcement along the base of the wall should be sufficient to resist a direct tension equal to one-quarter of the total load carried by the wall. The total load must include all other loads supported by the wall. These loads may be due to the roof or other superstructure or machines mounted above the bunker and to the weight of the wall.

The effect of wind on large structures should be calculated. In silos the direct compressive force on the leeward walls due to wind pressure is one of the principal forces to be investigated. The stress due to the eccentric compression due to the proportion of the weight of the contents supported by friction on the walls of a silo (see *Table* 14) must be combined with the stresses produced by wind pressure, and at the base or at the top of the walls there may be additional bending stresses due to continuity with the bottom or the covers or roof over the compartments.

If a wall is thicker at the bottom than at the top it may taper uniformly from bottom to top or the reduction in thickness may be made in steps. The shuttering may be more costly for a tapered wall than for a stepped wall, especially for cylindrical containers. A stepped wall, however, may be subjected to high secondary stresses at the change of thickness where also the day's-work joints generally occur. Stepping on the outside is often objectionable as it provides ledges for the collection of dust. Stepping on the inside may interfere with the free flow of the contents when emptying the container.

The size and shape of a bunker depend on the purpose which it is to serve, and the internal dimensions are therefore generally specified by the owner. Typical calculations are given on page 320 for a design in which the walls span vertically and horizontally. When the walls span horizontally the reinforcement varies from a maximum at the bottom to a nominal amount at the top; the vertical reinforcement need only be sufficient to keep the horizontal bars in place, and generally $\frac{3}{8}$-in. bars at 12-in. to 15-in. centres are satisfactory for this purpose. In the case of tall bunkers each lift of vertical reinforcement should not exceed about 10 ft., although if continuously-moving shuttering is used the vertical bars should be 4 ft. to 6 ft. long.

Hopper Bottoms.—The design of sloping hopper bottoms in the form of inverted truncated pyramids consists of finding, for each sloping side, the centre of pressure, the intensity of pressure normal to the slope at this point, and the mean span. The bending moments at the centre and edge of each slope are then calculated. The horizontal direct tension is next computed and combined with the bending moment to determine the amount of horizontal reinforcement required. The direct tension acting in the line of the slope at the centre of pressure and the bending moment at this point are combined to find the reinforcement necessary in the underside of the slab at this point. At the top of the slope the bending moment and the component of the hanging-up force are combined to determine the reinforcement required in the upper face at the top of the slope.

The centre of pressure and the mean span can be found by inscribing on a normal plan of the sloping side a circle touching three of the sides. The diameter of this circle is the mean span and the centre is the centre of pressure. The total intensity of load normal to the slope at this point is the sum of the normal components of the vertical and horizontal pressures at the centre of pressure and the dead weight of the slab. Values for the pressure on an inclined slab are given in *Table* 11, and expressions for the bending moments and direct tensions along the slope and horizontally are given in *Table* 99. When using this method it should be remembered that, although the horizontal span of the sloping side is considerably reduced towards the outlet, the amount of reinforcement should not be reduced below that determined for the centre of pressure, since in determining the bending moment based on the mean span adequate transverse support from the reinforcement towards the base is assumed.

The "hanging-up" force in the direction of the slope has a vertical and a horizontal component, the former being resisted by the walls acting as beams. The horizontal component, acting inwards, will produce horizontal bending moments on the beam at the top of the slope, but often this bending moment is neglected as the inward horizontal force is generally resisted by a corresponding outward pressure from the contained material. When there is insufficient head of material above the top of the slope to create the necessary counter-pressure, it is necessary to calculate and to make provision for the horizontal bending moment.

An example of the calculations by this method for a pyramidal hopper bottom is given on the page facing *Table* 99.

Chimneys.

Dimensions.—The height and internal sizes of a chimney are usually specified by the engineers responsible for the boiler installation. The reinforced concrete designer has to determine the thickness of, and reinforcement in, the shaft. The two principal forces on a chimney are the wind pressure and the dead weight. At any horizontal section the cantilever bending moment due to the wind is combined with the direct force due to the weight of the chimney and lining above the section considered to find the maximum stresses. Suitable values for wind pressures on circular and other shafts are given in *Table* 8, and an example showing a graphical method of determining the stresses is given in *Table* 85. Generally the preliminary determination of the thickness of the wall of the shaft is a matter of trial and error, but the thickness of concrete in inches and the total area of reinforcement required can be found approximately for a circular shaft from the expressions given in the table on the page facing *Table* 100. These expressions, which were established by Messrs. Taylor, Glenday, and Faber, are for stresses of 400 and 600 lb. per square inch in the concrete and 12,000, 14,000, and 16,000 lb. per square inch in the reinforcement. Low stresses are desirable for the combination of bending moment and weight only, as a margin is left thereby for increases in stresses due to rise of temperature. Having determined the trial dimensions for the shaft, the values of the bending moment and weight can be calculated more closely and the actual stresses obtained either by the graphical method described in *Table* 85 or by one of the analytical methods given in books on the design of chimneys; the stresses due to thermal effects have to be combined with those due to moment and weight, and the method of so doing is given in other publications. Some notes on thermal stresses are given on page 107.

Maximum Longitudinal Stresses.—The maximum stresses on any horizontal plane of a chimney shaft should be investigated for the following conditions. (*a*) When subjected to direct load only, that is the weight of the concrete shaft and the lining, the maximum compressive stress should not exceed the values given for direct stress in *Table* 56. For this purpose, the value of the modular ratio used in calculating the effect of the reinforcement can be assumed to be 15. (*b*) The stresses due to combining the bending moment due to the wind with the maximum direct load should be ascertained either by comparison with the factors given in the table on the page facing *Table* 100 or by the graphical analysis given in *Table* 85. To the maximum compressive stress at the inner face of the wall should be added the stress, c_T, due to the change of temperature, and to the maximum tensile stress in the reinforcement on the outer face should be added the stress t_T due to the change of temperature, thereby giving approximately the maximum stresses; a method of calculating t_T and c_T is given in the following (page 107). The maximum compressive stress in the concrete should not exceed the values given in *Table* 56 for bending.

The various stresses are inter-related, and the addition of, say, the temperature stresses to the combined bending and direct stresses may alter the basis of calculating these stresses by altering the position of the neutral plane, subjecting more of the concrete to tensile stresses in excess of the tensile strength, and thereby causing cracking.

Transverse Stresses.—The preceding remarks deal solely with stresses normal

to a horizontal plane. Stresses normal to a vertical plane are also produced both by wind pressure and differences of temperature. In chimney shafts of ordinary dimensions the transverse bending moment resulting from wind pressure is generally negligible, but this is not necessarily so in tanks and cooling towers of large diameter. A uniform pressure of p lb. per square foot produces a maximum bending moment of $\pm \frac{pD^2}{12}$ ft.-lb. on a height of 1 ft. of a cylinder of external diameter D ft. This bending moment causes a compressive stress at the outer face of a wall normal to the line of the action of p and tensile at the inner face. An equal bending moment, but of opposite sign, acts on the wall parallel to the line of action of p. If the distribution of wind pressure is as illustrated in *Table* 7 for cylinders, the bending moment at the end of the diameter normal to the line of action of p may be about three times that given in the foregoing.

A difference in temperature between the two faces of a concrete wall produces a transverse bending moment equal to $\frac{T\varepsilon_c E_c I_c}{d_c}$. (See pages 107 and 108 for notation.) If the shaft is unlikely to be cracked vertically, the maximum stress in the concrete due to this bending moment is about $0{\cdot}5\,T\varepsilon_c E_c$, being compressive on the face subjected to the higher temperature and tensile on the opposite face.

Industrial Structures.

In addition to the ability of the various members to sustain the forces and moments to which they are subjected, there are other considerations peculiar to each type of industrial structure. Vibration must be allowed for in the substructures for crushing and screening plants. Provision against overstressing a reinforced concrete pit-head frame is obtained by designing for various conditions of working and accidental loading such as described on page 148. Watertightness is essential in slurry basins, coal draining bunkers, settling tanks, and similar hydraulic structures, while airtightness is essential in gas purifiers and in airlock structures in connection with colliery work; the suction in airlocks is generally equivalent to a head of 5 in. to 10 in. of water, that is, 26 lb. to 60 lb. per square foot. The resistance of concrete to corrosion from fumes that are encountered in some industrial processes is one of the properties that recommends the material for industrial construction, but protection of the concrete is needed with other fumes and some liquids (see Appendix I). Provision should be made for expansion in structures in connection with steel-works, coke ovens, gas retorts, and other structures where great heat is experienced. Boiler foundations, especially on clay, should be made sufficiently thick to prevent undue heating and drying out of the subsoil or an insulating layer should be interposed between the foundation and the ground. Firing floors, coke-benches, and rolling-mill floors should be protected from extreme temperatures and abrasion by being covered with steel plates or bricks.

On floors where dust, rubbish, or slime may collect, as in coal washeries, it is advantageous to make a fall in the top surface of 3 in. in 10 ft. to facilitate the cleaning of the floors, but it must be ascertained that such a slope will not be inconvenient to the users of the floor; otherwise suitable channels must be provided to ensure that washing down will be effective.

Structures in mining districts should be designed for the possibility of subsidence of the ground upon which they stand. Thus raft foundations that have at any part equal resistance to negative and positive moments are commonly adopted for small

structures. If isolated foundations are provided for long structures such as gantries, the longitudinal beams should be designed as if freely supported.

Stresses due to Temperature.

The following consideration of stresses due to temperature can be applied to chimneys, tanks containing hot liquids, retort foundations, and other structures where there is a difference of temperature between two faces of the concrete.

The first stage is to determine the change of temperature T deg. F. through the concrete. The resistance to the transmission of heat through a wall of different materials, the successive thicknesses of which (in inches) are d_1, d_2, d_3, etc., is given by

$$\frac{1}{k} = \frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{d_3}{k_3} + \ldots + a_i + a_a + a_0 = \sum\frac{d}{k} + a_i + a_a + a_0$$

where k_1, k_2, k_3, etc., are the conductivities of the various materials of which the wall is made, a_i and a_0 represent the drop in temperature at the internal and external faces respectively, and a_a is the drop due to a cavity in the wall. Values of the conductivities, in B.th.u. per square foot per hour per degree F., are 6·7 for ordinary concrete, 7·4 for firebrick, and 0·3 for slag wool. The values of a_0, a and a_a are not readily determinable; a_0 for example depends on the degree of exposure and a value of 0·5 deg. F. for the transmission of one B.th.u. per square foot per hour is a reasonable average value, although in sheltered positions facing south the value may be 0·75 and as low as 0·10 for conditions of severe exposure facing north. In a chimney or in a tank containing hot liquid, there may be little difference between the temperature of the flue gas or liquid and the temperature of the face of the concrete or lining in contact with the gas or liquid. Hence a_i may often be neglected or, if some fall of temperature at the internal face is expected, a value of 0·7 may be used. The value of a_a depends on the amount of ventilation of the cavity and may be about unity.

If the temperature of the flue gas or hot liquid is T_G deg. F. and the external air temperature is T_A deg. F., the change of temperature through a concrete wall d_c in. thick is $T = \frac{(T_G - T_A)kd_c}{k_c}$ deg. F., where k_c is the conductivity of the concrete. Owing to the numerically indeterminate nature of many of the terms used in the calculation of T, extreme accuracy cannot be expected. Therefore only an approximate assessment of the stresses due to a difference in temperature is worth while.

In an uncracked reinforced concrete wall, or in a cracked wall that is entirely in compression, the change in the compressive stress in the concrete due to a temperature difference of T deg. F. is $c_T = \pm 0{\cdot}5T\varepsilon_cE_c$, where ε_c is the coefficient of linear expansion of concrete, say, 0·0000055 per degree F., and E_c is the modulus of elasticity of concrete. For E_c, a value of 3,000,000 lb. per square inch should be used.

In a cracked wall subject to tension, the concrete being neglected except as a covering for the reinforcement, the change in stress in the reinforcement is $t_T = \pm 0{\cdot}5(1 - f)T\varepsilon_sE_s$. The term $(1 - f)$ is such that $(1 - f)d_c$ is the distance between the centres of the reinforcement on opposite faces of the wall; ε_s is the coefficient of linear expansion for steel, say, 0·000006 per degree F., and E_s is the modulus of elasticity of steel, say, 30,000,000 lb. per square inch.

If the wall subjected to temperature strains is already stressed in tension on one face and compression on the other, as may occur in the wall of a tank containing hot liquid, then to the bending moment at any section a bending moment due to

change of temperature equal to $M_T = \frac{T\epsilon_c E_c I_c}{d_e}$ should be algebraically added, where I_c is the moment of inertia of the section expressed in concrete units and ignoring the area of any concrete that may be cracked. In impermeable construction designed to prevent the concrete cracking, the value of I_c would be based on the whole thickness of concrete together with an allowance for the reinforcement with a modular ratio corresponding to the value assumed for E_c.

It should be noted that the bending moment M_T due to change of temperature tends to produce compression on the face subject to the higher temperature.

Protection of Concrete.

Many structures, such as tanks, chimneys, and the floors of buildings used for certain purposes, have to resist the injurious effects of acids and other chemicals. In Appendix I a list is given which includes non-injurious and corrosive substances and liquids, and for injurious substances some protective treatments are described. Where corrosive liquids are stored in concrete containers the corrosion is generally most marked near the normal level of the liquid, and some liquids, which may not ordinarily affect the concrete, may cause corrosion when the concrete surface is alternately wet and dry.

The life of a reinforced concrete container exceeds that of a steel or timber structure, and can be further prolonged by careful design which reduces the liability to cracking, and thereby increases the resistance of the concrete to deterioration due to abrasion from falling and moving hard material. If attention is given to this matter and a dense concrete is used, lining the wearing faces of the structure with tiles or plates or rails may not be necessary except where coke is stored or where coal or stone is dropped into a bunker from a height. The top surfaces of ties and intermediate walls exposed to falling material should be made in the form of an inverted V and covered by a renewable metal shield. The bottoms of bunkers in which sticky coal or similar material is stored may be lined with glass to facilitate emptying.

Retaining Walls.

The weights of, and the methods of calculating the horizontal pressures due to, retained earth are given in *Tables* 10, 11 and 12. This section deals with the design of retaining walls including the calculation of the forces and bending moments produced by the pressure of the retained material. Other recommendations for the design of retaining walls, sheet-piled walls, and the like are given in Code No. 2, "Earth-Retaining Structures", published by the Institution of Structural Engineers.

Types of Retaining Walls.—A retaining wall is essentially a vertical cantilever, and when it is constructed in reinforced concrete it can be a cantilevered slab, a wall with counterforts, or a sheet-pile wall. A cantilevered slab is suitable for walls of moderate heights and has a base projecting backwards under the filling, as at (*b*) in the diagram at the top of *Table* 100, or a base projecting forward as at (*a*). The former type is generally the more economical. The latter type is only adopted when for reasons relating to buildings or other adjacent property it is not permissible to excavate behind the stem of the wall. If excavation behind the wall is permitted, but to a limited extent only, a wall with a base projecting partly backwards and partly forwards as at (*c*) can be provided. Any length of base projecting backwards

is advantageous as the weight of earth thereon assists in counterbalancing the overturning effects.

A wall provided with counterforts is suitable for a greater height than is economical for a simple slab wall. The slab spans horizontally between the vertical counterforts which are arranged as at (*d*). Where the net height of the wall is great, it is sometimes more economical to adopt the type of wall shown at (*e*), where the slab spans vertically between horizontal beams which bear against counterforts. By graduating the spacing of the beams the maximum bending moments in each span of the slab can be equal and the slab kept the same thickness throughout. When the shearing stresses allow, the web of the counterfort can be perforated; this saves concrete but complicates the shuttering and reinforcement.

Pressures behind Walls.—The value of the horizontal pressure due to retained earth is often assumed to be $27h$ lb. per sq. ft. at a depth h ft. below the level surface of the ground behind the wall. When the ground is compact a lower pressure is sometimes assumed, say, $22h$, which corresponds to an angle of repose of 40 deg. and a density of 100 lb. per cubic foot. The values $27h$ and $22h$ should be increased or reduced when the surface of the ground behind the wall slopes upwards or downwards or when a load is superimposed thereon. The pressure recommended by the Ministry of Transport is $30h$. In ground that may become accidentally waterlogged, it is often advantageous to design for a nominal factor of safety of 4 against ground pressure, and of $2\frac{1}{2}$ against the possible water pressure; that is, the equivalent pressure of the water alone after making allowance for the difference in factors of safety is $40h$

Cantilevered Retaining Walls.—The factors affecting the design of a cantilevered slab wall are usually considered per foot length of wall when the wall is of uniform height, but when the height of the wall varies a length of, say, 10 ft., or other convenient length, should be treated as a complete unit. For a wall with counterforts the length of a unit is the distance between two adjacent counterforts. The principal factors to be considered are stability against overturning, bearing pressure on the ground, resistance to sliding, and internal resistance to bending moments and shearing forces. Formulæ for the bending moments, forces, dimensions, and other factors relating to cantilevered-slab walls are given in *Table* 100, and typical details of cantilevered-slab walls of types (*a*) and (*b*) and of a wall with counterforts, and calculations for these designs, are given in Appendix II.

A factor of safety of not less than $1\frac{1}{2}$ against overturning should be allowed, rotation being assumed to be about the lowest forward edge of the base. Under the most adverse combination of vertical load and horizontal pressure, the maximum pressure on the ground should not exceed the pressure allowable on the ground upon which the wall is built.

To provide a factor of safety against forward movement of the wall as a whole the minimum total vertical load multiplied by a coefficient of friction should exceed the maximum horizontal pressure by at least 50 per cent. For dry sand, gravel, rock, and other fairly dry soils a coefficient of 0·4 is often used, but for clay, the surface of which may become wet, the frictional resistance to sliding may be zero. In this case the resistance of the earth in front of the wall must provide the necessary resistance to sliding, which can be increased by providing a rib on the underside of the base as shown at (*a*), (*b*) and (*c*) in *Table* 100. A rib is essential if the depth of earth in front of the wall is shallow. The plane of failure due to shearing in front of the wall is a curve sweeping upwards from the lowest forward edge of the wall. The

resistance of the earth in front of the wall is the passive resistance (see *Tables* 10 and 12). It is essential that earth be in contact with the front face of the base, otherwise a small, but undesirable, movement of the wall must occur before the passive resistance can operate. In walls of the form shown at (*a*), where the vertical load is small compared with the horizontal pressure, a rib should be provided either immediately below the wall stem or at the forward edge of the base to increase the resistance to sliding. If the theoretical passive resistance is depended upon to provide the whole of the resistance to sliding, the factor of safety should be at least two.

The foregoing movements of the wall, due to either overturning or sliding, are independent of the general tendency of the bank of a cutting to slip and to carry the retaining wall with it. The strength and stability of the retaining wall have no bearing on such failures, and the precautions necessary to be taken against the wall being carried away are outside the scope of the design of a retaining wall constructed to retain the toe of the bank and is a problem in soils mechanics.

The safe moment of resistance of the stem of the wall should be equal to the bending moment produced by the pressure on the slab. In a cantilevered slab, the critical bending moment may be at the top of the splay at the base of the stem. The base slab should be made the same thickness as the bottom of the wall and equal reinforcement should be provided. The base slab and the stem of the wall should be tapered.

When a single splay only is provided at the base of the stem of a cantilevered-slab wall, the critical bending moment may be at the bottom of the splay instead of at the top, since the increase in effective depth may not cause the moment of resistance to increase as rapidly as the bending moment increases. The effective depth should not be considered to increase more rapidly than is represented by a slope of one in three at each splay.

In walls with counterforts, the slab, which spans horizontally, can also taper from the bottom upwards as the pressure and consequently the bending moments decrease towards the top. Fixity with the base slab near the bottom will produce a certain amount of vertical bending requiring vertical reinforcement near the back face of the slab near the bottom. The horizontal negative and positive bending moments can be assumed to be $\frac{pL^2}{12}$, where p is the intensity of horizontal pressure and L is the distance between the centres of adjacent counterforts. If horizontal beams are provided the slab is designed as a continuous slab spanning vertically, requiring reinforcement near the front face between the beams and near the back face at the beams.

Counterforts are designed as vertical cantilevers, the main tensile reinforcement being in the back sloping face. Owing to the great width at the bottom, reinforcement for resistance to shear is seldom necessary, and when required it is generally most conveniently provided by horizontal binders. Only in the case of very high walls are inclined bars necessary for resistance to shearing.

Expansion and Contraction.—Long walls should be provided with expansion joints, suitable designs for which are given in *Table* 95. To reduce the risk of cracking due to shrinking of the concrete, sectional methods of construction should be specified, and as a further precaution against contraction and temperature cracks appearing on the front face of a wall reinforced near the back face, a mesh of reinforcement, say, ⅜-in. diameter bars spaced at 12-in. centres horizontally and vertically, should be provided near the front face if the thickness of the wall exceeds 8 in.

Drainage behind Walls.—Sloping the base slab in front of and behind the wall not only economises in concrete but also assists drainage, the provision for which is important, especially for a wall designed for a low pressure. Where the filling behind the wall is gravel or sand, a drain of clean loosely-packed rubble should be provided along the base of the back of the wall, and weep-holes, 3 in. to 6 in. in diameter, provided at intervals of about 10 ft. A weep-hole should be provided in every space between the counterforts, and the top surface of any intermediate horizontal beams should be given a slight slope away from the back of the wall. With backings of clay or other soil of low porosity, hand-packed rubble placed behind the wall for almost the whole height and draining to weep-holes, assists effective drainage of the filling. The filling behind the wall should not be tipped from a height, but should be carefully deposited and consolidated in thin horizontal layers.

Sheet-pile Walls.

When a satisfactory bearing stratum is not encountered at a reasonable depth below the surface in front of the earth to be retained, then a sheet-pile wall is provided. Precast reinforced concrete sheet-piles are driven into the ground sufficiently far to obtain an anchorage for the vertical cantilever and security against sliding and spewing. This type of wall is particularly suitable for waterside works, and in the simplest form the sheet-piles simply cantilever out of the ground, the heads of the piles being generally stripped and bonded into a cast-insitu capping beam.

Designs for typical sheet-piles and for the interlocked type of joint necessary to maintain alignment during driving, are given in the lower part of *Table* 101, where are also illustrated shapes of the shoes for starting piles and following piles.

If the height of the wall and the pressure on the sheet-piles are such that an excessively thick pile is required, the provision of a tie at the level of the capping beam reduces the maximum bending moment. The tie can be constructed in reinforced concrete or it can be a mild steel bar anchored into the capping beam and wrapped with bituminised hessian to protect it from corrosion. The capping beam must be designed to span between the ties and to transfer the horizontal forces from the top of the sheet-piles to the ties. The end of the tie remote from the wall should be anchored behind the natural slope of the ground, behind one of the lines shown in the upper part of *Table* 102. The anchorage should be provided by a block of mass concrete, by a concrete wall, by a vertical concrete plate, or by an anchor pile. Although the force in the tie is increased, bending moments on the sheet-piles can be further reduced by placing the tie at some point below the top of the wall, a horizontal beam being provided at the level of the tie. The provision of a tie reduces the depth to which it is necessary to drive the sheet-piles.

Cantilevered Sheet-pile Wall.—The forces on a simple cantilevered sheet-pile wall are indicated in *Table* 101, where P_1 is the active pressure due to the filling and surcharge behind the wall and P_2 and P_3 are passive pressures producing the necessary restraint moment to resist the overturning effect of P_1. The shaded diagram illustrates the probable variation of pressure, but the accompanying straight-line diagram is a practical approximation. The sheet-piles tend to rotate about the point X. The maximum bending moment on the sheet-piles occurs at some point D, and the distance L can be calculated approximately from the factors k_1' given in the column headed "free" in the tabulation in *Table* 100 for different angles of repose of the ground in which the pile is embedded. The bending moment on the

sheet-piles is P_1x, the value of P_1 being conveniently represented by the area of the trapezium ABCD in *Table* 101; P_1 can be determined from *Table* 11. The distance x indicates the centroid of the area. The embedded length of the sheet-piles must be great enough to enable sufficient passive pressures to be produced, and the factors k_2' (*Table* 100) enable this length to be calculated approximately.

The foregoing procedure using the factors k_1' and k_2' given in *Table* 100 is suitable for the preparation of a preliminary design for a simple cantilevered sheet-pile wall. The final design should be checked by the formulæ and procedure given in *Table* 101; the initial formula is derived by equating the forces acting behind the sheet-piles with those acting on the front of the sheet-piles, and by equating to zero the moments of these forces about E.

The value of p_a, the increase in active pressure per each foot of depth behind the wall, may be different at different depths if various classes of soil are encountered behind the wall and may be affected by waterlogged conditions. No general formula is serviceable under such conditions, and the designer should deal with such problems with caution and adopt safe values for the pressure factors. The two conditions which must be satisfied are that the algebraic sum of the horizontal forces must be zero and that the algebraic sum of the moments of these forces about the bottom of the sheet-piles must also be zero. The available theoretical passive resistance should be in excess of that required by a sufficient margin to allow for over-estimating the passive resistance.

Sheet-pile Wall with Ties.—When a tie is provided at the top of the wall the forces acting on the wall are as shown in *Table* 102; they are similar to those in *Table* 101 except for the introduction of the horizontal force in the tie. It is not possible to determine the variations of the pressure with any precision, but the diagram shows the probable variation. It is therefore recommended that the following procedure be adopted for preliminary designs. The factor k_2' in the column headed "hinged" in *Table* 100 gives the minimum value for h to produce sufficient restraint moment. The embedded length h must, however, be not less than the minimum length required to resist forward movement of the toe and not less than the length required to prevent spewing. The wall will be stable if $1{\cdot}5P_4x' \ngtr P_5y$, where P_4 is the total active pressure on the whole depth of the wall as shown and P_5 is the total passive pressure in front of the wall. The values of P_4 and P_5 can be computed from the data in *Tables* 10 and 11. The factor 1·5 is introduced in order to allow a margin between the theoretically calculated passive resistance and that actually required.

To prevent spewing in front of the wall the embedded length should not be less than $\frac{k_2^2 p_w}{w}$, where p_w is the weight per square foot at point E due to the earth and surcharge above this point, w is the weight per cubic foot of the earth in front of wall, and k_2 is the pressure factor taken from *Table* 10 or 11.

The bending moment on the wall can be calculated by first determining L from the factors given in the column headed "hinged" in *Table* 100. The sheet-piles can be considered as a propped cantilever of span L built in at D and propped at A and subjected to a trapezoidal load represented by the area ABCD. This load can be divided into a uniformly-distributed load and a triangularly-distributed load, the bending moment coefficients for which are given in *Table* 16, where is also given the reaction of the prop, which is the force in the tie. Since the security of this type of wall depends on the efficiency of the anchorage, no risk of underrating the force in

the tie should be incurred; it is better to increase the force to be resisted from the value represented by the theoretical reactions to $0{\cdot}5P_1$. The value of this force should certainly not be less than $P_4 - \frac{2}{3}P_5$, which is the part of the outward active pressure not balanced by the reduced passive pressure. The forces in anchor-piles are given in the lower part of *Table* 102.

Sheet-pile Wall with Tie below Head.—In a wall as in *Table* 102 it is assumed that the connection between the tie and the head of the wall is equivalent to a hinge, that is, the bending moment at A is zero. If the wall is extended above A, as shown, either by continuing the sheet-piles or by constructing a cast-insitu wall, a bending moment is introduced at A equal to $P_6 z$, where P_6 is the total active pressure on the extended portion of the wall AF. This bending moment introduces a negative bending moment on the wall at A, but reduces the positive bending moment on the sheet-piles between D and A and also reduces the negative bending moment at D. If the bending moment at A is large enough to produce conditions amounting to complete fixity at A, then the span L can be calculated by the factors k_1' given in the column headed " fixed " in *Table* 100. The factor k_2' in the same column gives the minimum embedded length h, but at the same time h must be sufficient to prevent spewing and forward movement as already described. The equation for stability is given in *Table* 102. The force in the tie is $P_4 - \frac{2}{3}P_5 + P_6$ or $0{\cdot}5P_4 + P_6$ whichever is the greater. The bending moment on the wall is calculated from the pressure represented by the area of the trapezium ABCD, considering the beam as fixed at both ends and using the appropriate coefficients given in *Table* 17. When the bending moment at A is insufficient to provide complete fixity, the bending moments, forces, and values of L and h are intermediate between those for hinged and fixed conditions at A.

A horizontal slab supported on king piles, as sometimes provided at A as shown in the diagram at the bottom of *Table* 102, has a sheltering effect on the piles since if the slab is carried far enough back it can completely relieve the wall below A from any active pressure due to the earth or surcharge above the level of A.

When a preliminary design has been prepared by the foregoing procedure, using the factors given in *Table* 100 and the formulæ in *Table* 102, the final design of the sheet-pile wall with anchored ties should be checked by one of the analytical or graphical methods given in text-books on this subject.

Reduced Bending Moments on Flexible Walls.—The pressures behind a flexible retaining wall adjust themselves in such a way that the bending moments on the wall are reduced. Stroyer suggested a formula applicable to reinforced concrete sheet-pile walls with ties. The reduction factors, which are not applicable to simple cantilevered walls, are given in *Table* 100.

Foundations.

The design of the foundations for a structure comprises three stages, the first of which is to determine from an inspection of the site the nature of the ground and, having selected the stratum upon which to impose the load, to decide the safe bearing pressure. The second stage is to select the type of foundation, and the suitability of one or more types may have to be compared. The third stage is to design the selected foundation to transfer and distribute the loads from the structure to the ground. Reference should be made to Code No. 4, " Foundations," issued by the Institution of Civil Engineers.

Inspection of the Site.—The object of an inspection of the site is to determine the nature of the top stratum and of the strata below in order to detect any weak strata that may impair the load-carrying capacity of the stratum selected for the foundation. Generally the depth to which knowledge of the strata should be obtained should be not less than one and a half times the width of an isolated foundation or the width of a structure with closely-spaced footings.

The nature of the ground can be determined by digging trial holes, sinking bores, or driving piles. A trial hole can be taken down to only a moderate depth, but enables the undisturbed soil to be examined, and the difficulty or otherwise of excavating and the need or otherwise of timbering and pumping to be determined. A bore can be taken very much deeper than a trial hole. A test pile does not indicate the kind of soil it has been driven through, but the driving data combined with local information may give the necessary particulars. A test pile is useful in showing the thickness of the top crust or the depth below poorer soil at which a firm stratum lies. A sufficient number of any of these tests should be made until the engineer is satisfied that he is certain of the nature of the ground under all parts of the foundations.

Reference should be made to British Standard Code of Practice No. 200.

Safe Bearing Pressures on Ground.—The pressures that can be safely imposed on thick strata of soils commonly met with are in some districts the subject of by-laws. *Table* 104 gives some pressures which are often recommended as a guide, but these pressures must be considered as maxima since there are several factors that may necessitate the use of lower values. Permissible pressures may generally be exceeded by an amount equal to the weight of earth between the foundation level and adjacent ground level but, if this increase is allowed, any earth carried on the foundation must be included in the foundation load. For a soil of uncertain resistance a study of local existing buildings on the same soil may be useful, as may also be the results of a ground bearing test.

Failure of a foundation may be due to consolidation of the ground causing settlement, or rupture of the ground due to failure in resistance to shearing. The shape of the surface along which shearing failure occurs under a strip footing is an almost circular arc extending from one edge of the footing and passing under the footing and continued then as a tangent to the arc to intersect the ground surface at an angle depending on the angle of internal friction of the soil. The average safe resistance of soil therefore depends on the angle of internal friction of the soil, and on the depth of the footing below the ground surface. In a cohesionless soil the resistance to bearing pressure not only increases as the depth increases but is proportional to the width of the footing. In a cohesive soil there is also an increase in resistance to bearing pressure under wide footings, but it is less than in non-cohesive soils. Graphical solutions, such as that attributed to Krey, are sometimes used to find the bearing resistance under a footing of known width and depth. The theoretical formulæ, based on Rankine's formula for a cohesionless soil and Mr. Bell's formula for clay, for the maximum bearing pressure on a foundation at a given depth, although giving irrational results in extreme cases, for practical cases give results that are well on the safe side. These formulæ are given in *Table* 104.

Unless they bear on rock, foundations for all but single-story buildings or other light structures should be taken down at least 3 ft. below the ground surface since, apart from the foregoing considerations, it is seldom that undisturbed soil which is sufficiently consolidated is reached at a less depth. In a clay soil a depth of

about 5 ft. is necessary in Britain to ensure protection of the bearing stratum from weathering.

Eccentric Load.—When a foundation is subjected to a concentric load, that is when the centre of gravity of the superimposed load coincides with the centroid of the foundation, the bearing pressure on the ground is for practical purposes uniform and its intensity is equal to the total applied load divided by the total area. When the load is eccentrically placed on the base the pressure is not uniformly spread, but varies from a maximum at the side nearer the centre of gravity of the load to a minimum at the opposite side, or to zero at some intermediate point. The variation of pressure between these two extremes depends on the magnitude of the eccentricity and is assumed to be linear. The maximum and minimum pressures are given by the formulæ in *Table* 104. For large eccentricities there may be a part of the foundation under which there is no bearing pressure. Although this state may be satisfactory for transient conditions (such as those due to wind), it is preferable for the foundation to be designed so that there is bearing pressure throughout under ordinary working conditions.

Blinding Layer.—For reinforced concrete footings or other construction where there is no mass concrete at the bottom forming an integral part of the foundation, the bottom of the excavation should be covered with a layer of lean concrete in order to provide a clean surface on which to place the reinforcement. The thickness of this layer depends upon the compactness and wetness of the bottom of the excavation, and is generally from 1 in. to 3 in. The safe compressive stress in the concrete should be not less than the maximum bearing pressure on the ground.

Types of Foundations.—The most suitable type of foundation depends, primarily, on the depth at which the bearing stratum lies and the safe bearing pressure which determines the area of the foundation. Data relating to common types of separate and combined reinforced concrete foundations, suitable for sites where the bearing stratum is near the surface, are given in *Tables* 104 and 105. Some types of combined bases are also given in *Table* 103. In selecting a type of foundation suitable for a particular purpose the type of structure should be considered. Sometimes it may have to be decided whether the risk of settlement can be taken in preference to providing a more expensive foundation. In the case of silos and fixed-end arches, risk of unequal settlement of the foundations must be avoided, but for gantries and bases for large steel tanks a simple foundation can be provided and probable settlement allowed for in the design of the superstructure. In mining districts, where subsidence is reasonably anticipated, a rigid raft foundation should be provided for small structures in order that the structure may move as a whole; as a raft may not be economical for a large structure, the latter should be designed as a flexible structure or as a series of separate structures each of which, on independent raft foundations, can accommodate itself to movements of the ground without detriment to the structure as a whole.

Separate Bases.—The simplest form of foundation for a reinforced concrete column or steel stanchion is the common pyramidal base (*Table* 104). Such bases are suitable for concentric or slightly eccentric loads if the area exceeds about 10 sq. ft. For smaller bases, and for some bases on rock or other ground of high bearing capacity, a rectangular block of plain concrete is probably more economical; the thickness of the block must be sufficient to enable the load to be transferred to the ground under the entire area of the base at an angle of dispersion through the block of not less than 45 deg. to the horizontal.

To reduce the risk of unequal settlement, the sizes of separate bases for the columns of a building founded on a compressible soil should be in proportion to the dead load carried by each column. Bases for the columns of a storage structure should be in proportion to the total load, excluding the effects of wind. In all cases the pressure on the ground under any base due to the combined dead and live load, including wind load and any bending at the base of the column, must not exceed the safe bearing resistance of the ground.

In the design of a separate base the area of a concentrically-loaded base (as in *Table* 104) is determined by dividing the maximum load on the ground by the safe bearing resistance. The thickness of a reinforced concrete footing of the common pyramidal shape is determined from consideration of resistance to shearing force and bending. The critical shearing stresses are assumed to occur on a plane at a distance equal to the effective depth of the base from the face of the column. This is in accordance with the recommendations of the B.S. Code, as is the requirement that the maximum bending moment at any section shall be the sum of the moments of all the forces on one side of the section. The critical section for the bending moment on a base supporting a reinforced concrete column is at the face of the column, but for a base supporting a steel stanchion it is at the centre of the base. The appropriate formulæ are given in *Table* 104. The moment of resistance of pyramidal bases cannot be determined with precision; the formulæ are rational, but conservative.

If the size of the base relative to its thickness is such that the load from the column can be spread by dispersion at 45 deg. over the entire area of the base, no bending moment need be considered and only nominal reinforcement need be provided. If the base cannot be placed centrally under the column, the pressure on the ground is not uniform, but varies as shown in *Table* 104. The base is then preferably rectangular in plan and the modified formulæ for bending resistance are given in *Table* 104. A special case of an independent base with the equivalent of eccentric loading is a chimney foundation.

A separate base may be subjected to moments and horizontal shearing forces in addition to a vertical concentric load. Such a base should be made equivalent to a concentrically-loaded base by placing the base eccentrically under the column to such an extent that the eccentricity of vertical load offsets the equivalent of the moments and shearing forces. This procedure is impracticable if the moments and shearing forces can act either clockwise or anti-clockwise at different times, in which case the base should be provided centrally under the column and designed as an eccentrically-loaded base complying with the two conditions.

Tied Bases.—Sometimes, as in the case of the bases under the towers of a trestle or gantry, the bases are in pairs and the moments and shearing forces act in the same sense on each base at the same time. In such conditions the bases can be designed as concentrically-loaded and connected by a tie-beam which relieves them of effects due to eccentricity. Such a pair of tied bases is shown in *Table* 105, which also gives the formulæ for the bending moment and other effects on the tie-beam.

Balanced Foundations.—When it is not possible to place an adequate base centrally under a column or other load owing to restrictions of the site, and when under such conditions the eccentricity would result in inadmissible ground pressures, a balanced foundation as shown in *Tables* 103 and 105 is provided. This case is common in the external columns of buildings on sites in built-up areas.

Combined Bases.—If the size of the bases required for adjacent columns is

so large that independent bases would overlap, two or more columns can be provided with a common foundation. Suitable types for two columns are shown in *Table* 105 for concentrically-loaded bases and for a base that cannot be arranged relative to the columns so as to be concentrically loaded. It may be that, under some conditions of loading on the columns, the load on the combined base may be concentric, but, under other conditions, the load on the same base may be eccentric; alternative conditions must be taken into account. Some notes on combined bases are given on the page facing *Table* 103.

Strip Bases.—When the columns or other supports of a structure are closely spaced in one direction, it is common to provide a continuous base similar to a footing for a wall. Particulars of the design of strip bases are given in *Table* 105. Some notes on these bases are given, in relation to the diagrams in *Table* 103, on the page facing the latter table, and an example is given on the page facing *Table* 105.

Rafts.—When the columns or other supports of a structure are closely spaced in both directions, or when the column loads are so high, and the safe ground pressure so low, that a group of independent bases almost or totally covers the space between the columns, a single raft foundation of one of the types shown in (i) to (iv) in *Table* 103 should be provided. Notes on these designs are given on the page facing the table.

The analysis of a raft foundation supporting a series of symmetrically arranged equal loads is generally based on the assumption of uniformly-distributed pressure on the ground, and the design is similar to an inverted reinforced concrete floor upon which the imposed load is the ground pressure due to the superimposed loads only. Notes on the design of a raft when the columns are not symmetrically disposed are given on the page facing *Table* 103.

Basements.—A basement, a typical cross-section of which is shown at (v) in the lower part of *Table* 103, is partly a raft, since the weights of the ground floor over the basement, the walls, and other structure above the ground floor, and the weight of the basement itself, are carried on the ground under the floor of the basement. For watertightness it is common to construct the wall and floor of the basement monolithically. In most cases the average ground pressure is low, but owing to the large span the bending moments are high and consequently a thick floor is required if the total load is assumed to be distributed uniformly over the whole area. Since the greater part of the load is transmitted through the walls of the basement, it is more economical to consider the load to be spread on a strip immediately under the walls if by so doing the ground pressure does not exceed the maximum allowable. The bending moment at the edge of the wall due to the cantilever action of this strip determines the thickness of the strip, and the remainder of the floor can generally be less thick.

Where basements are in water-bearing soils the effect of water pressure must be taken into account. The upward water pressure is uniform below the whole area of the basement floor, which must be capable of resisting the pressure less the weight of the slab. The walls must be designed to resist the horizontal pressure of the water-logged ground. It is necessary to prevent the basement from floating. There are two critical stages. When the structure is complete the total weight of the basement and all superimposed dead loading must exceed the maximum upward pressure of the water by a substantial margin. When the basement only is complete, there must also be an excess of downward load. If these conditions are not present, one

of the following steps should be taken. (1) The level of the ground-water near the basement should be controlled by pumping or other measures; (2) Temporary vents should be formed in the floor or at the base of the walls to enable water freely to enter the basement, thereby equalising the external and internal pressures; the vents should be sealed when sufficient dead load from the superstructure is obtained; (3) The basement should be temporarily flooded to a depth such that the weight of water in the basement, together with the dead load, exceed the total upward force on the structure. During the construction of the basement, method (1) is generally the most convenient, but when the basement is complete method (3) is preferable on account of its simplicity. The designer should specify the depth of water required, and a suitable rule for ascertaining this depth in a large basement is to assume 1 ft. for every foot head of ground water less 1 ft. for every 5 in. thickness of concrete in the basement floor above the waterproof layer. The omission of the weights of the basement walls and the ground floor provides a margin of safety.

Foundation Piers.—When a satisfactory stratum is found at 5 ft. to 15 ft. below the natural ground level a suitable foundation can be made by building up piers from the low-level to ground level, and commencing the construction of the columns or other supports on these piers at ground level The piers are generally square in cross-section, and can be constructed in brick, masonry, or plain or reinforced concrete. The maximum bearing pressure of the construction on the top of the pier depends on the material of the pier. Safe pressures on plain concrete, brickwork, and masonry, in accordance with B.S. Code 111, are given in *Table* 104; safe pressures on plain concrete in accordance with the London By-laws are given in *Table* 56.

The economical size of the pier is when the load it carries is great enough to require a base to the pier equal in area to the smallest hole in which men can conveniently work; otherwise unnecessary excavation has to be taken out and refilled. For example, if a man can conveniently work in a hole one yard square at a depth of 12 to 15 ft., the total load would be 36 tons on a stratum capable of sustaining 4 tons per square foot. Generally it is better to provide as few piers as possible and to impose as much of the load as practicable on to each pier, thus making each pier of generous proportions. It may not be necessary to dig a hole larger than is required for the stem of the pier, as the ground at the bottom may be firm enough to be undercut for a widening at the base. In the case of concrete piers this procedure reduces the amount of shuttering.

Reinforced concrete columns can sometimes be economically taken down to moderate depths, but to avoid slender columns it is generally necessary to provide lateral support at ground level.

When piers are impracticable, either by reason of the depth at which a firm bearing statrum occurs or due to the nature of the ground requiring timbering or continuous pumping, piles are adopted.

Wall Footings.—When the load on a strip footing is uniformly distributed throughout its length, as in the general case of a wall footing, the principal bending moments are due to the transverse cantilever action of the projecting portion of the footing. If the wall is of concrete and is built monolithically with the footing, the transverse bending moment at the face of the wall is the critical bending moment. If the wall is of brick or masonry the maximum bending moment occurs under the centre of the wall. Expressions for these bending moments are given in *Table* 105. When the projection is less than the thickness of the base the transverse bending

moments can be neglected, but in all cases the thickness of the footing should be such that the safe shearing stress is not exceeded.

Whether wall footings are designed for transverse bending or not, if the safe ground pressure is low, longitudinal reinforcement should be inserted to resist possible longitudinal bending moments due to unequal settlement and non-uniformity of the load. One method of providing the amount of longitudinal reinforcement required for unequal settlement is to design the footing to span over a cavity (or area of soft ground) 3 ft. to 5 ft. wide, according to the nature of the ground. The longitudinal bending moment due to non-uniform load is calculated in the same way as for combined footings.

Foundations for Machines.—The area of a concrete base for a machine or engine must be sufficient to spread the load on to the ground without exceeding the safe bearing pressure. It is an advantage if the shape of the base is such that the centroid of the bearing area coincides with the centre of gravity of the loads when the machine is working. This reduces the risk of unequal settlement. If vibration from the machine is transmitted to the ground the bearing pressure should be considerably lower than that generally assumed for the class of ground upon which the base bears, especially if the ground is clay or contains a large proportion of clay. It is often essential that the vibration of the machine shall not be transmitted to adjacent structures either directly or through the ground. In such cases a layer of cork or similar insulating material is placed between the concrete base carrying the machine and the ground. Sometimes the base is enclosed in a pit lined with insulating material. When transmission of vibration is particularly undesirable the base may stand on springs, or more elaborate damping devices may be installed. In all cases, however, the base should be separated from surrounding concrete ground floors.

With light machines the bearing pressure on the ground may not be the factor that decides the area of the concrete base, since the area occupied by the machine and its frame may require a base of larger area. The position of the holding-down bolts generally determines the width and length of the base, which should extend 6 in. or more beyond the outer edges of holes left for the bolts.

The depth of the base must be such that the bottom is on a satisfactory bearing stratum and that there is sufficient thickness to accommodate the holding-down bolts. If the machine exerts an uplift on any part of the base, the dimensions of the base must be such that the part subjected to uplift has sufficient weight to resist the uplift with a suitable margin of safety. A single base should be provided under all the supports of one machine, and sudden changes in depth and width of the base should be avoided. This reduces risk of fractures that might result in unequal settlements that might throw the machine out of alignment. If the load from the machine is irregularly distributed on the base, the dimensions of a plain concrete base should be sufficient to resist the bending moments produced therein without overstressing the concrete in tension. If there is any risk of so overstressing the concrete, or if the operation of the machine would be adversely affected by cracking and deformation of the base, reinforcement should be provided to resist all tensile forces.

Reinforced Concrete Piles.

Precast Piles.—Reinforced concrete piles are either precast or cast insitu. Precast piles have been driven in lengths exceeding 100 ft., although when more than 60 ft. long it is necessary to give special consideration to the design of the pile and

of the lifting and driving plant. Piles less than 15 ft. long are seldom economical. For ordinary work precast piles are generally square or octagonal and are from 8 in. to 18 in. wide. For their support, piles depend either on direct bearing on a firm stratum or on frictional resistance in soft strata, or more often on a combination of both resistances. The safe load on a pile depends on the load that the pile can safely carry as a column and on the load that produces settlement or further penetration of the pile into the ground. So many factors affect the load causing settlement for any particular pile that calculated loads are not very reliable unless associated with loading tests on driven piles. Such tests are often inconvenient and expensive, and frequently an engineer has to rely on computed loads and a large factor of safety.

Formulæ for calculating the safe load on a pile are either impact formulæ which are applicable to bearing piles, or friction formulæ which are applicable to piles that are supported by the frictional resistance of the ground in which they are embedded. The formula of Mr. Hiley includes most of the variants occurring in pile driving such as the weight and the type of the hammer, the fall of the hammer, the penetration per blow, the length of the pile, the type of helmet, the nature of the ground, and the material of which the pile is made. A modified form of this formula is given in *Table* 106, in which the constant c takes into account the energy absorbed in temporarily compressing the pile, the helmet, and the ground. Since the quake of the ground below the pile shoe is included, it follows that the nature of the ground in which the toe of the pile is embedded affects the value of c, and the tabulated values apply to firm gravel; c must be increased if the pile is driven by a long dolly. The dimension $2c$ is a quantity that is measurable on a pile while being driven, since it represents the difference between the permanent penetration for one blow and the greatest instantaneous depression of the pile head as measured at the top of the helmet. The efficiency of the blow depends on the ratio of the weight of the pile (including the weight of the helmet, dolly, cushioning, and the stationary parts of the hammer resting on the pile head) to the weight of the moving parts of the hammer. Values for the efficiency of the blow are given in *Table* 106, together with values of the effective fall which allow for the freedom or otherwise of the fall of the hammer. The resistance to driving as calculated by Mr. Hiley's formula is subject to a factor of safety of 1½ to 3. If a pile is driven into clay or soils in which clay predominates, or into fine saturated sand, the resistance to further penetration may increase after the pile has been at rest for a while. This increase is due to the frictional resistance of the soil settling around the pile, but may be in part offset by a reduction in the bearing resistance, if on clay, which takes place in the course of time. Impact formulæ are not therefore very reliable for piles driven into clay, nor for piles that are driven into sand with the assistance of a water-jet.

When piles are driven into soft ground and depend solely upon friction between the sides of the pile and the ground for their support, the safe load can only be estimated approximately by considering the probable frictional resistance offered by the strata through which the pile is driven and the probable bearing resistance of the ground under the toe of the pile. Formulæ may be of little assistance in this case; a test load on an isolated pile or on a group of piles is the most satisfactory means of determining the settlement load.

A formula by which an estimate of the safe load on a pile driven entirely into clay can be derived is given in *Table* 106. An alternative formula is

$$\text{Safe load (lb.)} = \frac{1}{F}\left[\frac{A_s C}{2} + A(7{\cdot}5C + w_s L_1)\right]$$

The notation is as follows. C is the cohesive strength (lb. per sq. ft.) and w_s is the density (lb. per cu. ft.) of the clay; A is the cross-sectional area (sq. ft.) of the pile, the embedded length of which is L_1(ft.); F is a factor of safety (see *Table* 106).

Precast piles should be designed to withstand the stresses due to lifting, driving, and loading, with appropriate factors of safety. Overstressing the concrete during handling and slinging can be guarded against by arranging the position and number of the points of suspension so that the stresses due to bending moments produced by the weight of the pile are within safe limits. For square piles of 1 : 1½ : 3½ concrete and having the ordinary percentage of longitudinal steel the maximum bending moment due to bending about an axis parallel to one side of the section should not exceed, say, $60D^3$ in.-lb., where D is the length of the side of the pile in inches, if cracking is to be avoided.

The moment of resistance of a square pile bending about a diagonal is only about two-thirds of that when bending about an axis parallel to one of the sides. For this reason bending about a diagonal should be avoided where possible. If lifting holes are provided there is some assurance that the pile will not be lifted so as to bend about a diagonal. The lifting holes or the points of suspension should be arranged so that the smallest bending moments are experienced during lifting, and the positions for this condition for lifting at one or at two points are given in *Table* 106.

The greatest compressive stress in a pile is generally that due to the driving and occurs near the head. If driving is severe helical binders should be provided at the top of the pile. Octagonal piles generally have helical binders throughout their length.

In *Table* 106 is shown the reinforcement in a square precast reinforced concrete pile, in which helical binders at the head of the pile are provided. The arrangement of the lifting holes and spacers is also indicated. For driving into clay, gravel or sand, a pile shoe having an overall taper of about 2 to 1, as shown, is generally satisfactory, but for other types of soil other shapes of shoe are necessary. If the pile has to be driven through soft material to bear on gravel overlying softer ground it is necessary to have a blunter shoe to prevent punching through the thin stratum. For friction piles driven into soft material throughout a shoe is not absolutely necessary, and a blunter end should be formed as shown in *Table* 106. When driving through soft material to a bearing on soft rock or stiff clay, the form of pile end shown for this case is satisfactory so long as driving ceases as soon as the firm stratum is reached or is only just penetrated. When driving down to hard rock, or where heavy boulders are anticipated, a special shoe or point as shown should be fitted.

Irrespective of the load a pile can carry before settlement occurs, the stresses produced by the load on the pile acting as a column should be considered. For calculation of the reduction of load due to slenderness (see *Table* 83) the effective length of the pile can be considered as two-thirds of the length embedded in soft soil, or one-third of the length embedded in a fairly firm ground, plus the length of pile projecting above the ground. The end conditions of a pile are generally equivalent to one end fixed and one end hinged.

Arrangement of Piles.—In preparing plans of piled foundations attention must be given to the practicability of driving as well as to effectiveness for carrying

loads. In order that each pile in a group shall carry an equal share of the load the centre of gravity of the group should coincide with the centre of the superimposed load. The clear distance between any two piles should generally be not less than 2 ft. 6 in. So far as possible piles should be arranged in straight lines in both directions throughout any one part of a foundation, as this form reduces the amount of movement of the driving frame. The arrangement should also allow for driving to proceed in such a way that any displacement of earth due to consolidation in the piled area shall be free to take place in a direction away from the piles already driven.

Pile-caps.—Pile-caps should be designed primarily for punching shear around the heads of the piles and around the column base and for bending moment due to transferring the load from the column to the piles. The thickness of the cap should also be sufficient to provide adequate bond length for the bars projecting from the pile and for the dowel bars for the column. If the thickness is such that the column load can all be transmitted to the piles by dispersion no bending moments need be considered, but generally when two or more piles are placed under one column it is necessary to reinforce the pile-cap for the bending moments produced.

Loads on Piles in a Group.—If the centre of gravity of the total load W_T on a group of N vertical piles is at the centre of gravity of the piles, each pile will be equally loaded, and will be subjected to a load $\frac{W_T}{N}$. If the centre of gravity of the load is displaced a distance e from the centre of gravity of the group of piles, the load on any one pile is $W_T\left(\frac{1}{N} \pm \frac{ed_1}{\Sigma d^2}\right)$, where Σd^2 is the sum of the squares of the distance of each pile measured from an axis passing through the centre of gravity of the group of piles and at right angles to the line joining this centre of gravity to the centre of gravity of the applied load; d_1 is the distance of the pile considered from this axis, and is positive if it is on the same side of the axis as the centre of gravity of the load and negative if it is on the opposite side.

If the structure supported on the group of piles is subject to a bending moment M, which is transmitted to the foundations, the expression given for the load on any pile can be used by substituting $e = M/W_T$.

The total load that can be carried on a group of piles is not necessarily the safe load calculated for one pile multiplied by the number of piles, as allowance must be made for the overlapping of the zones of stress in the soil supporting the piles. The reduction due to this effect is greater in a group of piles that are supported mainly by friction. For piles supported entirely or almost entirely by bearing the maximum safe load on a group cannot greatly exceed the safe load on the area of the bearing stratum covered by the group.

Cast-insitu Piles.—The following advantages are obtained with cast-insitu piles, although all are not applicable to any one system.

The length of each pile conforms to the depth to the bearing stratum and no pile is too long or too short; cutting-off surplus lengths or lengthening insitu is not therefore required. The top of a pile can be at any level below ground, and in some systems at any level above ground. The formation of an enlarged foot giving a greater bearing area is possible with some types of piles. With tube-driven or mandrel-driven piles it is possible to punch through a thin intermediate hard stratum. Boring shows the class of soil through which the pile passes and the nature of the bearing stratum can be observed. A bored pile has little frictional resistance, but greater frictional resistance in soils such as compact gravels is obtained in tube-driven types

where the tube is withdrawn. Bored piles have no ill-effect on adjacent piles or on the level of the ground due to consolidation of ground when several piles are driven in a constricted atea. Boring piles is less noisy and is vibrationless; only a small headroom is required.

Some of the advantages of a precast pile over a cast-insitu pile are that hardening of the concrete is unaffected by deleterious ground waters; that the pile can be inspected before being driven into the ground; that the size of the pile is not affected by water in the ground (this applies also to cast-insitu types with a central core); and that the pile can be driven into ground that is below water. In neither the precast pile nor the cast-insitu pile are damage to, or faults in, the pile visible after it is driven or formed. The designer must consider the conditions of any problem, and select the pile which complies with the requirements.

Groups of Inclined and Vertical Piles.—*Table* 107 and the examples on the pages facing *Tables* 106, 107 and 108, relate to the loads on piles in a group that project above the ground, as in a wharf or jetty. For each probable condition of load the external forces are resolved into horizontal and vertical components H and W, the points of application of which are also determined. If the direction of action and position are opposite to those shown in the diagrams, the signs in the formulæ must be changed. It is assumed that the piles are surmounted by a rigid pile-cap or superstructure. The effects on each pile when all the piles are vertical are based on a simple, but approximate, statical analysis. Since a pile has little resistance to bending, structures with vertical piles only are not suitable when H predominates. The resistance of an inclined pile to horizontal force is considerable. In groups containing inclined piles, the bending moments and shearing forces on the piles are negligible. The ordinary theoretical analysis, upon which *Table* 107 is based, assumes that each pile is hinged at the head and toe. Although this is not an accurate assumption, the theories based thereon accord fairly well with the behaviour of actual groups of piles.

Wharves and Jetties.

The loads, pulls, blows, and pressures to which wharves and jetties and similar waterside and marine structures may be subjected are dealt with on page 14. Such structures may be a solid wall of plain or reinforced concrete, as are most dock walls and some quays, in which case the pressures and principles described on pages 19 *et seq.* and in *Tables* 10 to 13 and 100 for retaining walls apply. A quay or similar waterside wall is more often a sheet-pile wall, which is dealt with in *Tables* 101 and 102, or it may be an open-piled structure similar to a jetty. Piled jetties and the piles for such structures are considered in *Tables* 106 and 107. If the piles in a group containing inclined piles are arranged symmetrically, the summations in *Table* 107 are simplified thus: $\Sigma_2 = 0$; $G = \Sigma_1\Sigma_5$; $y_0 = -(\Sigma_4 \div \Sigma_5)$; Σ_3 is not required since $x_0 = 0{\cdot}5x_n$. Three designs of the same typical jetty are given on the pages facing *Tables* 106, 107 and 108.

SECTION 6

SPECIFICATIONS AND QUANTITIES

Contract Documents.

DOCUMENTS relating to a contract are the Form of Tender, Form of Agreement, Conditions of Contract, Specification, and in most cases Bills of Quantities, a Schedule of Prices, and Contract Drawings. The drawings are later amplified by working drawings and bar-bending schedules. Standard forms of tender and agreement should be used. Since the Conditions of Contract include clauses of a legal nature, it is advisable to use a standard form such as those of the Royal Institute of British Architects for building contracts under the supervision of an architect, of the Institution of Structural Engineers for a contract of purely structural nature, or of the Institution of Civil Engineers for structural work combined with civil engineering. The Specification is generally divided into one part dealing with materials and another with workmanship and manufactured goods. A specification suitable for ordinary reinforced concrete construction follows. It should not be used without making sure that all the requirements apply. Generally some clauses or parts of clauses will have to be omitted or altered and other requirements may have to be inserted.

As British Standards are often revised, the year of publication of the latest Standard should be inserted where required. The engineer should read each British Standard quoted to ensure that his decisions on all optional requirements are expressed in the Specification for the works.

A Specification for Reinforced Concrete Construction.

MATERIALS.—Materials used in the Works shall be new, of the qualities and kinds specified herein and equal to approved samples. Delivery shall be made sufficiently in advance to enable further samples to be taken and tested if required. No materials shall be used until approved, and materials not approved shall be immediately removed from the Works at the Contractor's cost.

Materials shall be transported, handled, and stored on the site or elsewhere in such a manner as to prevent damage, deterioration, or contamination.

REINFORCEMENT.—Mild steel bars shall be plain round hot-rolled steel bars complying with B.S. No. 785. Cold-drawn wire shall be plain round cold-drawn hard mild steel wire complying with B.S. No. 785. Twisted square and twin-twisted bars shall be rolled mild steel bars of suitable sizes which have been twisted when cold and shall comply with B.S. No. 1144. Welded wire fabric, twisted square-bar fabric, or expanded metal shall comply with the appropriate part of B.S. No 1221. Twisted ribbed bars and other reinforcement for which there is no British Standard shall comply with the Engineer's requirements.

Reinforcement shall be free from pitting, loose rust, mill scale, paint, oil, grease, adhering earth, ice, or any other material that may impair the bond between the concrete and the reinforcement or that may cause corrosion of the reinforcement or disintegration of the concrete. Adhering limewash or cement grout shall be permitted.

Neither the size nor length of a bar or wire shall be less than the size or length described in the bar schedule or elsewhere, and the length shall be not more than 2 in. in excess of the length so described.

CEMENT.—The cement shall be ordinary-setting cement of approved manufacture and shall comply with B.S. No. 12 for ordinary and rapid-hardening Portland cement, B.S. No. 146 for Portland-blastfurnace cement, B.S. No. 915 for high-alumina cement,

or B.S. No. 1370 for low-heat Portland cement. Compression tests on cement-sand cubes shall be made instead of tensile tests. Sulphate-resistant and other special cements shall comply with the Engineer's requirements.

All cement shall be fresh when delivered. Cements of different types shall not be mixed one with another. Consignments shall be used in the order of delivery. Admixtures shall be used only if approved.

AGGREGATE.—Materials used as aggregates shall be obtained from a source known to produce aggregates satisfactory for concrete and shall be chemically inert, strong, hard, durable, of limited porosity, and free from adhering coatings, clay lumps, coal and coal residues, and organic or other impurities that may cause corrosion of the reinforcement or may impair the strength or durability of the concrete. Aggregates shall be tested in accordance with the requirements of B.S. No. 812 and the results of such tests shall be as hereinafter specified, the percentages being by weight unless the context indicates otherwise.

FINE AGGREGATE.—Fine aggregate shall be natural sand or sand derived by crushing gravel or stone and shall be free from coagulated lumps. Sand derived from stone unsuitable for coarse aggregate shall not be used as fine aggregate.

The caustic-soda test for organic impurities shall show a colour not deeper than that of the standard solution. The amount of fine particles as ascertained by the laboratory sedimentation test shall not exceed 10 per cent. for crushed stone and 3 per cent. for natural sand or crushed gravel. In the settling test, and after being allowed to settle for three hours, the thickness of the layer of silt deposited on the coarser material shall not exceed 6 per cent. by volume.

The grading of fine aggregate shall be such that not more than 10 per cent. shall exceed $\frac{3}{16}$ in. in size. For a natural sand or crushed gravel not more than 10 per cent., and for crushed stone not more than 20 per cent., shall pass B.S. sieve No. 100. Between these limits the grading shall conform to the grading for either Zones 1, 2, or 3 (B.S. 882), or Zone 4 if so instructed.

COARSE AGGREGATE.—Coarse aggregate shall be crushed or uncrushed gravel or crushed stone. The pieces shall be angular or, except for concrete surfaces subject to abrasion, rounded in shape and shall have granular or crystalline or smooth (but not glassy), non-powdery surfaces. Friable, flaky, and laminated pieces, mica, and shale shall only be present in such quantities as not to affect adversely the strength and durability of the concrete.

The "aggregate crushing value" shall not exceed 45 per cent. and for concrete surfaces subject to abrasion shall not exceed 30 per cent. The amount of fine particles occurring in a free state or as a loose adherent shall not exceed 1 per cent. when determined by the laboratory sedimentation test. After twenty-four hours' immersion in water, a previously-dried sample of the coarse aggregate shall not have gained in weight more than 10 per cent. or not more than 5 per cent. if for use in impermeable construction.

The grading of coarse aggregate shall be such that not more than 5 per cent. shall be larger than $\frac{3}{4}$-in. and not more than 10 per cent. shall be smaller than $\frac{3}{16}$ in. and not less than 25 per cent. or more than 55 per cent. shall be smaller than $\frac{3}{8}$ in.

"ALL-IN" AGGREGATE.—All-in aggregate shall comply in all respects except grading with the requirements for fine and coarse aggregates, and for the purposes of all tests except the grading test shall be separated into material smaller than $\frac{3}{16}$ in. and material $\frac{3}{16}$ in. and over; these materials shall be considered as fine and coarse aggregates respectively.

The grading of all-in aggregate shall be such that not more than 5 per cent. shall exceed $1\frac{1}{2}$ in. and not more than 6 per cent. shall pass B.S. sieve No. 100, and not less than 45 per cent. or more than 75 per cent. shall be smaller than $\frac{3}{4}$ in. and not less than 25 per cent. or more than 45 per cent. shall be smaller than $\frac{3}{16}$ in.

WATER.—Water shall be clean and fresh and free from organic or inorganic matter in solution or suspension in such amounts that may impair the strength or durability of the concrete. Water shall be obtained from a public supply where possible, and shall be taken from any other source only if approved. No sea-water or water from excavations shall be used. Only water of approved quality shall be used for washing out shuttering, curing concrete, and similar purposes.

HOLLOW CLAY BLOCKS.—Hollow clay blocks for use in the construction of slabs shall be accurate in shape, free from twist, and with right-angles at all external corners. The external dimensions shall be as described on the drawings or elsewhere within a tolerance of 5 per cent. under or over. The thickness of material shall nowhere be less than ⅜ in. The blocks shall be sound, free from cracks and lime, well and evenly burnt, of uniform density, and with a crushing strength not less than 2,500 lb. per square inch. Tests for shape and strength shall be made in compliance with B.S. No. 1190 and the results of such tests shall be within the limits specified therein. The average weight of any twelve blocks shall not exceed the weight given on the drawings or elsewhere. All faces of blocks intended to receive plaster or rendering shall be scored or roughened in an approved manner to form a key.

FILLING MATERIAL.—Materials for filling shall be uniform in character throughout and free from substances that by decay or otherwise may cause the formation of hollows or cavities or otherwise affect the stability of the filling.

Earth filling shall be selected material obtained from the excavation or other approved source. No soft chalk or clay or earth with a predominating clay content shall be used. Hardcore shall be selected hard clean gravel, broken brick, broken concrete, broken or crushed stone, quarry waste, or similar approved material. Concrete for filling shall have the proportions described.

WORKMANSHIP.—The workmanship shall be of the quality specified, and all persons employed on the Works or elsewhere in connection with the Works shall be competent and skilled in their respective occupations.

EXCAVATION.—The site of the Works shall be cleared and excavated or filled to the levels given on the drawings or elsewhere. Excavations shall be of the width and length necessary for the construction of the foundations or other work below ground. The depths of all excavations shall be decided by the Engineer. The Contractor shall record on a suitable plan, which shall be deposited with the Engineer, the depth of every foundation as constructed.

Any obstacle encountered during the excavation shall be reported to the Engineer and shall be dealt with as then instructed.

Material taken out of the excavation shall be disposed of as instructed.

The Contractor shall supply and maintain and remove any necessary planking and strutting, sheet-piling, or cofferdams, and shall by pumping or other approved means maintain the excavations free from water.

Excavations shall be left open for as short a period as practicable. Immediately before foundations or other work be constructed therein, the sides of the excavation shall be trimmed if necessary, and the bottom shall be cleaned and if in loose or disturbed ground shall be well rammed, and the whole shall be approved.

DEPOSITING FILLING.—Natural hollows or pockets where soft ground has been taken out below the level of foundations or ground slabs or similar shall be filled with hardcore or 1 : 6 concrete or earth filling as instructed. Earth filling shall be in layers not exceeding 8 in. thick, each layer being well rammed to the same degree of consolidation as the surrounding ground.

Filling behind or at the sides of foundations or other work under ground shall consist entirely of earth surplus from the excavations and shall be deposited as the work proceeds in layers not exceeding 1 ft. thick, each layer being well rammed. Filling behind retaining walls or similar shall consist of approved selected earth deposited gently against the back of the wall in layers not exceeding 3 ft. in depth unless otherwise instructed. The earth shall be prevented from mixing with the rubble backing (if any) behind the wall, and the earth and rubble shall be brought up together.

PREPARATION OF GROUND BELOW PERMANENT CONSTRUCTION.—Plain concrete in foundations or site concrete shall be placed in direct contact with the bottom of the excavation, the concrete being deposited in such a manner as not to be mixed with the earth.

The bottom of excavations for reinforced concrete work shall be covered with a "blinding" layer of 1 : 6 concrete not less than 2 in. thick, finished to a smooth surface. The required cover of concrete under the reinforcement shall be entirely above the blinding layer.

BENDING REINFORCEMENT.—Reinforcement bars shall be bent by machine or other approved means producing a gradual and even motion. Bars shall be bent cold unless the Engineer shall approve bars of over 1 in. in size being bent hot. Bars bent hot shall not be heated beyond cherry-red colour and after bending shall be allowed to cool slowly without quenching. Bars dependent on cold-working for their strength shall always be bent cold.

Bars incorrectly bent shall be used only if the means used for straightening and rebending be such as shall not injure the material. No reinforcement shall be bent when in position in the Works without approval, whether or not it is partially embedded in hardened concrete.

Bends shall comply with the dimensions given in the bending schedule. Dimensions of bent bars and internal dimensions of binders and the like shall not exceed the dimensions given on the bar schedule or elsewhere. Unless described otherwise, bending dimensions shall conform to B.S. 1478.

The internal radii of bends shall be not less than twice the size of the bar unless described to the contrary. The internal radii of the bends at corners of binders or the like shall be half the size of the bar embraced by the binder. For the purpose of this requirement the size of a bar shall mean the diameter of a plain round bar or wire, or one-and-a-half times the nominal dimension of the side of a twisted square bar, or the nominal diameter of a deformed bar of any other type.

FIXING REINFORCEMENT.—Reinforcement shall be accurately fixed and by approved means maintained in the position described on the drawings or elsewhere. Bars intended to be in contact at passing points shall be securely wired together at all such points with No. 16 gauge annealed soft-iron tying wire. Binders and the like shall tightly embrace the bars with which they are intended to be in contact and shall be securely wired or if approved welded thereto.

Immediately before concreting, the reinforcement shall be examined for accuracy of placing and cleanliness and corrected if necessary.

Reinforcement projecting from work being concreted or already concreted shall not be bent out of its correct position for any reason unless approved and shall be protected from deformation or other damage.

The cover of concrete to the reinforcement shall be as described on the drawings and shall be provided and maintained within a tolerance $\frac{1}{8}$ in. under and over (except where specified as a minimum) by means of distance pieces of cement mortar or other approved material.

The vertical distances required between successive layers of bars in beams or similar members shall be maintained by the provision of mild steel spacer bars inserted at such intervals that the main bars do not perceptibly sag between adjacent spacer bars.

WELDING REINFORCEMENT.—If butt jointing of reinforcement bars by electric-arc welding be approved, the Engineer's requirements or the regulations of the local or other authority shall be complied with and all operations connected therewith shall be done only by men skilled thereat. These requirements shall not apply to the spot-welding of binders or the like to main bars where approved or to electrically-welded fabric. No welding shall be done in connection with twisted bars or other reinforcement (excepting cold-drawn wire) the strength of which depends on cold-working.

SHUTTERING.—Shuttering for concrete shall be rigidly constructed of approved material and shall be true to the shape and dimensions described on the working drawings. Timber shall be well seasoned, free from loose knots and, except where otherwise approved, wrought on all faces. Faces in contact with concrete shall be free from adhering grout, projecting nails, splits, or other defects. Joints shall be sufficiently tight to prevent the leakage of cement grout and to avoid the formation of fins or other blemishes. Faulty joints shall be caulked. Where described on the working drawings or elsewhere the position and direction of the joints shall be as so described. Unless described on the working drawings or elsewhere to the contrary, 1-in. by 1-in. chamfers shall be formed on the external corners of concrete members. Openings for inspection of the inside of the shuttering and for the escape of water used for washing out shall be formed so that they can be conveniently closed before placing the concrete.

Connections shall be constructed to permit easy removal of the shuttering and shall be either nailed, screwed, bolted, clamped, wired, or otherwise secured so as to be strong enough to retain the correct shape during consolidation of the concrete. Bolt holes in concrete shall be made good after removal of the bolts. Wire ties passing through concrete shall be used only where approved, and the ends of the wires shall be concealed and measures taken to prevent rust stains on the concrete.

Shuttering shall be provided for the top faces of sloping work, and anchored to prevent flotation, where the slope exceeds 1 in 2½.

Shuttering shall be true to line and braced and strutted to prevent deformation under the weight and pressure of the wet concrete, constructional loads, wind, and other forces. The deflection shall not exceed ⅛ in. Bottoms of beam boxes shall be erected with an upward camber of ¼ in. for each 10 ft. of span. If so instructed the designs for the shuttering shall be submitted for approval before construction.

The shuttering for beams and slabs shall be erected so that the shuttering on the sides of the beams and of the soffits of slabs can be removed without disturbing the beam bottoms. Re-propping of beams shall not be done except when, with the approval of the Engineer, props be reinstated in anticipation of loads in excess of the design load. Vertical props shall be supported on wedges, or other measures shall be taken whereby the props can be gently lowered vertically when commencing to remove the shuttering. Props for an upper story shall be placed directly over those in the story immediately below and the lowest prop shall bear upon work sufficiently strong to carry the load.

If the shuttering for a column is erected to the full height of the column, one side shall be left open and shall be built up in sections as placing the concrete proceeds.

Before placing the concrete, bolts and fixings shall be in position, and cores and other devices used for forming openings, holes, pockets, chases, recesses, and other cavities shall be fixed to the shuttering. No holes shall be cut in any concrete unless approved.

An approved mould oil or other material shall be applied to faces of shuttering in contact with wet concrete to prevent adherence of the concrete. Such coatings shall be insoluble in water, non-staining, and not injurious to the concrete, and shall not become flaky or be removed by rain or wash-water. Liquids that retard the setting of concrete shall be used only when approved. Mould oil, retarding liquid, and similar coatings shall be kept from contact with the reinforcement.

PROPORTIONS OF CONCRETE.—The aggregate shall be measured by volume (unless instructed to be measured by weight) in an approved container which shall be filled without compacting with the aggregate to a predetermined uniform depth, accurate allowance being made for bulking due to the moisture in the fine aggregate. The cement shall be measured by weight. If stored in bags, one or more complete bags containing 112 lb. of cement shall be used for each batch of concrete. If stored loose the cement shall be measured by weight by approved means.

If the proportions of cement and aggregate are not otherwise described on the drawings or elsewhere, the concrete shall be mixed in the proportions of 1 cwt. of Portland cement to 2½ cu. ft. of sand (measured when dry) and 5 cu. ft. of coarse aggregate. These quantities or others as described shall be altered if instructed and any alteration between the proportions of 1 part of fine aggregate to 1½ parts of coarse aggregate and 1 part of fine aggregate to 2 parts of coarse aggregate shall be made without any alteration in the charge made by the Contractor.

Only sufficient water shall be added to the cement and aggregate during mixing to produce a concrete having sufficient workability to enable it to be well consolidated, to be worked into the corners of the shuttering and around the reinforcement, to give the specified surface finish, and to have the specified strength. When a suitable amount of water has been determined the resulting consistency shall be maintained throughout the corresponding parts of the work, and approved tests shall be conducted from time to time to ensure the maintenance of this consistency.

If difficulty be experienced in placing concrete of the specified proportions and approved consistency between and below the reinforcement bars in the bottom of beams and similar members, the bars shall be embedded in concrete of improved

workability by increasing the amount of cement as approved and by using aggregates of approved smaller maximum size than specified.

MIXING CONCRETE.—The cement and aggregates shall be thoroughly mixed together in the proportions described in a batch-type mechanical mixer, unless otherwise approved. The water shall not be admitted to the drum of the mixer until all the cement and aggregate constituting the batch are in the drum. Mixing shall continue until the concrete is uniform in colour and for not less than two minutes after all the materials and water are in the drum. The entire contents of the drum shall be discharged before the materials for the succeeding batch are fed into the drum. No partly-set or retempered concrete shall be used. Partly-set or excessively wet concrete shall not be used on the Works and shall be immediately removed therefrom.

STRENGTH OF CONCRETE.—The compressive strength of the concrete at twenty-eight days shall be not less than 3000 lb. per square inch or such other strength as is described on the drawings or elsewhere, except that if rapid-hardening Portland cement is used the required strength shall be attained in seven days.

The compressive strength shall be ascertained by crushing 6-in. cubes of concrete, the cubes being made on the Works and tested as instructed. The Contractor shall pay all costs incurred in supplying the material for, and in making, maturing, delivering, and testing the cubes and shall be reimbursed for the costs of cubes that attain the required strength.

DISTRIBUTION OF CONCRETE.—The concrete shall be distributed from the mixer to the position of placing in the Works by approved means which do not cause separation or otherwise impair the quality of the concrete.

Mixing and distributing equipment shall be clean before commencing mixing and distribution of the concrete and such equipment shall be kept free from set concrete.

PLACING CONCRETE.—Before proceeding to place the concrete, the shuttering shall be re-aligned if necessary and water and rubbish therein shall be removed by approved means. Immediately prior to placing the concrete, the shuttering shall be well wetted except in frosty weather and inspection openings shall be closed.

The interval between adding the water to the dry materials and the completion of the placing of the concrete shall not exceed twenty-five minutes.

Except where otherwise approved concrete shall be placed in the shuttering by shovels or other approved implements and shall not be dropped from a height or handled in a manner which will cause separation. Accumulations of set concrete on the reinforcement shall be avoided. Concrete shall be placed directly in its permanent position and shall not be worked along the shuttering to that position.

Each layer of concrete while being placed shall be consolidated by approved methods of ramming, tamping, or mechanical vibration to form a dense material with all surfaces free from honeycombing and tolerably free from water and air holes or other blemishes. Any water accumulating on the surface of newly-placed concrete shall be removed by approved means, and no further concrete shall be placed thereon until such water be removed.

No unset concrete shall be brought into contact with unset concrete containing cement of a different type.

Unless otherwise approved, concrete shall be placed in a single operation to the full thickness of slabs, beams, and similar members, and shall be placed in horizontal layers not exceeding 3 ft. deep in walls, columns, and similar members. Concrete shall be placed continuously until completion of the part of the work between construction joints as specified hereinafter or of a part of approved extent. At the completion of a specified or approved part a construction joint of the form and in the position hereinafter specified shall be made. If stopping of concrete placing be unavoidable elsewhere, a construction joint shall be made where the work is stopped.

Tremies, bottom-opening skips, or other equipment used for placing concrete under water shall be of approved design and shall be used as instructed by the Engineer. No concrete shall be placed if the temperature of the water is below 35 deg. F. Concrete placed between tides shall be shuttered on all faces, the top face being closed immediately after completion of placing and before subsequent submersion. No pumping that may adversely affect the concrete being placed shall be done while placing or within twenty-four hours of placing the concrete.

PLACING CONCRETE IN COLD WEATHER.—No concrete shall be mixed or placed while the temperature is below 34 deg. F. on a rising thermometer or below 36 deg. F. on a falling thermometer. The Contractor shall supply an accurate maximum and minimum thermometer and hang it in an approved position on the Works. Aggregates that have been exposed to frost shall not be used until completely thawed. Concrete shall be maintained by approved means at a temperature of not less than 40 deg. F. during placing and for a period of three days thereafter, or for a period of one day thereafter if rapid-hardening Portland cement or high-alumina cement is used. All concrete placed during cold weather or when a frost is predicted or is likely to occur or occurs contrary to expectation, shall be protected from freezing by approved means.

HOLLOW CLAY BLOCK SLABS.—Hollow clay blocks shall be laid true to line and accurately spaced. Joints between blocks shall be pointed and faces of blocks in contact with concrete shall be given a thick coat of cement slurry immediately before placing the concrete. Broken blocks shall not be used and shall be removed from the Works. Approved stop-ends shall be inserted to close the open ends of each line of blocks. If so described, clay-tile slips of approved thickness shall be provided at the bottom of each concrete rib. Blocks shall be well soaked in water before placing the concrete and slurry.

The concrete in the ribs between the blocks, in the slab over the blocks, and in the solid slab around the edges of the area laid with blocks shall be placed in one operation and shall be mixed in the nominal volumetric proportions of $1 : 1\frac{1}{2} : 3$ using coarse aggregate having a maximum size of $\frac{1}{2}$ in.

CONSTRUCTION JOINTS.—Construction joints shall be rebated and of an approved shape and shall be vertical or horizontal, as required, except that in an inclined or curved member the joint shall be at right-angles to the axis of the member.

Construction joints shall be provided in the positions described on the drawings or elsewhere and where not so described shall be in accordance with the following. A joint shall be formed horizontally at the top of a foundation and 3 in. below the lowest soffit of the beams meeting at the head of a column. A joint shall be formed in the rib of a large tee-beam and ell-beam 1 in. below the soffit of the slab. Concrete in a haunch or a splay on a beam or a brace, and in the head of a column where one or more beams meet, shall be placed without a joint at the same time as that in the beam or beams or brace. Concrete in the splay at the junction of a wall and a slab shall be placed without a joint at the same time as that in the slab. Concrete in a beam shall be placed throughout without a joint, but if the provision of a joint is unavoidable the joint shall be vertical and at the middle of a span. A joint in a slab shall be vertical and parallel to the principal reinforcement; where it is unavoidably at right-angles to the principal reinforcement the joint shall be vertical and at the middle of the span.

Before placing new concrete against concrete that has already hardened, the face of the old concrete shall be cleaned and roughened and scum and loose aggregate removed therefrom, and immediately before placing the new concrete the face shall be thoroughly wetted and a coating of neat cement grout applied thereto. The new concrete shall be well rammed against the prepared face before the grout sets.

STRUCTURE JOINTS.—Expansion joints, contraction joints, hinges, or other permanent structure joints shall be provided in the positions and of the form described in the drawings or elsewhere.

PROTECTION OF CONCRETE.—Newly-placed concrete shall be protected by approved means from frost, rain, sun, and drying winds. Exposed faces of concrete shall be kept moist by approved means for seven days after placing or for three days if rapid-hardening Portland cement is used, except if there is a likelihood of curing water or damp coverings freezing, when the period shall be as instructed by the Engineer.

Concrete placed below the ground shall be protected from falling earth during and after placing. Concrete placed in ground containing deleterious substances shall be kept free from contact with such ground and with water draining therefrom during placing and for a period of three days or as otherwise instructed thereafter. The ground-water around a structure below the ground shall be kept to an approved level by pumping, or the Works shall be flooded or other approved means taken to prevent

flotation. Approved means shall be taken to protect immature concrete from damage by debris, ice, excessive loading, vibration, abrasion, deleterious ground-water, mixing with earth or other materials, flotation, and other influences that may impair the strength and durability of the concrete.

Removal of Shuttering.—Shuttering shall be removed by gradual easing without jarring. Before removal of the shuttering the concrete shall be examined and removal shall proceed only in the presence of a competent supervisor and if the concrete has attained sufficient strength to support its own weight and any load likely to be imposed upon it. If the imposition of a load exceeding the design load is anticipated, props shall be provided in an approved manner after removal of the shuttering and before the imposition of the load. The Contractor shall record on the drawings or elsewhere the date upon which the concrete is placed in each part of the work and the date upon which the shuttering is removed therefrom. The assessment of the period elapsing between placing the concrete and removing the shuttering and consequences arising therefrom shall be the Contractor's entire responsibility.

The shuttering for a part of a structure suspended from concrete placed subsequently to that in or on the shuttering concerned shall not be removed until the supporting concrete has matured, and such shuttering shall be prominently marked as a warning against premature removal.

Finishes.—Honeycombed surfaces shall be made good immediately upon removal of the shuttering, and superficial water and air holes shall be filled in. Unless instructed to the contrary the face of exposed concrete placed against shuttering shall be rubbed down immediately upon removal of the shuttering to remove fins or other irregularities. The face of concrete for which shuttering is not provided, other than slabs, shall be smoothed with a wooden float to give a finish equal to that of the rubbed-down face where shuttering is provided. The top face of a slab which it is not intended to cover with other materials shall be levelled and floated while unset to a smooth finish at the levels or falls shown on the drawings or elsewhere. The floating shall be done so as not to bring an excess of mortar to the surface of the concrete. Indentations in the surface of the concrete shall be formed by approved implements to the depths and patterns described. The top face of a slab intended to be surfaced with mortar, granolithic, or similar material shall be left with a spaded finish.

Concealed concrete faces shall be left as from the shuttering except that honeycombed surfaces shall be made good. Faces of concrete intended to be rendered shall be roughened by approved means to form a key. Faces of concrete that are to have finishes other than those specified shall be prepared in an approved manner as instructed.

Precast Piles.—Shoes for precast piles shall have cast-iron points and wrought-iron or mild steel straps and shall be of the pattern and weight described. Forks for retaining the longitudinal reinforcement in position shall be of cast iron, pressed steel, or other approved material and of approved shape.

Shoes, reinforcement, forks, links, toggle-hole tubes, and other fittings shall be fixed accurately in the position described on the working drawings or elsewhere. Shoes shall be fitted with the point in line with the axis of the pile. One end of the longitudinal bars shall bear on the top of the shoe and the upper ends of such bars shall all be at the same level.

The moulds shall be so made that the pile is not distorted during casting or at any other stage of manufacture. The piles shall be protected from damage from any cause during handling, and shall be slung only from the points described on the working drawings or elsewhere.

The period that shall elapse between making and driving a pile, or between placing the concrete in a pile lengthened insitu and recommencing driving, shall be not less than six weeks if ordinary Portland cement is used and not less than two weeks if rapid-hardening Portland cement is used.

The weight, type, and the height of fall of the driving hammer to be used, and the final set or depth to which each pile shall be driven, shall be as described on the drawings or as instructed. Driving shall proceed without cessation until the pile is driven to the set or depth described. If cessation of driving is unavoidable, or when

the driving of a pile lengthened insitu is recommenced, the amount of driving that shall be done before the set is measured shall be as instructed.

During driving, the head of the pile shall be protected by approved means. Piles shall be driven in the sequence described on the working drawings or elsewhere. Sheet piles shall be driven together and shall be keyed together as described on the drawings and elsewhere.

If a pile is driven in an incorrect position, or if a pile intended to be vertical is driven out of plumb, or if an inclined pile is not driven to the correct angle, the Contractor shall take at his own cost such remedial measures as instructed.

Where piles bond with pile-caps or other reinforced concrete work, or if the driven pile be of excessive length, or if a driven pile is to be increased in length, the concrete shall be cut away for a distance below the head of the pile as instructed. The cutting away shall be in an approved manner without damage to the rest of the pile left insitu. Unless instructed to the contrary, the main reinforcement shall not be cut, but shall be embedded into concrete subsequently placed, any bending being done without damage to the concrete in the pile.

PRECAST CONCRETE.—The concrete in one precast piece shall be placed in one operation. No piece shall be removed from the mould or erected until sufficiently matured to ensure that no damage shall be done to the piece. A piece shall be suspended or supported only at the points described on the working drawings or elsewhere. A piece that is cracked or otherwise damaged during, before, or after erection shall be removed from the Works and replaced by the Contractor free of charge. Pieces shall be bedded or otherwise fixed in their permanent positions as instructed.

STRUCTURE TESTS.—The Engineer shall instruct the Contractor to make a loading test on the Works or any part thereof if in the Engineer's opinion such a test is necessary.

If the test be instructed to be made solely or in part for the reason that the site-made concrete cubes fail to attain the specified strength, the test shall be made at the Contractor's own cost. If the test be instructed to be made because of one or more circumstances attributable to alleged negligence on the part of the Contractor, the Contractor shall be reimbursed for the cost of the test if the result thereof is satisfactory. If the test be instructed to be made for any reason other than the foregoing, the Contractor shall make the test and shall be reimbursed for all costs relating thereto irrespective of the result of the test.

For the purpose of testing floors, roofs, and similar structures and their supports, the test load shall be equivalent to one-and-a-quarter times the live load for which the Works or part thereof to be tested has been designed. The test load shall not be applied within twenty-eight days of the completion of placing of the concrete in the part of the Works to be tested, and the latter shall be unsupported during the test by the shuttering or other non-permanent support. The test shall be made as instructed.

For a test on a floor, roof, or similar construction, the result shall be deemed to be satisfactory if upon removal of the load the residual deflection does not exceed one-quarter of the maximum deflection after maintaining the load in position for twenty-four hours. If the residual deflection exceeds this amount, the test shall be repeated, and the result shall be deemed to be satisfactory if the residual deflection after removal of the load for the second time does not exceed one-quarter of the maximum deflection occurring during the second test.

If the result of the test is not satisfactory, the Engineer shall instruct that the part of the Works concerned shall be taken down or cut out and reconstructed to comply with this Specification, or that other measures shall be taken to make the Works secure. If in accordance with this Specification the Contractor is liable to conduct the test at his own cost, he shall also at his own cost take down or cut out and reconstruct the defective work or shall execute remedial measures as instructed.

Quantities.

The bills of quantities for a reinforced concrete structure can be divided into six principal divisions (each of which is subdivided into items), such as: Excavation and filling; Plain concrete in foundations and elsewhere; Reinforced concrete; Shuttering; Reinforcement; and Sundries (including asphalt, fixing machinery and pipes, glazing, plumbing, waterproofing, refractory linings, joinery, and other works). The method of measurement and the bills of quantities should be in accordance with a standard method in order to avoid ambiguities. If the contract is for a building under the direction of an architect, the " Standard Method of Measurement of Building Works " should be used. For a civil engineering contract, the " Standard Method of Measurement of Civil Engineering Quantities " issued by the Institution of Civil Engineers should be used. For small reinforced concrete structures, a short bill of quantities, similar to that given later, may be sufficient.

Measurement of Quantities.—The amount of plain and reinforced concrete, shuttering, piling, earthwork, and similar work can generally be measured from the general drawings.

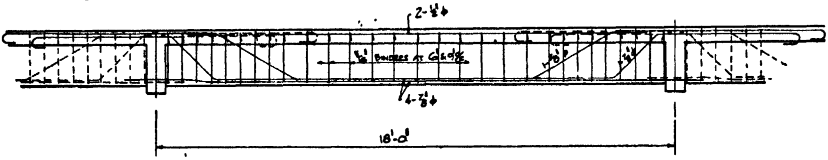
For the measurement of the quantities from the drawings, a system should be followed to ensure accuracy and facilitate checking. The following is recommended for measuring the quantities of concrete and shuttering. In the first column of measurement is entered the number of identical concrete members being measured. In the second column the dimensions of the member in the order of length, breadth and depth are entered. In the third column the volume of concrete represented by the product of the three preceding dimensions multiplied by the number of members is given. The dimensions of the surfaces requiring shuttering are entered opposite the corresponding item for the concrete. The example given later demonstrates the method. The members are measured in order downwards from the top. The quantities for similar members are then abstracted and added to similar items for other parts of the structure before transferring them to the bill of quantities.

Reinforcement.—The total weight can be roughly estimated from the type of structure and volume of concrete. The amount of steel in an ordinary reinforced concrete structure varies from 1 ton in 5 cu. yd. of concrete to 1 ton in 30 cu. yd. The following are approximate ratios of cubic yards of concrete to a ton of reinforcement. Warehouses, girder bridges, rectangular tanks, water towers, silos, circular bunkers, pits, and pit-head gears: 10 to 15; factories, residential buildings, retaining walls, culverts, swimming baths, and open circular tanks: 15 to 20; arch bridges, rectangular bunkers, pyramidal tanks in the ground, and elevated conical tanks: 8 to 10. Exceptional loads and other circumstances may affect the ratios considerably.

If the working drawings or typical details are available, but no bar-bending schedules, a more accurate estimate can be made from consideration of the principal reinforcement. The principal reinforcement is that near the lower edge at midspan in a beam or slab, and the main vertical bars in a column. The area A sq. in. of the principal reinforcement can be computed from *Table* 59 or 60 and the approximate average weight (W lb. per foot of beam or column, or per square foot of slab) can be calculated from $W = KA$, where K has the following values: freely-supported beams, 3 to 6; continuous beams, 6 to $8\frac{1}{2}$; cantilevers, 4 to 6; columns, 5 to 6; continuous slabs spanning in one direction, 5 to $8\frac{1}{2}$; continuous slabs spanning in two directions, $4\frac{1}{2}$ to $7\frac{1}{2}$. For slabs and beams the smaller values apply in cases where there is no

compression reinforcement or no special reinforcement to resist shearing force; the larger values apply where there are equal amounts of compression and tension reinforcement or where a large amount of reinforcement is provided to resist shearing. The smaller values for columns apply to those with independent lateral ties, and the larger for columns with helical binding or where loose splice bars are provided. For slabs spanning in two directions, A is the sum of the areas of the reinforcement in each direction.

The unit weight multiplied by the length (ft.) of a beam or column or by the area (sq. ft.) of a slab, gives the approximate total weight (lb.) of reinforcement. For members such as important beams and arch ribs, an approximate list of the bars should be made and the total length of the bars of each size multiplied by the weight per foot (*Table* 59). The weight of reinforcement in walls or other slabs with uniform reinforcement throughout can be found by multiplying the area (sq. yd.) of the wall or slab by the weight per square yard and adding an amount for laps. If bar-bending



BENDING SCHEDULE FOR 1 No BEAM	No	DIA	LENGTH	BENDING
	2	⅞"	20'-0"	2'-0" 1'0" 17'-0"
	1	⅞"	28'-9"	4'-10" 1'-11" 15'-9" 1'-11" 4'-10" 9" 9"
	1	⅞"	31'-0"	7'-6" 3'-6" 3'-11" 8'-3" 7'-6" 9" 9"
	2	½"	16'-0"	STRAIGHT
	29	5/16"	4'-6"	20" 4" AT 6" AND 9" CRS

DIAM	No	UNIT LENGTH	TOTAL LENGTH	WEIGHT LB
⅞ in	2	20 ft 0 in	40 ft	
	1	29 · 9 ·	29 ·	
	1	31 · 0 ·	31 ·	
			100 × 2·044 =	204
½ in	2	16 ft 0 in	32 × 0·677 =	22
5/16 in	29	4 · 6 ·	131 × 0·261 =	34
				260
				say, 2½ cwt.

schedules are available, the weight of reinforcement can be calculated exactly from *Table* 59. To demonstrate the methods, the weight of reinforcement in the beam shown in the accompanying diagram will be calculated.

The approximate weight is estimated by considering the principal reinforcement, which is four ⅞-in. diameter bars (A = 2·41 sq. in.). Since there is no compression reinforcement at midspan and inclined bars are provided for resistance to shearing force, a suitable value for K is 6. The average weight of reinforcement per foot of beam is therefore $6 \times 2{\cdot}41 = 14$ lb. per foot. The length of the span is 18 ft., and therefore the estimated total weight of reinforcement in one beam is $18 \times 14 = 252$ lb.; say, 2¼ cwt.

A bar-bending schedule giving particulars of each bar is given on the diagram. The total length of bars of each size is calculated and is multiplied by the weight per foot. The system of calculating the weights as given on the diagram avoids confusion and facilitates checking.

The quantities of reinforcement bars are given by weight. Round bars ⅜ in. diameter and over are generally given in one item, but the weights of each size of bar less than ⅜ in. are given separately. The weights of bent bars need not generally be separated from straight bars unless there is a predominating weight of the latter. Some indication of the length of bars and type of structure should be given, and of the basis of measurement, for example whether tying wire, the rolling margin, or the

like are included in the weight or must be allowed for in the price. Bars over 40 ft. in length and less than 5 ft. should be separately measured. A comprehensive clause describing the reinforcement and enabling a contractor to give a keen price is as follows:

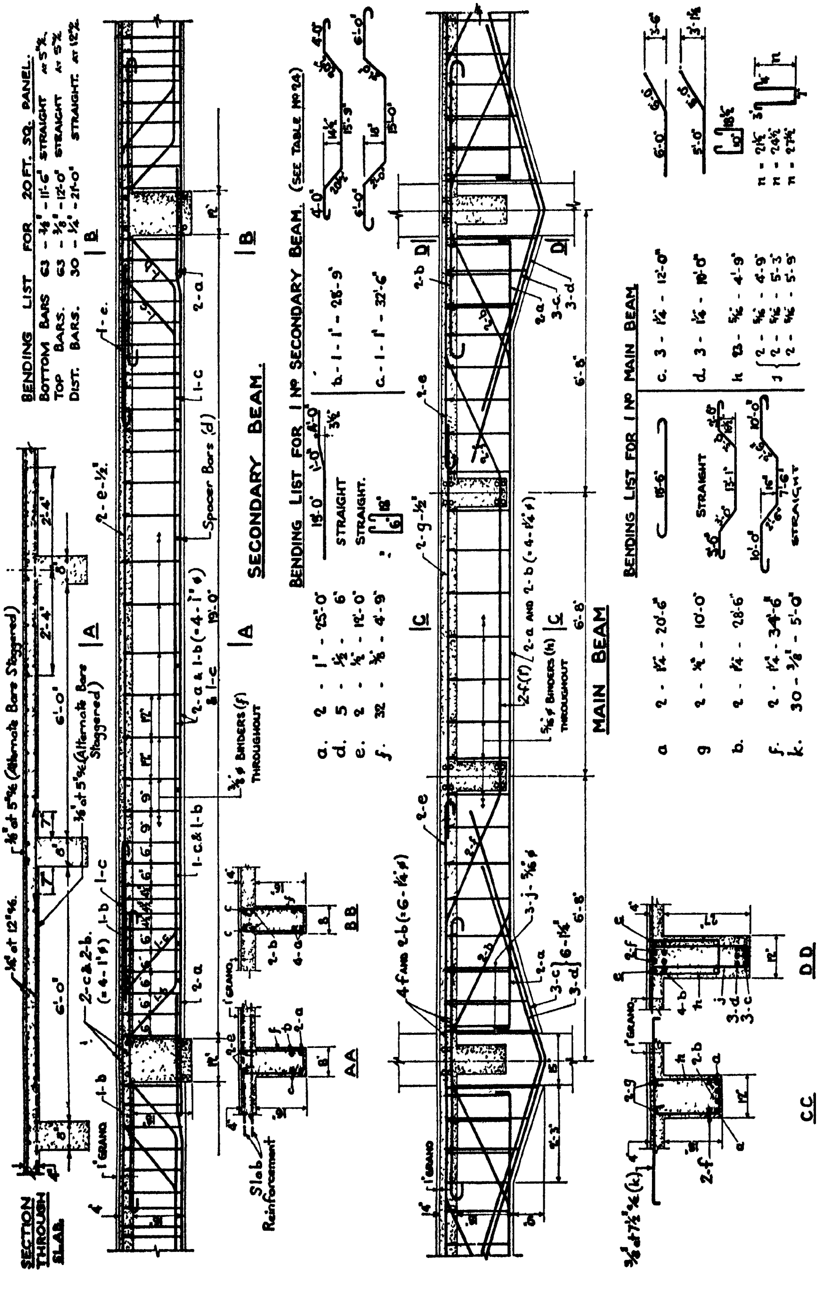
> Supply, handle, cut, bend, crank, and fix mild steel bars $\frac{5}{8}$-in. diameter and over, but not exceeding $1\frac{1}{2}$ in. in diameter, in main reinforcement in beams and columns to all floors and roofs and in foundations, in lengths not less than 5 ft. and not exceeding 40 ft. The price shall include for tying wire. No allowance in the weight paid for shall be made for the rolling margin. The weight of any bar shall be computed from the length given on the drawings or bar-bending schedules, the weight in pounds per foot of bar being calculated as 2·67 times the square of the specified diameter of the bar . . x cwt.

Similar items should be given for smaller bars and for longer and shorter bars. When the supply of the reinforcement, or the fixing or bending, or both, is a sub-contract, it should be stated who does the cutting and hoisting and supplies the tying wire.

The "Building Works" method of measuring reinforcement requires bars over 30 ft. long to be given in separate items for each 5 ft. advance in length. Separate items must be given for bent and straight bars, stirrups, helical binding, and bars bent to a large radius. Reinforcement for floors, roofs, walls, beams, and columns must be separate. Bars of each size less than $\frac{5}{8}$ in. and more than 1 in. must be given separately.

Estimating Costs.

In a contracting firm the data and methods for making an estimate of the cost of a reinforced concrete structure are generally available. Otherwise an approximate estimate can be made by pricing each item in the bill of quantities at the average prevailing rates for similar construction. Unless local rates are known, however, this may give misleading results. If a recent priced bill of quantities for previous work in the locality is not at hand, the prices given in technical periodicals might be used to obtain an approximate estimate. A better method is to calculate the probable prices, basing the computation on the cost of labour, materials, and general costs. The wages payable to different classes of workers are given in technical periodicals, and the man-hours required to perform a given piece of work are given in text-books on this subject. The trend of the general productivity of labour should, however, be considered when using published data. The cost of labour can vary considerably. The costs of materials can be obtained from current prices, and the amount of each material required can be determined from the bills of quantities. In *Table* 56 are given the quantities of stone, sand, and cement required for a cubic yard of concrete. To the net quantities of aggregate must be added 10 per cent. for tolerances on measurement and for waste, and 5 per cent. to the net quantities of cement for waste and for making grout. If the price does not include delivery, the cost of transport must be added. The general costs, which include overhead charges, profit, and site staff, depend principally upon the contract time and the plant likely to be used. For an approximate estimate for an ordinary reinforced concrete structure, allowance for the general costs can be made by increasing the net cost of materials by, say, 15 per cent. and of labour by, say, 50 per cent. For small structures and for very large works this method is not dependable, and such work should be estimated on a more precise basis.



BEAM-AND-SLAB CONSTRUCTION.—EXAMPLE FOR MEASUREMENT OF QUANTITIES.

Example of Measuring Quantities.

In the following the quantities are measured for one 20-ft.-square panel of the floor of beam-and-slab construction shown on page 136, the columns being excluded.

	Concrete.		Shuttering.	
Slab.	20′ 0″ 20′ 0″ 4″	133·3	20′ 0″ 20′ 0″	400
Secondary Beams.	2/19′ 0″ 1′ 4″ 8″	33·9	2/2/19′ 0″ 1′ 4″	101
	18′ 9″ 8″ 1′ 4″	16·7	2/18′ 9″ 1′ 4″	50
Main Beams.	18′ 9″ 1′ 0″ 1′ 6″	28·1	2/18′ 9″ 1′ 6″	56
	2/½/ 2′ 3″ 1′ 0″ 9″	1·7	2/2/½/ 2′ 3″ 6″	2
	Total	213·7 cu. ft.		609
	say = 8·0 cu. yard.		Deduct 1′ 3″ 1′ 3″	2
			Total	607 sq. ft.
			say =	68 sq. yd.

1-in. granolithic:	20′ 0″ 20′ 0″	400
Deduct	1′ 3″ 1′ 3″	2
		398 sq. ft. = 45 sq. yd.

Reinforcement (see bending schedule on page 136).

Diameter	¼ in.	5/16 in.	⅜ in.	½ in.	1 in.	1¼ in.	
Slab (20 ft. sq.)	630		735 756				
Secondary beams (3 No.)			408	108 8	150 86 98		
Main beam: (1 No.)		109 9 11 12	150	20		41 57 69 36 30	
Total length	603	141	2049	136	316	233 ft.	
Weight per ft.:	0·167	0·261	0·376	0·668	2·67	4·173 lb.	(See *Table* 59)
Total weight (net)	105	37	779	91	865	971 lb.	
Gross weight	1	7½		1	16½ cwt.		

The foregoing quantities are abstracted and the totals transferred to the bill (see page 138). Price rates can be taken from published data. The basis of the method is unaltered by any fluctuations in the prices.

BILL OF QUANTITIES.

Beam-and-Slab Construction.

Item No.	Description.	Quantities.	Rate.	Total. £ s. d.
1.	Mixing and placing 1 : 2 : 4 concrete in slabs and beams (rough screeding only).	cu. yd. 8		
2.	Close-boarded shuttering to soffits of slabs, and sides and soffits of beams and haunches.	sq. yd. 68		
3.	Provide and lay 1-in. granolithic surfacing.	sq. yd. 45		
4.	Supply, cut if necessary, bend, and fix reinforcement bars $\frac{5}{8}$ in. diameter and over, including tying wire.	cwt. $16\frac{1}{2}$		
5.	Ditto $\frac{1}{2}$ in. diameter.	cwt. 1		
6.	Ditto $\frac{3}{8}$ in. and $\frac{5}{16}$ in. diameter.	cwt. $7\frac{1}{4}$		
7.	Ditto $\frac{1}{4}$ in. diameter.	cwt. 1		
	Total estimated cost for one panel 20 ft. square .			

PART II

TABLES AND EXAMPLES

CONTENTS

(For complete List of Tables, see pages v and vi.)

DEAD LOADS.

Weight of Concrete.—The primary dead load is usually the weight of the reinforced concrete, which for design purposes can be conveniently assumed to be 144 lb. per cu. ft., that is 1 lb. per lin. ft. for each square inch of cross-sectional area; but the weight of reinforced concrete is rarely less than 150 lb. per cu. ft. and varies with the density of the aggregate and the amount of reinforcement. Some typical weights of plain and reinforced concrete are given in *Table* 1, together with the weights of solid concrete slabs and hollow clay-block slabs. For members where the dead load predominates, a weight of not less than 150 lb. per cu. ft. should be assumed for ordinary reinforced concrete, this being the minimum weight recommended in the B.S. Code, London By-laws, and by other authorities. The weights of various lightweight concretes are also given in *Table* 1. Heavy concrete for use as kentledge and nuclear-radiation shielding is made by using aggregates of greater density than ordinary stone, such as barytes, limonite, magnetite, and other iron ores and steel shot or punchings; the weights of such concretes are also given in *Table* 1.

Other Structural Materials and Finishes.—Dead loads include such permanent weights as those of the finishes and linings on walls, floors, stairs, ceilings, and roofs; asphalt and other applied waterproofing layers; partitions; doors, windows, roof lights, and pavement lights; superstructure of steelwork, masonry, brickwork, or timber; concrete bases for machinery and tanks; fillings of earth, sand, puddled clay, plain concrete, hardcore; cork and other insulating materials; rail-tracks and ballasting; refractory linings; and road surfacing. In *Table* 1 are given the basic weights of structural materials. In *Table* 2 are given the weights of glazed, sheeted, and slated roofs, and the average equivalent weights of steel trusses, such weights being useful in estimating the loads imposed on a concrete substructure. Rules for estimating the total weight of structural steelwork based on adding to the sum of the nominal weights of the members an allowance for cleats, connections, rivets, bolts and the like are given in *Table* 2; extra allowances should be made for stanchion caps, bases, and grillages. The smaller allowances permissible for welded steelwork are also given.

Where concrete lintels support brick walls it is not necessary to consider the lintel as carrying the whole of the wall above it; it is sufficient to allow only for the triangular areas indicated in the diagrams in *Table* 2.

Partitions.—The weights of partitions should be included in the dead loads of floors and it is convenient to consider such weights as equivalent uniformly-distributed loads w_e. It is usual to consider a minimum load of 20 lb. per sq. ft. of floor area for partitions in offices and buildings of similar use, but this load is only sufficient for timber or glazed partitions. The material of which the partition is constructed and the story-height will determine the weight of the partition, and in the design of floors the actual weight and position of a partition, when known, should be allowed for when calculating shearing forces and bending moments on the slab and beams. Expressions are given in *Table* 2 for the equivalent uniformly-distributed load if the partition is at right-angles to the direction of the span of the slab and is placed at the middle of the span, or if the partition is parallel to the direction of the span. According to the B.S. Code, the equivalent uniformly-distributed load per sq. ft. of floor for partitions, the positions of which are not known, should be not less than 10 per cent. of the specified minimum imposed load (see *Table* 3), and in no case less than 20 lb. per sq. ft. for office floors.

In the case of brick or similarly-bonded partitions some relief of loading on the slab occurs due to arching action of the partition if it is continuous over two or more beams, but the presence of doorways or other openings destroys this relieving action.

The uniformly-distributed load on a beam due to partitions can be considered as the proportion of the total weight of the partitions carried by the beam adjusted to allow for non-uniform incidence.

CONCRETE		LB. PER CUBIC FOOT
PLAIN CONCRETE (1:6 OR 1:4)		AVERAGE 144
GRAVEL OR CRUSHED STONE AGGREGATE		140 TO 155
BROKEN BRICK AGGREGATE	(AV. 120)	105 TO 135
CRUSHED LIMESTONE AGGREGATE		135 TO 150
REINFORCED CONCRETE		MINIMUM 144
FOR GENERAL DESIGN PURPOSES (B. S. CODE)		150
WITH 1% REINFORCEMENT		144 TO 154
" 2% "	(AVERAGE 150)	148 TO 158
" 5% "		160 TO 170

LIGHTWEIGHT CONCRETE	STRENGTH LB./SQ.IN.	LB. PER CUBIC FOOT
CLINKER (1:8)	300 TO 900	65 TO 95
PUMICE (1:6 SEMI-DRY)	200 TO 550	45 TO 70
FOAMED BLASTFURNACE SLAG	200	60
	800	95
STRUCTURAL	2000 TO 5000	105 TO 130
EXPANDED CLAY OR SHALE	800 TO 1200	60 TO 75
STRUCTURAL	2000 TO 5000	85 TO 115
VERMICULITE (EXPANDED MICA)	70	25
	500	70
PULVERISED-FUEL ASH (FLYASH)	400 TO 1000	70 TO 80
STRUCTURAL	2000 TO 5000	85 TO 110
NO-FINES GRAVEL AGGREGATE		100 TO 120
CLINKER "		55 TO 80
CELLULAR (AERATED OR "GAS")	200	25
	1500	90
	1500 TO 2250	100
"YTONG" AND SIMILAR		25 TO 40

AIR-ENTRAINED CONCRETE — FOR DESIGN USE SAME WEIGHTS AS PLAIN OR REINFORCED CONCRETE

HEAVY CONCRETE.	
BARYTES	200
LIMONITE, MAGNETITE, AND SIMILAR ORES	AV. 224
STEEL SHAVINGS OR PUNCHINGS	AV 336

REINFORCED CONCRETE SLABS TOTAL THICKNESS		LB. PER SQUARE FOOT SOLID	HOLLOW CLAY TILE CONSTRUCTION WITH 1-IN. TOPPING (AV.)
	3 IN	38	
	4 "	50	
	4½ "	57	
	5 "	63	42
	6 "	75	45
	7½ "	94	51
	9 "	113	57
	10½ "	132	63
	12 "	150	70
CLINKER BLOCK PARTITIONS (3 IN. THICK)		25	
CONCRETE TILES. PLAIN = 15; INTERLOCKING		8	
GRANOLITHIC CONCRETE (1 IN. THICK)		12	
TERRAZZO (1 IN. THICK)		12	
WOOD-WOOL (1 IN. THICK)		2 TO 6	
CEMENT-MORTAR SCREED (1 IN. THICK)		10 TO 12½	
PLASTER LIME 9; HARD PLASTER		10	
LATH AND PLASTER		8	
TERRACOTTA (1 IN THICK)		10	
SOIL-CEMENT (PER INCH THICK)		100	

OTHER MATERIALS	LB. PER CUBIC FOOT
NATURAL STONE (SOLID)	
GRANITE, BASALTS, DOLERITES	165
LIMESTONE, SANDSTONE, YORKSTONE	140
SLATE	180
CRUSHED STONE	80 TO 140
GRAVEL, BALLAST, ETC. (LOOSE)	100
HARDCORE AVERAGE	120
SAND (DRY TO WET)	90 TO 120
PACKED STONE RUBBLE	140
BROKEN BRICK	70 TO 100
ASH FILLING	60 TO 80
EARTH FILLING (COMPACTED)	100
PUDDLED CLAY	100 TO 120
CRUSHED SLAG	90
TIMBER AVERAGE	50
YELLOW PINE	30
RED PINE, SPRUCE	30 TO 45
ENGLISH OAK	45 TO 60
LARCH, ELM	35
PITCH PINE	40 TO 45
TEAK	40 TO 55
JARRAH	60
GREENHEART	65 TO 75
QUEBRACHO	80
CORK (SOLID; NOT GRANULAR)	15
WATER (SEAWATER 64) FRESH	62·4
SNOW (COMPACT 15 TO 50) LOOSE	5 TO 12

METALS (PER INCH THICK)	LB. PER SQUARE FOOT
STEEL (ROLLED OR CAST)	40¾
SHEET LEAD	60
COPPER, BRASS, BRONZE, GUNMETAL	45 TO 47
CAST OR WROUGHT IRON	38 TO 40
TIN, SPELTER, ZINC	37 TO 38
ALUMINIUM	14
BITUMEN (PER INCH THICK)	7½
MACADAM (PER INCH THICK) WATERBOUND	13½
TAR-MACADAM (")	12
DOORS (WOODEN) AVERAGE	8
GLASS (PER INCH THICK)	14
WINDOWS (METAL OR WOODEN FRAMES)	5
BRICKWORK (PER INCH THICK)	
ENGINEERING, IN P.C. MORTAR	12
COMMON (IN LIME 9) IN P.C.	10
REFRACTORY	6
HOLLOW-BLOCK PARTITIONS (2 IN. TO 4 IN.)	9 TO 15
EARTHEN WARE TILES	14
SLATES	8
ASPHALT (1 IN. THICK)	12
FIBRE-BOARD, PLASTER BOARD (½ IN. THICK)	¾
HARDBOARD (½ IN. THICK)	1½
TIMBER BOARDING (1 IN. THICK)	3
RUBBER PAVING AND SIMILAR (1 IN.)	5
GRANITE SETTS (PER INCH THICK)	14
WOOD-BLOCK PAVING, (PER INCH THICK)	5

See also *Table* 2.

LOADS.

Examples of the Use of Tables 1, 2 and 3.

Determine the design loads for the following cases.

(*a*) A 4½-in. flat roof slab, with an average of 1½ in. of screeding and ½ in. of asphalt. (Note: Effect of wind on a flat roof is suction.)

Imposed load (*Table* 3)	= 30 lb. per sq. ft.
½-in. asphalt, 0·5 × 12 (*Table* 1)	= 6 ,, ,,
1½-in. screeding, 1·5 × 10 (*Table* 1)	= 15 ,, ,,
4½-in. slab (*Table* 1)	= 57 ,, ,,
Total load	= 108 ,, ,,

(*b*) A 4½-in. floor slab spanning 7 ft. and surfaced with 1-in. boards in residential flats. Since the span is less than 8 ft., the minimum total imposed load will control if the design is to be in accordance with the London By-laws (Class No. 2) or the B.S. Code (Class No. 40); (see *Table* 3).

Hence the imposed load is 320 lb. ÷ 7 ft. =	46 lb. per sq. ft.
4½-in. slab (*Table* 1) =	57 ,, ,,
1-in. boards (*Table* 1) =	3 ,, ,,
Allow for bearers =	1 ,, ,,
½-in. plaster on soffit (*Table* 1) say =	5 ,, ,,
	112 ,, ,,

(*c*) A 3-in. hollow-block partition 9 ft. high plastered on both faces and supported on a continuous floor slab spanning 10 ft., the partition being placed at midspan and at right-angles to the direction of the span:

Weight of partition, 25 lb. (*Table* 1) × 9 ft. = 225 lb. per lin. ft. = w_p.

Equivalent distributed load (*Table* 2), $w_e = 1{\cdot}5 \times \frac{225}{10} = 34$ lb. per sq. ft.

(Minimum, according to the B.S. Code: $w_e = 0{\cdot}1 w_p = 0{\cdot}1 \times 225 = 23$ lb. per sq. ft.)

(*d*) A secondary beam in an office ground floor; 5-in. slab surfaced with ¾-in. granolithic and ½-in. plaster on underside. Beams 12 in. net depth by 6 in. wide; span 20 ft.; at 10-ft. centres.

Imposed load on 10 ft. width of slab, 10 ft. × 60 lb. (*Table* 3) = 600 lb. per lin. ft. of beam.

(Check total imposed load = 600 × 20 = 12,000 lb., which exceeds the minimum of 3840 lb.—see *Table* 3.)

Dead load: 5-in. slab (*Table* 1)	= 63 lb. per sq. ft. of slab
Partitions (minimum per *Table* 2)	= 20 ,, ,, ,,
¾-in. granolithic (*Table* 1)	= 9 ,, ,, ,,
½-in. plaster (*Table* 1)	= 5 ,, ,, ,,
Total	= 97 ,, ,, ,,
Dead load on beam from slab, 10 ft. × 97 lb.	= 970 lb. per lin. ft.
Beam rib, 12 × 6 × 150 ÷ 144 (*Table* 1)	= 75 ,, ,,
Plaster (hard) on sides of rib, 2 × 1 ft. × ½ × 10 (*Table* 1)	= 10 ,, ,,
	1055 ,, ,,
Total dead plus live load	= 600 + 1055
	= 1655 lb. per lin. ft. of beam.

(*e*) A 9-in. by 9-in. lintel spanning a clear opening of 7 ft. and supporting a 9-in. brick wall (cement mortar); height of wall exceeds 6 ft.

Height of 60 deg. triangle on 7-ft. base, ½ × 7√3 (*Table* 2)	= 6·06 ft.
Weight of wall at 10 lb. per in. thick. (*Table* 1)	= 90 lb. per sq. ft.
Weight supported by lintel, ½ × 6·06 × 7 × 90	= 1910 lb.
Weight of lintel, 9 × 9 × 7 × 150 ÷ 144	= 590 ,,
	2500 ,,

PARTITIONS

B. S. CODE.

IF POSITION OF PARTITION IS NOT KNOWN:

ADDITIONAL DEAD LOAD PER SQ. FT. OF FLOOR $\nless$ 0·1 (WT. PER LIN. FT. OF PARTITION)
$\nless$ 20 LB./SQ. FT. FOR OFFICES.

IF POSITION OF PARTITION IS KNOWN: DESIGN FLOOR TO SUPPORT ACTUAL WEIGHT OF PARTITION.

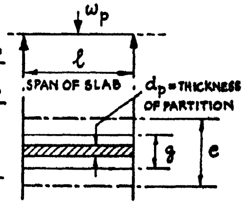
EQUIVILENT UNIFORMLY-DISTRIBUTED DEAD LOAD w_e LB. PER SQ. FT.

WEIGHT OF PARTITION = w_p LB. PER LIN. FT.

PARTITION AT RIGHT-ANGLES TO SPAN OF SLAB.

SLAB FREELY SUPPORTED: $w_e = 2\dfrac{w_p}{\ell}$

SLAB CONTINUOUS OVER BOTH SUPPORTS: $w_e = 1{\cdot}5\dfrac{w_p}{\ell}$



PARTITION PARALLEL TO SPAN OF SLAB.

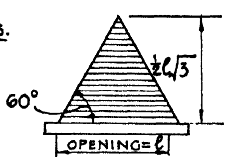
$g = d_p + 2$ (THICKNESS OF SLAB)

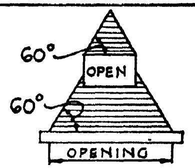
$e = g + 0{\cdot}6\ell \nless g + 3$ FT. $\quad w_e = \dfrac{w_p}{e}$

LINTELS

LOAD ON LINTELS SUPPORTING BRICK (AND SIMILARLY BONDED) WALLS.

SHADING DENOTES EXTENT OF WALL CONSIDERED TO BE SUPPORTED BY LINTEL





ROOFS AND WALLS

	LB. PER SQUARE FOOT
CORRUGATED SHEETING (WITH BOLTS, ETC.):- GALVANISED IRON OR ASBESTOS-CEMENT	3 TO 3½
LIGHT STEEL-FRAMED WALLS WITH 4½" BRICK PANELS, WINDOWS, ETC. AVERAGE	50
ROOF GLAZING (ASTRAGALS, GLASS, ETC.), STEEL PURLINS AND CONNECTIONS	8
SLOPING ROOFS. (FOR WEIGHTS OF TRUSSES - SEE BELOW)	LB./SQ. FT. OF PLAN AREA
SLATES OR TILES, BATTENS, STEEL PURLINS, ETC.	12 TO 16
DITTO DITTO WITH BOARDING, FELT, ETC.	15 TO 20
CORRUGATED SHEETING (GAL. IRON OR ASBESTOS-CEMENT), STEEL PURLINS, ETC.	7 TO 9
ASBESTOS-CEMENT SHEETING, STEEL PURLINS & TRUSSES, WIND AND LIVE LOAD AVERAGE	30

STRUCTURAL STEELWORK

RIVETTED STEELWORK = NOMINAL WEIGHT OF STEEL MEMBERS PLUS 10 % FOR CLEATS, RIVETS, BOLTS, ETC.

WELDED STEELWORK = DITTO PLUS 2½% TO 5%

ROLLED SECTIONS: BEAMS = DITTO PLUS 2½%

STANCHIONS = DITTO PLUS 5% (EXTRA FOR CAPS & BASES)

PLATE-WEB GIRDERS = DITTO PLUS 10% FOR RIVETS, STIFFENERS, ETC.

CONVEYOR GANTRIES (STEEL FRAMING; CORRUGATED SHEETING; WOODEN FLOOR) 5 TO 6 CWTS./LIN. FT.

STEEL WALL FRAMING TO TAKE SHEETING OR BRICK PANELS 5 TO 7 LB./SQ. FT. OF WALL.

STEEL STAIRS (3 FT. INDUSTRIAL TYPE) 56 LB. PER LINEAR FOOT.

ROOF TRUSSES	SPAN OF TRUSS (FT.)	25	30	40	50	60	80	SPACING OF TRUSSES
	APPROXIMATE WEIGHT LB. PER SQUARE FOOT OF PLAN AREA	2	2½	2¾	3	4¼	5	10 FT.
		1½	1½	1¾	2¼	3	3½	15 FT.

SMALL ROOF TRUSSES, PURLINS, ETC. (EXCLUDING COVERING) 6½ LB./SQ. FT. OF PLAN AREA

MISCELLANEOUS

RAIL TRACKS (STANDARD GAUGE): RAILS, SLEEPERS, BALLAST, ETC. AVERAGE = 2 CWT. PER SQ. FT.

RAILS (BULL-HEAD OR FLAT-BOTTOM), CHAIRS, TRANSVERSE SLEEPERS, ETC. = 180 LB/LIN. FT. OF TRACK

BRIDGE RAILS, LONGITUDINAL SLEEPERS, CONNECTIONS, ETC. = 110 " " " " "

STEEL-TUBES 2 IN. BORE = 2½ LB. PER LIN. FT. ¾ IN. GAS PIPE = 1¼ " " "

BELT CONVEYORS FOR CEMENT, GRAIN, COAL, CRUSHED STONE, ETC. = 1½ TO 2½ CWTS./LIN. FT.

SCREENING PLANT (SHAKER TYPE FOR COAL) INCLUDING SUPPORTING STEELWORK = 1½ CWTS./SQ. FT.

See also *Table* 1.

LOADS.

Classification of Floors.—For assessing the imposed (or live) load on floors in buildings, the B.S. Code (No. 3; Chapter V) and the London By-laws classify the floors according to the use to which they are to be put; these classifications and the corresponding minimum imposed loads are given in *Table* 3. The imposed loads on garage floors are given in *Table* 5.

Reduction of Imposed Loads on Multiple-story Buildings.—The scales of the reduction of imposed loads on the floors of multiple-story buildings for the design of the columns, walls, and foundations according to B.S. Code (Chapter V, 1952) and the London By-laws (1952) are given in *Table* 3. No reduction is to be made for factories and workshops designed for 100 lb. per square foot or less, warehouses, garages, and buildings used for storage. For factories and warehouses designed for more than 100 lb. per sq. ft., the reductions given in *Table* 3 apply, but the total imposed load after reduction is to be not less than 100 lb. per sq. ft. per floor.

Examples.—(*a*) Calculate the total load on a column supporting a flat roof and ten floors, including the ground floor, of an office building. The total dead weight of the roof is 60 lb. per sq. ft., and of each floor 120 lb. per sq. ft. including partitions. The story-height is 10 ft. and the average size of each column is 18 in. square. Each column supports a panel of floor and roof 20 ft. by 16 ft.

Dead load:	Roof	= 60	lb. per sq. ft.
	Nine upper floors 9×120 lb.	= 1080	,, ,,
	Ground floor	= 120	,, ,,
	Total dead load	= 1260	,, ,,
Imposed load:	Roof	= 30	,, ,,
	Nine upper floors $= 9 \times 0{\cdot}6 \times 50$ lb.	= 270	,, ,,
	Ground floor $= 0{\cdot}6 \times 60$ lb.	= 36	,, ,,
Total uniformly-distributed load		= 1596	,, ,,
Load on column $= 1596 \times 20 \times 16$		= 510,720	lb.
Weight of column $= (18 \times 18 \times 150 \div 144) \times 10 \times 10$ ft.		= 33,750	,,
Total load per column (below ground floor)		= 544,470	,,

(*b*) Assuming that the building in Example (*a*) is a factory, each floor of which is designed for 150 lb. per sq. ft., calculate the total load on one column.

The normal reduced load for ten floors is $0{\cdot}6 \times 10 \times 150$ lb., that is 900 lb. per sq. ft., which is less than the specified minimum load of 10×100 lb. = 1000 lb. per sq. ft.; the minimum load therefore applies. The total load is therefore

Dead load (as before)		= 1260	lb. per sq. ft.
Imposed load:	Roof	= 30	,, ,,
	Floors	= 1000	,, ,,
Total uniformly-distributed load		= 2290	,, ,,
Load on column $= 2290 \times 20 \times 16$		= 732,800	lb.
Weight of column (as before)		= 33,750	,,
Total load on column		= 766,550	,,

FLOORS	CLASS Nº	B. S. CODE CP. 3(V) 1952	30	40	50	60	80	100	150	200
		LONDON BY-LAWS 1952	1	2	3	4	5	6	7	8
	IMPOSED LOAD (LB. PER SQ. FT.)		30	40	50	60	80	100	150	200
	MINIMUM TOTAL LOAD	SLABS SPAN LESS THAN 8 FT.	240	320	400	480	640	800	TOTAL LOAD (LB.) PER FT. WIDTH.	
		BEAMS SUPPORTING LESS THAN 64 SQ. FT.	1920	2560	3200	3840	5120	6400	TOTAL LOAD (LB.) ON BEAM.	

	TYPE	USE OF FLOOR	B. S. CODE CLASS Nº.	LOND. BY-LAWS CLASS Nº.
FLOORS	RESIDENTIAL	HOUSES: > TWO STORIES (ONE OCCUPANT)	30	1
		OTHERS (INCLUDING FLATS)	40	2
		HOSPITAL WARDS; DORMITORIES	40	2
		HOTELS: BEDROOMS; PRIVATE SITTING-ROOMS	40	2
		PUBLIC ROOMS	100	–
	COMMERCIAL	OFFICES: ENTRANCE FLOOR AND BELOW	60	4
		OTHER FLOORS	50	3
		STORAGE AND FILING ROOMS	100	6
		BANKING HALLS	60	–
		RETAIL SHOPS	80	5
		BOOK AND STATIONERY STORES	200	8
		WAREHOUSES (DEPENDING ON TYPE OF GOODS)	100, 150, 200	6, 7, 8
	INDUSTRIAL	WORKSHOPS AND FACTORIES (DEPENDING ON USE)	100, 150, 200	6, 7, 8
		WORKROOMS: WITH CENTRAL POWER-DRIVEN MACHINES	80	5
		WITHOUT DITTO OR STORAGE	50	3
		POWER STATIONS & MACHINERY HALLS: CIRCULATION SPACE	80	–
	PLACES OF ASSEMBLY	SCHOOL CLASSROOMS	60	4
		WITHOUT FIXED SEATING; DANCE HALLS	100	–
		WITH FIXED SEATING; CHURCHES, ETC.	80	–
		RESTAURANTS	80	–
	MISCELLANEOUS	PUBLIC PAVEMENTS OVER BASEMENTS	200	–
		CORRIDORS. — SAME IMPOSED LOAD AS FLOOR OF WHICH THEY FORM PART.		
		BALCONIES — SAME IMPOSED LOAD AS FLOOR TO WHICH THEY GIVE ACCESS. MINIMUM TOTAL LOAD APPLIES IF PROJECTION OF CANTILEVERED BALCONY IS LESS THAN 8 FT.		

	SLOPE (θ DEG.)	IMPOSED LOAD (LB PER SQUARE FOOT OF HORIZONTAL AREA)	
ROOFS	FLAT NOT STEEPER THAN 10 DEG.	WITHOUT ACCESS 15 LB. WITH ACCESS 30 LB. (MINIMUM TOTAL LOADS AS FOR CLASS No 1 OR 20)	PER B.S. CODE No 3, CHAP. V (1952) AND LONDON BY-LAWS (1952) — COMBINE WITH WIND PRESSURE TO GIVE MOST ADVERSE EFFECTS.
	NOT STEEPER THAN 30 DEG / STEEPER THAN 30 DEG.	WITHOUT ACCESS 15 LB. NOT STEEPER THAN 75°: $25 - \frac{\theta^\circ}{3}$ LB. STEEPER THAN 75°: NIL	
	CURVED	DIVIDE INTO FOUR EQUAL SEGMENTS.— IMPOSED AND WIND LOADS ARE THOSE APPLICABLE TO THE SLOPE OF THE CHORD OF EACH SEGMENT.	
	NOT STEEPER THAN 45 DEG.	ROOF COVERING (EXCEPT GLASS) TO BE DESIGNED FOR A LOAD OF 200 LB. CONCENTRATED ON 5 IN. SQUARE	PER B.S CODE

		B. S CODE	LONDON BY-LAWS	IMPOSED LOAD	
STAIRS AND LANDINGS	CLASS OF FLOOR SERVED	No. 30	No. 1	30 LB /SQ FT	MINIMUM TOTAL LOAD DOES NOT APPLY
		No. 40, 50 OR 60	No. 2, 3 OR 4	60 " " "	
		OTHER CLASSES	OTHER CLASSES	100 " " "	
		SEPARATE CANTILEVERED STEP	—	300 LB. ON UNSUPPORTED END.	

COLUMNS WALLS AND FOUNDATIONS	NUMBER OF FLOORS SUPPORTED	1	2	3	4	5 OR MORE	NOT APPLICABLE TO STORES, WAREHOUSES, GARAGES, WORKSHOPS AND FACTORIES.
	PROPORTION OF IMPOSED LOAD ON ALL FLOORS SUPPORTED	1·0	0·9	0·8	0·7	0·6	
	WORKSHOPS AND FACTORIES DESIGNED FOR MORE THAN 100 LB /SQ FT.— REDUCTION FACTORS AS ABOVE APPLY BUT IMPOSED LOAD NOT TO BE REDUCED TO LESS THAN 100 LB /SQ.FT.						
	BEAM SUPPORTING 500 SQ FT. OR MORE OF FLOOR.— IMPOSED LOAD MAY BE REDUCED BY 5% (MAX. REDUCTION 25%) FOR EACH 500 SQ.FT. OF FLOOR SUPPORTED.						

See also *Table* 4 for loads on garage floors; *Table* 2 for loads due to partitions; and *Tables* 7 and 8 for wind pressure on roofs.

LIVE LOADS.

Live loads on structures include the weights of stored solid materials and liquids (see *Table* 4) and the loads imposed by vehicles and moving equipment, the weights of which are given in *Tables* 5 and 6; notes on the latter tables are given in the following.

Secondary Effects on Railway Bridges.—Weights and other particulars of typical rail vehicles are given in *Table* 6. To the static forces, such as wheel loads, the following secondary dynamic effects must be added.

Impact on a bridge used by steam locomotives is more severe than on a bridge carrying electric or diesel-operated trains. The effect of impact on the structure is greatly reduced by the provision of ballast, and also depends on rail joints and irregularities in the track and the wheels. Formulæ suitable for conditions comparable with main lines in Great Britain and elsewhere, and allowances for less-well-laid and maintained tracks, are given in B.S. No. 153 (Part 3A).

Lurching is allowed for on a bridge carrying a main line by assuming that five-eighths of the total axle load may be imposed on any one rail, the remaining three-eighths being carried on the other rail forming the track; these fractions apply to British conditions and can be assumed to be more nearly equal for trains running at low speeds, but more unequal for rolling stock with light springing.

Nosing is allowed for by assuming that a single force of 10 tons acts horizontally at rail level in either direction at right-angles to the track, irrespective of the number of tracks.

The centrifugal effect is considered as a force F_c lb. per lin. ft. acting horizontally at 6 ft. above the level of the rails, where $F_c = \frac{wV^2}{R} \cdot \frac{n}{15}$ in which w lb. per lin. ft. is the live load on one track expressed as an equivalent uniformly-distributed load, V m.p.h. is the speed of the train, R ft. is the mean radius of the track, and n is the number of tracks.

The longitudinal force due to braking and tractive effort is allowed for by introducing at rail level a longitudinal force equal to 10 per cent. of the net live load on one track or 20 per cent. of the total of the net loads on the driving wheels, whichever is greater. For a bridge carrying more than one track the force is calculated assuming two tracks to be occupied, both forces acting in the same direction.

Railway Sleepers.—The various tracks in *Table* 5 are classified as (i) main lines upon which a frequent service of trains with axle loads up to 23 tons and travelling at speeds exceeding 70 m.p.h. is maintained; (ii) secondary lines upon which the weights and speeds are similar to those for main lines but on which the service is less frequent; (iii) tertiary lines which include lines where speeds not exceeding 45 m.p.h. are reached by trains with axle loads not exceeding 20 tons, and upon which the frequency is less than two-thirds of that on main lines; (iv) refuge sidings, which include heavily-worked sidings, goods loops, and similar lines where the speed of trains does not exceed 30 m.p.h.; (v) sidings which include lightly-worked and storage sidings.

Example of Loading on Deck of Railway Bridge.—To find the maximum total load on a 12-in. concrete slab supporting a single line of standard railway over which can pass at slow speeds any size of locomotive.

Load from ballast, sleepers, chairs, rails, etc. (*Table* 2)	= 224	lb. per sq. ft.
Concrete slab, 12 in. thick (*Table* 1)	= 150	,, ,,
Total dead load	= 374	,, ,,

Dispersion of wheel loads (*Table* 6):

D = say, 12-in. ballast + 10½-in. effective depth of slab = 1 ft. 10½ in.

$A = 2D$ + distance over two sleepers (minimum at joints = 3 ft.) = 3 ft. 9 in. + 3 ft. = 6 ft. 9 in. (This dimension should not exceed the distance between the axles of the locomotive.)

$B = 2D$ + length of sleeper = 3 ft. 9 in. + 9 ft. = 12 ft. 9 in.

Axle load of 22½ tons (*Table* 6), $\frac{22 \cdot 5 \times 2240}{12 \cdot 75 \times 6 \cdot 75} = 586$ lb. per sq. ft.

For infrequent passage and slow speed of such a heavy axle load, allow 50 per cent. for impact and other dynamic effects.

Total load, $374 + (1 \cdot 5 \times 586) = 1253$ lb. per sq. ft.

(In main-line bridge construction, longitudinal rail-beams might be provided, thereby relieving the slab of loads from the rails and wheels.)

SOLIDS IN GRANULAR FORM UNLESS STATED OTHERWISE

		LB. PER CU. FT.			LB. PER CU. FT.
ASHES	GENERAL	40 TO 70	HAY	PRESSED IN BALES	8
	PULVERISED-FUEL		ICE		57
BRICK	BROKEN, COMMON	70 TO 80	IRON ORE	GENERAL	150
	ENGINEERING	80 TO 100		SWEDISH	230
CEMENT	GENERAL	90	LIME	SLAKED	25 TO 35
	AIR-AGITATED	75	PAPER		60
CLAY	LOOSE IN LUMPS	63	SALT	DRY, LOOSE	90 TO 105
CLINKER	AVERAGE	60	SAND	DRY, LOOSE	100
COAL	SOLID	80		DAMP-WET	110 TO 120
	LOOSE IN LUMPS	56	SLAG	CRUSHED	90 TO 112
COKE, BREEZE, ETC.		35	STONE, ETC. (CRUSHED)	GRANITE	100 TO 130
COLLIERY "DIRT" (SHALE, ETC.)		70		BASALT, DOLERITE	110 TO 140
COTTON	IN BALES	15 TO 40		LIMESTONE, SANDSTONE	80 TO 120
EARTH	LOOSE, DRY	90		CHALK	60 TO 80
	COMPACT, DAMP	100		SHALE	70
FLOUR		40 TO 45		QUARRY WASTE	90
GRAIN	BARLEY	25 TO 40	STRAW		15 TO 20
	OATS	25 TO 30	SUGAR		50
	WHEAT	40 TO 45	TEA		28
GRAVEL	LOOSE (AVERAGE)	100	TIMBER	SOFT-WOODS	30 TO 45
	WITH SAND	120		HARD-WOODS	45 TO 80

LIQUIDS AND SEMI-LIQUIDS

	LB. PER CU. FT.		LB. PER CU. FT.		LB. PER CU. FT.
ALCOHOL	53	LINSEED OIL	56	SEWAGE	62 TO 75
AMMONIA	56	MILK	65	SULPHURIC ACID (CONC.)	115
BEER	63	MINERAL OILS (AV.)	58	TAR	64
BITUMEN	87	NAPTHA	47	TURPENTINE	62
BENZINE	43	PARAFIN (KEROSENE)	50	WATER FRESH	62·4
CLAY SLURRY	76	PETROL (GASOLENE)	44	SEA-WATER	64
CLAY-CHALK SLURRY	100	PETROLEUM OIL	51	WINE	62

SOLID MATERIALS IN LIQUIDS

w_l = DENSITY OF LIQUID (LB. PER CU. FT.)

p_v = PRESSURE ON BOTTOM OF CONTAINER (LB. PER SQ. FT.)

h = TOTAL DEPTH OF LIQUID (FT.)

h_m = THICKNESS OF LAYER OF SUBMERGED LIQUID (FT.)

w_m = WEIGHT OF SOLID MATERIAL (= DENSITY IN LB. PER CU. FT.)

V = VOLUME OF VOIDS IN 1 CU. FT. OF DRY GRANULAR MATERIAL

MATERIAL FLOATING IN LIQUID.

$$p_v = w_l h$$

MATERIAL SUBMERGED IN LIQUID OF LESS DENSITY THAN MATERIAL.

$$p_v = h_m\left[w_m(1 - V) + w_l V\right] + w_l(h - h_m)$$

VALUES OF $[w_m(1-V)+62{\cdot}4V]$ FOR MATERIALS SUBMERGED IN WATER	MATERIAL	WEIGHT IN SOLID (w_m) LB PER CU. FT.	PERCENTAGE OF VOIDS (= 100V) 25	30	35	40	45	50%
	CRUSHED COAL	80	77	75	74	73	72	71
	CRUSHED STONE	160	136	131	126	121	116	111
	SAND	160						

Other Materials.—Stone, timber, etc., in solid (no voids): see *Table* 1. Granular materials such as sand, crushed stone and coal, gravel, etc., see *Table* 11. Cohesive materials such as clay, earth, etc., see *Table* 12.

MOVING LOADS.

Garage Floors: Example.—To find the equivalent uniformly-distributed imposed load at the support section of a beam 12 in. wide, spacing 10 ft., span 10 ft., fully continuous, in a garage floor; the maximum wheel load is $1\frac{1}{2}$ tons. From *Table* 5, $f = 0{\cdot}95$ and $w = 330$ lb. per ton. With $P = 1\frac{1}{2}$ tons, the equivalent uniform load is $1{\cdot}5 \times 330 \times 0{\cdot}95 \times 1{\cdot}5 = 705$ lb. per lin. ft.; compare with the normal imposed load of 150 lb. per sq. ft. $= 10 \times 150 = 1500$ lb. per lin. ft. Hence the normal load controls; to the load of 1500 lb. per ft. must be added the dead load of the slab and beams. This combined load, say 2200 lb. per lin. ft., only affects the bending moment, which is about $\dfrac{2200 \times 10^2 \times 12}{12} = 220{,}000$ in.-lb.

The greatest shearing force is $1{\cdot}5 \times 1\frac{1}{2} = 2{\cdot}25$ tons plus the shearing force due to the dead load, that is about 8540 lb.

Overhead Travelling Cranes.—To allow for vibration, acceleration, and impact, the maximum static wheel loads, as given in *Table* 5, of electric overhead travelling cranes should be increased by 25 per cent. Braking or travelling under power produces in the rail-beam a horizontal thrust which is transferred to the supports. The traversing of the crane and load produces a horizontal thrust transversely to the rail-beam. Therefore the additional forces acting on the supporting structure when the crane is moving are (*a*) a horizontal force acting transversely to the rail and equal to 10 per cent. of the weight of the crane and the load lifted, it being assumed that the force is equally divided between the two rails; (*b*) a horizontal force acting along each rail and equal to 5 per cent. of the total weight of the crane, crane bridge, and the load lifted (when in the position nearest to the rail). The forces (*a*) and (*b*) are not considered to act simultaneously, but the effect of each must be combined with that of the increased maximum vertical wheel loads. The foregoing recommendations are in accordance with B.S. No. 449.

For a crane operated by hand, the vertical wheel loads should be increased by 10 per cent.; for force (*a*), the proportion of the weight of crane and load can be 5 per cent.

Loads on Colliery Pit-head Frames.—The loads to which a pit-head frame is subjected are as follows. (These notes do not apply to the direct vertical winding type of pit-head tower.)

Dead Loads.—The dead loads include the weights of (i) the frame and any stairs, housings, lifting beams, etc., attached to it; (ii) winding pulleys, pulley-bearings, pedestals, etc.; (iii) guide and rubbing ropes plus 50 per cent. for vibration.

Live Loads.—The live loads are the resultants of the tensions in the ropes passing over the pulleys and (unless described otherwise) are transmitted to the frame through the pulley bearings and may be due to the following conditions (*a*) Retarding of descending cage when near the bottom of the shaft, this force is the sum of the net weight of the cage, load, and rope, and should be doubled to allow for deceleration, shock and vibration. (*b*) Force due to overwinding the cage which is then dropped on to the over-wind platform; this force acts only on the platform (and not at the pulley bearings) and is the sum of the net weights of the cage and attachments and the load in the cage, which sum should be doubled to allow for impact. (*c*) Force causing rope to break due to cage sticking in shaft or other causes; the force in the rope just before breaking is the tensile strength of the rope. (*d*) Tension in rope when winding up a loaded cage.

Combined Loads.—For a frame carrying one pulley, the conditions to be designed for are the total dead load combined with either live load (*a*), (*b*), (*c*) or (*d*). Generally condition (*c*) gives the most adverse effects, but it is permissible in this case to design for stresses, say, double the ordinary permissible working stresses because of the short duration of the maximum force; the procedure would be to design the frame for dead load plus half of force (*c*) and adopt the ordinary working stresses. If the frame carries two pulleys, the conditions to be investigated are: dead load plus (*a*) on one rope and (*d*) on the other (this is the ordinary working condition); dead load plus (*a*) on one rope and over-wind (*b*) on the other; dead load plus (*a*) on one rope and breaking force (*c*) on other rope (this is generally the worst case; force (*c*) can be halved as explained for a single-pulley frame).

The weights of the ropes, cages, etc., and the strength of the ropes would be obtained for any particular pit-head frame from the colliery authorities, and vary too greatly for typical values to be of any use.

Structures Supporting Lifts.—The effect of acceleration must be considered in addition to the static loads when calculating the load due to lifts and similar machinery. If a net static load of W_D is subject to an acceleration of a feet per second per second the load on the supporting structure is given approximately by $W_M = W_D(1 + 0{\cdot}03a)$. The average acceleration of a passenger lift may be about 2 ft. per second per second, but the maximum acceleration will be considerably greater. An equivalent load of $1{\cdot}25W_D$ should be taken as the minimum to allow for dynamic effects. The load for which lift supports, pit-head

(*Continued on page* 150.)

GARAGE FLOORS

GROSS WEIGHT OF VEHICLE	USE OF FLOOR	MEMBER	ORDINARY IMPOSED LOAD	ALTERNATIVE MINIMUM IMPOSED LOAD
NOT GREATER THAN 2½ TONS	PARKING ONLY	SLABS AND BEAMS	LB. PER SQ. FT. 80	MOST ADVERSE ARRANGEMENT OF ACTUAL WHEEL LOADS
	GARAGE	SLABS	80 *	80 LB./SQ.FT. OR MOST ADVERSE ARRANGEMENT OF ACTUAL WHEEL LOADS WHICHEVER IS GREATER
		BEAMS	80 *	50 " " OR MOST ADVERSE ARRANGEMENT OF ACTUAL WHEEL LOADS WHICHEVER IS GREATER
NOT GREATER THAN 4 TONS	GARAGE	SLABS AND BEAMS	150 *	2000 LB. (MINIMUM)* OR NOT LESS THAN 1½ x ACTUAL WHEEL LOAD (WHICHEVER IS GREATER) DISTRIBUTED ON AREA 2 FT. 6 IN. SQUARE.

PER B.S. CODE No. 3, CHAP. V (1952) AS REVISED 1958

*CONFORMING TO LONDON BY-LAWS (1952)

EQUIVALENT UNIFORMLY-DISTRIBUTED LOAD (w) FOR B. M. CALCULATION DUE TO ONE TON DISTRIBUTED OVER AREA 2 FT. 6 IN. SQUARE

SLABS

SPAN OF SLAB		4 FT.	6 FT.	8 FT.	10 FT.
Simply supported	MIDSPAN	310	235	190	160
One end fixed	MIDSPAN	320	270	230	200
	FIXED SUPPORT	300	220	170	140
Both ends fixed	MIDSPAN	340	285	240	205
	SUPPORT	295	270	160	130

EQUIVALENT LOADS w ARE IN LB. PER SQUARE FOOT OF SLAB PER TON LOAD FROM ONE WHEEL.

LOAD IS ASSUMED TO ACT ON STRIP 2 FT. 6 IN. WIDE.

BEAMS

SPAN OF BEAM		10 FT.	12 FT. 6 IN.	15 FT.	20 FT.
Simply supported	MIDSPAN	390	320	270	205
One end fixed	MIDSPAN	500	430	360	280
	FIXED SUPPORT	340	280	240	170
Both ends fixed	MIDSPAN	510	440	370	290
	SUPPORT	330	270	230	160
REDUCTION FACTORS	SPACING OF BEAMS	5 FT.	10 FT.	15 FT.	OVER 15 FT.
	f	0·90	0·95	0·975	1·0

EQUIVALENT LOADS w ARE IN LB. PER FOOT LENGTH OF BEAM PER TON LOAD FROM ONE WHEEL.

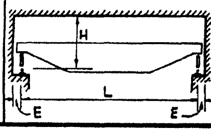
EFFECT OF WHEEL LOADS MUST BE COMPARED WITH EFFECT OF ORDINARY IMPOSED LOAD AND MOST ADVERSE CASE CONSIDERED.

EQUIVALENT UNIFORM LOAD PER FOOT OF BEAM $= 1\cdot 5 wPf$ (LB.) P = MAX. WHEEL LOAD (TONS).

OVERHEAD TRAVELLING CRANES (ELECTRIC)

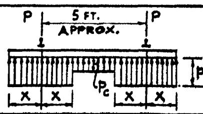
LIFTING CAPACITY TONS	MINIMUM WHEEL BASE OR NOT LESS THAN SPAN/5	WEIGHT OF CRANE (EXC. LOAD) TONS	MAX LOAD (TONS) ON PAIR OF WHEELS — SPAN (L) OF CRANE 30 FT	40 FT	50 FT	60 FT	CLEARANCES HEIGHT H	END E
2	6 FT	6 TO 9	5½	6	7	-	5·5 FT.	8 IN
5	8·5 "	11½ TO 16	-	11½	13	14	6·0 "	9 "
10	10 "	14½ TO 19	-	18	20	21½	6·75 "	9½ "
20	10·5 "	21 TO 26½	-	31	33	35½	7·5 "	11 "
30	12 "	25 TO 33	-	46	48	51	8·5 "	12 "
50	13 "	43 TO 51½	-	70	72	81	10·25 "	14 "

NOTE. THE DATA GIVEN IN THIS TABLE ARE TYPICAL AND MAY VARY DUE TO MAKER AND USE.



RAILWAY SLEEPERS (REINFORCED CONCRETE) STANDARD GAUGE (PER B. S. No. 986)

TYPE OF LINE (SEE CHAP. II)	LOAD (P) FROM EACH RAIL — SLEEPERS AT JOINTS.	INTERMEDIATE SLEEPERS	MINIMUM MOMENT OF RESISTANCE AT CENTRE	DISTRIBUTION OF PRESSURE ON BALLAST (FOR PURPOSES OF DESIGN) P_c/P
(i) MAIN LINE	14 TONS	11 TONS	± 45,000 IN.-LB	P_c/P = ¼ TO ⅓
(ii) SECONDARY LINE	12 "	10 "	± 45,000 " "	⅓
(iii) TERTIARY LINE	10 "	9 "	± 30,000 " "	½
(iv) GOODS LINES	7½ "	7½ "	± 30,000 " "	½
(v) SIDINGS	5½ "	5½ "	± 30,000 " "	½



MOVING LOADS (*continued from page facing Table* 5).

frames, and similar structures are designed should be related to the load on the ropes. If the latter is W_M and the ropes have a factor of safety of ten, the design load on the supports should not be less than $2\frac{1}{2}W_M$ to ensure that the structure is as strong as the ropes.

Weights and Dimensions of Road Vehicles.—The data in *Table* 6 relating to heavy motor vehicles, trailers, public-service vehicles, and road locomotives are abstracted from "The Motor Vehicles (Construction and Use) Regulations 1955" (issued on behalf of the Ministry of Transport). As there are many varieties of vehicle only the maximum loads and dimensions permissible are given. In general the Regulations apply to vehicles registered in or after 1955; vehicles registered before that date may have greater dimensions and weights. Supplementary data are as follows. (Street-cleaning and road-repair vehicles are excluded from *Table* 6.)

Load imposed by the wheels of a vehicle on a strip of road 2 ft. wide and transverse to the longitudinal axis of the vehicle must not exceed 11 tons.

Tractors.—Overall width must not exceed 7 ft. 6 in. and overhang must not exceed 6 ft.

Motor cars.—Overall length must not exceed 30 ft.; overall width must not exceed 7 ft. 6 in.; overhang must not exceed half the wheel-base. For vehicles not exceeding 20 ft. long, overhang may be 9 in. plus half the wheel-base and not more than $\frac{7}{24}$ of overall length. For rear-tipping lorries, overhang need not conform to the foregoing if not exceeding 3 ft. 9 in.

Trailers.—Total weight of trailers drawn by one locomotive must not exceed 40 tons.

Articulated vehicles.—Overall length must not exceed 35 ft. (except for a vehicle for an indivisible load). Laden weight of trailer must not exceed 20 tons if having not more than four wheels or 24 tons if more than four wheels.

Standard Live Loads for Road Bridges.

Ministry of Transport.—The uniformly-distributed load applicable to the "loaded length" of a bridge or of a structural member forming part of a bridge is selected from *Table* 6.

The "loaded length" is the length of member which should be considered to be carrying load in order to produce the greatest stresses. On a freely-supported span, the "loaded length" would thus be (*a*) for bending moment: the entire span; (*b*) for shearing force at the support: the entire span; and (*c*) for shearing force at any intermediate section: from the section to the farther support. In arches and continuous spans the "loaded length" can be taken from influence lines.

The live load to be used is in two parts: (1) the uniformly-distributed load which varies with the loaded length; (2) an invariable knife-edge load of 2700 lb. per ft. of width applied at the section where it will, when combined with the uniformly-distributed load, be most effective, that is, on a freely-supported span: (*a*) for bending moment at midspan: at mid-span; (*b*) for shearing force at the support: at the support; and (*c*) for shearing force at any section: at the section.

On slabs, the knife-edge load of 2700 lb. per ft. width is assumed to act parallel to the supporting members (that is, at right-angles to the direction in which the slab spans). On longitudinal girders, stringers, etc., this load is assumed to act transversely to them (that is, parallel to their supports). On transverse beams the load is assumed to act in line with them (that is, 2700 lb. per ft. of beam).

If longitudinal or transverse members are spaced more closely than at 5-ft. centres, the live load allocated to them must be that calculated on a strip 5 ft. wide. With wider spacing the width of this strip will be equal to the spacing of the members.

B.S. No. 153 (Part 3a): Design Loads.—Slabs spanning in One Direction: uniformly-distributed load $= w_L$ lb. per sq. ft. (*Table* 6); knife-edge load = 2700 lb. per ft. at right-angles to span; transverse reinforcement = sufficient to resist 50 per cent. of live-load bending moment.

Slabs spanning in Two Directions and Cantilevered Slabs (not less than 4-ft. projection): two wheel loads each equal to W (*Table* 6).

Aircraft Runways.—Aircraft runways and aprons are designed to support a basic load designated as the Equivalent Single Wheel Load (E.S.W.L.), which is equal to the actual wheel load if each undercarriage of the aircraft has only one wheel and, in the case of multiple-wheel undercarriages, is the single wheel load that produces the same stresses in the slab as the group of wheels at the same tyre pressures. The E.S.W.L. varies with the thickness and type of paving, and for each type of aircraft corresponding values are specified by the authorities.

In British practice, a Load Classification Number (L.C.N.) is attributed to each thickness and type of paving and for each aircraft; a given paving can be used by aircraft having the same or lower L.C.N. as the paving. The L.C.N. of an aircraft is equal to the E.S.W.L. (lb.) divided by 1000. The International Civil Aircraft Organisation (I.C.A.O.) has a different load

(*Continued on page* 152.)

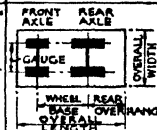
RAILWAY ROLLING STOCK (BRITISH RAILWAYS)

LOCOMOTIVES	HEAVY GOODS + TENDER (56′-0″)	EXPRESS + TENDER (59′-3″)	MIXED TRAFFIC (TANK) (52′-7″)	SHUNTING (TANK) (15′-6″)	EXPRESS (DIESEL) (67′-9″ 9⁄16)	NOTE.
TOTAL WT. ENGINE & TENDER	139	157	$86\frac{3}{4}$	56	106	MAX. AXLE LOAD $\ngtr$ 20 TONS ON RAILS 95/100 LB. YD. OR $\ngtr 22\frac{1}{2}$ TONS FOR MULTI-CYL. LOCOS.
AXLE LOAD MAX. (TONS)	$15\frac{1}{2}$	22	$17\frac{3}{4}$	$18\frac{1}{2}$	$\ngtr$ 20	

DESCRIPTION OF VEHICLE		WEIGHTS (TONS) MAX. AXLE LOAD	WEIGHTS (TONS) TOTAL WT. LADEN	REAR OVER-HANG	OVERALL WIDTH	OVERALL LENGTH	GAUGE	WHEEL BASE
OPEN MINERAL WAGONS	$24\frac{1}{2}$ TONS.-4-WHEEL	$17\frac{1}{4}$	$34\frac{1}{2}$ ($\ngtr$ 35)	6′-0″	8′-0″	24′-0″	4′-$8\frac{1}{2}$″	12′-0″
	56 TONS.-8-WHEEL HOPPERED (ORE)	$21\frac{1}{4}$	85	–	9′-0″	28′-0″	4′-$8\frac{1}{2}$″	5′-6″, 15′-6″, 5′-6″
MISC.	COLLIERY TUBS AND MINE CARS	$\frac{3}{8}$ TO $\frac{3}{4}$	$\frac{3}{4}$ TO $1\frac{1}{2}$	–	3′-6″	–	2′-0″ TO 2′-3″	1′-3″ MIN. TURNING RAD. = 12 FT.

ROAD VEHICLES

DESCRIPTION OF VEHICLE			MAX. AXLE LOAD	TOTAL WT. LADEN	REAR OVER-HANG	OVERALL WIDTH	OVERALL LENGTH	GAUGE	WHEEL BASE
STREET TRAM CAR (8 WHEELS)			$7\frac{1}{2}$ (DRIVING) $3\frac{1}{4}$ (PONY)	$22\frac{1}{2}$	–	–	–	–	–
ROLLERS	STEAM (20 TONS)		8 (FRONT ROLLER) 12 (DRIVING)	20	–	9′-0″	–	5′-9″ DRIVING WHEELS	10′-0″
	DIESEL	3-WHEEL	8	16 WITH SAND BALLAST	–	–	–	6 FT. ROLLING WIDTH	10′-8″
		TANDEM	10	5 WITH WATER BALLAST	–	–	–	4 FT. ROLLING WIDTH	10′-0″
HEAVY MOTOR VEHICLES			$\ngtr$ 9	MAX: $\ngtr$ 4 WHEELS: 14; > 4 " 20; > 6 " 24	$\ngtr \frac{B}{2}$	$\ngtr$ 7′-6″	$\ngtr$ 30′-0″	–	–
PUBLIC SERVICE VEHICLES			$\ngtr$ 8	4 " 12; > 4 " 14	HEIGHT $\ngtr$ 15′ 0″	–	$\ngtr$ 26′-0″ 4-WHEEL DOUBLE-DECK; $\ngtr$ 30′-0″ OTHER TYPES		
TRAILERS (EXCLUDING VEHICLE FOR INDIVISIBLE LOAD, PART OF ARTICULATED VEHICLE OR LAND IMPLEMENT.)			$\ngtr$ 9	< 6 " 14; TRAILER PLUS TOWING VEHICLE 24, 22, 32	– (CRAWLER) (TRAILER BRAKED)	$\ngtr$ 7′6″ OR 8′-0″ IF TOW-VEHICLE = 8′-0″	$\ngtr$ 22′-0″ EXCL DRAW-BAR		
ROAD LOCO (M.O.T.)			$\ngtr$ 11	$\ngtr 20\frac{1}{2}$ OR < 6 WHEELS: 22; 6 " 26; > 6 " 30 — WITH SPRINGS & TYRES		$\ngtr$ 9′ 0″	–		



AIRCRAFT

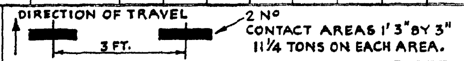
CONTACT AREA = E.S.W.L. ÷ TYRE PRESSURE	130	250	350	450	525	600	SQ. IN.
LOAD CLASSIFICATION No. (L.C.N. = $\frac{E.S.W.L.}{1000}$)	10	20	30	40	50	60	LB. ÷ 1000
I.C.A.O. (1953) No.	6		5		4	3	–

STANDARD LIVE LOADS ON ROAD BRIDGES

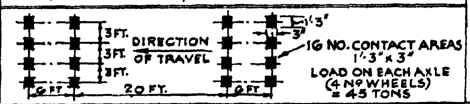
NORMAL UNIFORMLY-DISTRIBUTED LOAD W_L LB PER SQUARE FOOT.

LOADED LENGTH (FT)		3	4	5	6	8	10	12	15	20 TO 75	90	100	150
MINISTRY OF TRANSPORT		2420	1700	1225	872	444	220	220	220	220	212	208	192
R C SLABS ON STEEL BRIDGES (PER B.S. No. 153 PART 3A)	LONGITUDINAL	2420	1700	1225	850	520	400	325	250	220	208	200	168
	TRANSVERSE	2270	1180	770	580	390	310	260	220				

WHEEL LOADS W (B.S. 153)



ABNORMAL LOAD (M. O. T. & B.S. 153)

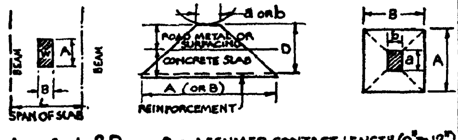


SEE NOTES ON PAGE OPPOSITE

B. S. No. 153
DISPERSION AT 45 DEG. MAY BE ALLOWED FROM EACH CONTACT AREA.
SPECIFIED LOADS INCLUDE IMPACT.
PERMISSIBLE STRESSES MAY BE INCREASED BY 25 PER CENT. WITH THESE LOADS.

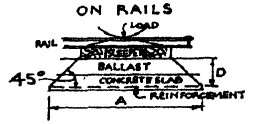
LOAD DISPERSION

DISPERSION OF WHEEL LOADS ON ROAD BRIDGES, ETC.



A = a + 2D   a = ASSUMED CONTACT LENGTH (0″ TO 12″)
B = b + 2D   b = TYRE WIDTH (6″ TO 18″)
FOR GARAGE FLOORS A = B = 2 FT. 6 IN.

DISPERSION OF WHEEL LOADS ON RAILS



FOR TOTAL AXLE LOAD:
A = 2D + WIDTH OVER TWO SLEEPERS
B = 2D + LENGTH OF SLEEPER.

Aircraft Runways (*continued from page* 150).

classification, and the equivalent classification and L.C.Ns. are given in *Table* 6. Design of a pavement depends upon the loading, the contact area (= E.S.W.L. ÷ tyre pressure), and the supporting power of the ground. Values of contact area and the corresponding L.C.N. in accordance with a standard loading curve are given in *Table* 6. Some examples of contact area (and in brackets the corresponding E S.W.L.) for various aircraft are as follows.

Dakota, 270 (12,000); Viscount 700, 270 (24,000); Comet 4, 225 (35,000); Constellation, 490 (37,500); Boeing 707, 360 (50,000); and D.C.8, 430 (56,000).

Reference should be made to "International Standards and Recommended Practices—Aerodromes. Annex 14" (issued by the International Civil Aviation Organisation=I.C.A.O.) For particular design data see " Design and Construction of Airfield Pavements " by J. Maxwell Watson (Journal of Institution of Structural Engineers, 1958), from which the foregoing data is abstracted.

EFFECTS OF WIND.

Wind Pressures on Buildings (*Table* 8).

Exposures.—Exposure D corresponds to a wind having a velocity of 72 m.p.h. and is applicable to exposed sites near the coast and on inland mountains in Great Britain. Exposure C corresponds to a velocity of 63 m.p.h. and applies to inland sites in open country not exceeding 800 ft. in altitude. Exposure B is for general use and is the condition generally accepted by the London By-laws; it corresponds to a velocity of 54 m.p.h. and applies to inland sites at altitudes not exceeding 500 ft. For sites well protected from the wind by natural features, exposure A (45 m.p.h.) applies.

Pressures.—The basic pressures p lb. per sq. ft. at any height and for any velocity V m.p.h. are calculated from $p = 0{\cdot}0001925V^2p_D$, in which p_D is the pressure for the same height given for exposure D ($V = 72$ m.p.h.) in *Table* 8. The height of the building on which the value of p is based is the distance from the ground to a point midway between the eaves and the ridge if the roof is sloping, or to the eaves or top of the parapet if the roof is flat. The value of p for projections above the general level of the roof is based on the total height of the projection above the ground.

General Formula for Wind Pressure. (*Table* 7).—The following example shows the application of the formulæ for the velocity and pressure of the wind as given in *Table* 7.

Example.—If meteorological records for a certain site show that the probable maximum velocity of the wind is 75 m.p.h. at a height of 33 ft. (10 metres), estimate the pressures on a circular chimney erected on this site, and having a height of 200 ft.

For $V_1 = 75$ m.p.h., $V_H = 17{\cdot}5 + 49{\cdot}3 \log 0{\cdot}3(H + 16)$.

Allowing a shape factor of 0·77, $p = 0{\cdot}0034V^2 \times 0{\cdot}77 = 0{\cdot}0026V_H^2$.

Substitution of various values of H gives

H	200	150	100	50 ft.
V_H	107	101	94	82 m.p.h.
p	30	27	24	18 lb. per sq. ft.

Example of Wind Effects on a Water Tower.—To find the overturning moment due to wind on a water tower comprising a tank of 20 ft. diameter and 12 ft. high supported on an enclosed tower 14 ft. square, the distance from the ground to the underside of the tank being 50 ft. The foundation is 3 ft. below ground. Assume a wind 60 m.p.h. at 40 ft.; therefore from *Table* 8 the average pressure on the tower ($h = 25$ ft. average) is about 13 lb. per sq. ft., and 15 lb. per sq. ft. on the tank ($h = 56$ ft. average).

Total wind pressure on tank = $20 \times 12 \times 15$ lb. $\times 0{\cdot}66 = 2400$ lb.

The factor 0·66 is the reduction due to circular shape (*Table* 7); ratio of H to B is about 4.

Centre of pressure = $3 + 50 + (0{\cdot}5 \times 12) = 59$ ft. above foundation.

Total wind pressure on tower (wind normal to face) = $14 \times 50 \times 13 \times 1{\cdot}1 = 10{,}000$ lb.

Ditto (wind normal to diagonal) = $14\sqrt{2} \times 50 \times 13 \times 0{\cdot}88 = 11{,}300$ lb.

The factor 0·88 is the reduction due to wind normal to a diagonal (*Table* 7).

Centre of pressure = $3 + (0{\cdot}5 \times 50) = 28$ ft. above foundation.

Overturning moment about axis parallel to a face of the tower

= $(2400 \times 59) + (10{,}000 \times 28) = 421{,}000$ ft.-lb.

Overturning moment about a diagonal = $(2400 \times 59) + (11{,}300 \times 28)$

= 458,000 ft.-lb.

BASIC DATA

RELATION OF VELOCITY TO HEIGHT.

MET. OFFICE FORMULA (MODIFIED).

$$V_H = 0{\cdot}2337\,V_1\left[1 + 2{\cdot}81 \log_e 0{\cdot}3(H+16)\right]$$

V_H = VELOCITY (M.P.H.) OF WIND AT HEIGHT H FT.

V_1 = VELOCITY (M.P.H.) AT HEIGHT OF 33 FT. (10 METRE)

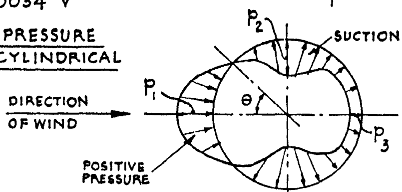
V = VELOCITY (M.P.H.) GENERAL

p = GROSS PRESSURE (LB./SQ.FT.)

RELATION OF GROSS PRESSURE TO VELOCITY.

$$p = 0{\cdot}0034\,V^2$$

VARIATION OF PRESSURE ON VERTICAL CYLINDRICAL SURFACE



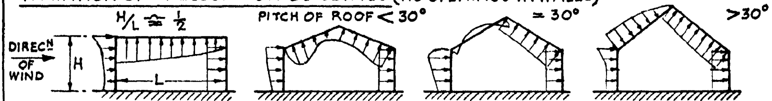
$p_1 = +1{\cdot}0p$

$p_2 = -2{\cdot}2p$

$p_3 = -0{\cdot}4p$

$\theta = 33°$

VARIATION OF PRESSURE ON BUILDINGS (NO OPENINGS IN WALLS)

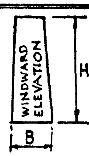


VELOCITY AND PRESSURE

No ON BEAUFORT SCALE	DESCRIPTION OF WIND	GENERAL EFFECTS	VELOCITY V MILES PER HOUR	PRESSURE $p = 0{\cdot}0034V^2$ LB. PER SQ.FT.
0	CALM	SMOKE RISES VERTICALLY.	0	LESS THAN 5 LB. PER SQ. FT.
1	LIGHT AIR	DIRECTION SHOWN BY SMOKE; WIND VANES NOT AFFECTED.	2	
2	LIGHT BREEZE	BREEZE FELT ON FACE; LEAVES RUSTLE; WIND VANES MOVE.	5	
3	GENTLE BREEZE	LEAVES AND SMALL TWIGS IN MOTION; LIGHT FLAGS EXTENDED.	10	
4	MODERATE BREEZE	SMALL BRANCHES MOVE; DUST AND PAPER RISE.	15	
5	FRESH BREEZE	SMALL TREES IN LEAF SWAY; CRESTED WAVELETS ON INLAND WATER.	21	
6	STRONG BREEZE	LARGE BRANCHES IN MOTION; TELEGRAPH WIRES WHISTLE.	27	
7	MODERATE GALE	WHOLE TREES IN MOTION; WALKING INCONVENIENT.	35	
8	FRESH GALE	TWIGS BREAK OFF; WALKING IMPEDED.	42	6
9	STRONG GALE	SLIGHT STRUCTURAL DAMAGE.	50	8¼
10	WHOLE GALE	TREES UPROOTED; STRUCTURAL DAMAGE.	59	12
11	STORM	WIDE-SPREAD DAMAGE.	68	16
12	HURRICANE	–	>75	>19
–	VIOLENT HURRICANE	–	100	34

REDUCTION FACTORS DUE TO SHAPE

RATIO OF HEIGHT TO BASE $\frac{H}{B}$		NOT GREATER THAN 4	FROM 4 TO 8	NOT LESS THAN 8
SHAPE OF STRUCTURE ON PLAN	CIRCLE	0·6(0·66	0·65(0·72)	0·7(0·77)
	OCTAGON	0·8(0·88)	0·9(·99)	1·0(1·1)
	SQUARE–WIND NORMAL TO DIAGONAL	0·8(0·88)	0·9(·99)	1·0(1·1)
	" " " " FACE	1·0(1·10)	1·15(1·77)	1·3(1·43)



OPEN FRAMES (UNCLAD STRUCTURES).	REDUCTION FACTORS	
MEMBERS OF NON-CIRCULAR CROSS-SECTION	1·0	B. S. CODE No.3 CHAP V
DITTO CIRCULAR DITTO	0·7	
SINGLE ISOLATED MEMBERS OF CIRCULAR CROSS-SECTION	0·6	
CHIMNEYS AND SHEETED TOWERS (CLAD STRUCTURES)	FACTORS IN BRACKETS	

WIND ON BUILDINGS.

Example.—A building having the cross-sectional dimensions shown in the accompanying diagram is on a site graded as exposure B. In the centre there is a tower 18 ft. wide in a longitudinal direction. The slope of the pitched roof is 3 : 1. Determine the wind forces for which the various parts of the building should be designed.

Stability and Foundations.

(i) The effective height of main part of building from ground to midpoint between eaves and ridge is $67{\cdot}5 + (0{\cdot}5 \times 5) = 70$ ft. The basic wind pressure (p) for this height (by interpolation) is 11·5 lb. per sq. ft., acting as a positive pressure of $0{\cdot}5 \times 11{\cdot}5 = 5{\cdot}75$ lb. per sq. ft. on the windward face and a negative pressure (suction) of 5·75 lb. per sq. ft. on the leeward face. The total horizontal force on windward (and on leeward) vertical face is

$$100 \times 67{\cdot}5 \times 5{\cdot}75 = 38{,}813 \text{ lb.}$$

(ii) The pressure normal to windward slope of roof (slope is about 20 deg.) is

$$0{\cdot}4 \times 11{\cdot}5 = -\,4{\cdot}6 \text{ lb. per sq. ft.}$$

The total pressure normal to windward slope is $(100 - 18)15{\cdot}8 \times (-\,4{\cdot}6) = -\,5970$ lb.

Horizontal component of ditto $= \dfrac{1}{\sqrt{10}}(-\,5970) = -\,1890$ lb.

(iii) The pressure normal to the leeward slope is $-\,0{\cdot}45 \times 11{\cdot}5 = -\,5{\cdot}2$ lb. per sq. ft. Horizontal component of pressure normal to leeward slope (by proportion to windward slope) $= -\,1890\left(\dfrac{5{\cdot}2}{4{\cdot}6}\right) = -\,2130$ lb.

(iv) The height of projection (tower) above ground is $67{\cdot}5 + 22{\cdot}5 = 90$ ft.

Basic wind pressure for 90 ft. is 12·5 lb. per sq. ft., that is 6·25 lb. per sq. ft. pressure on the windward face and 6·25 lb. per sq. ft. suction on the leeward face. The total horizontal force on windward (and on leeward) vertical face of tower is

$$6{\cdot}25 \times 22{\cdot}5 \times 18 = 2525 \text{ lb.}$$

The pressure on the flat roof of the tower has no horizontal component.

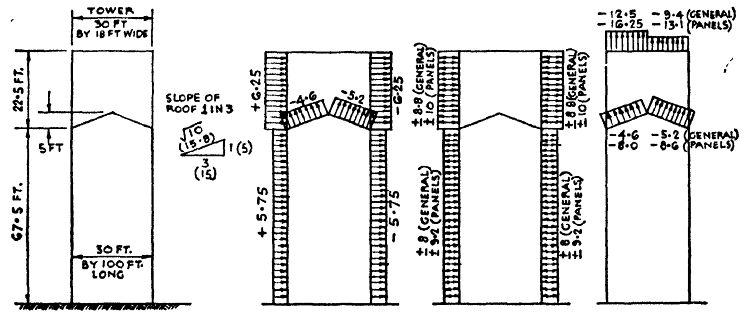
Design of Walls.

Assuming ordinary proportion of openings, the walls of the main building as a unit should be designed for a pressure of $\pm\,0{\cdot}7p = \pm\,0{\cdot}7 \times 11{\cdot}5 = \pm\,8$ lb. per sq. ft.

Individual wall panels should be designed for $\pm\,0{\cdot}8 \times 11{\cdot}5 = \pm\,9{\cdot}2$ lb. per sq. ft.

Walls of tower should be designed for $\pm\,0{\cdot}7 \times 12{\cdot}5 = \pm\,8{\cdot}8$ lb. per sq. ft.

Panels of walls of tower should be designed for $\pm\,0{\cdot}8 \times 12{\cdot}5 = \pm\,10$ lb. per sq. ft.



DIMENSIONS STABILITY WALLS . ROOFS

PRESSURES IN LB. PER SQUARE FOOT

Design of Roofs.

The windward slope of the pitched roof (about 20 deg.) should be designed for a general suction of $0{\cdot}4 \times 11{\cdot}5 = 4{\cdot}6$ lb. per sq. ft., and the leeward slope for a general suction of $0{\cdot}45 \times 11{\cdot}5 = 5{\cdot}2$ lb. per sq. ft.

Each panel of the pitched roof should be designed for the following suctions: Windward slope, $0{\cdot}7 \times 11{\cdot}5 = 8$ lb. per sq. ft. Leeward slope, $0{\cdot}75 \times 11{\cdot}5 = 8{\cdot}6$ lb. per sq. ft.

The windward half of the flat roof of the tower should be designed for a general upward pressure (suction) of $1{\cdot}0 \times 12{\cdot}5 = 12{\cdot}5$ lb. per sq. ft., and the leeward half for a general upward pressure of $0{\cdot}75 \times 12{\cdot}5 = 9{\cdot}4$ lb. per sq. ft.

Individual panels of the flat roof should be designed for the following upward pressures (suctions): Windward half, $1{\cdot}3 \times 12{\cdot}5 = 16{\cdot}25$ lb. per sq. ft.

Leeward half, $1{\cdot}05 \times 12{\cdot}5 = 13{\cdot}1$ lb. per sq. ft.

BUILDINGS

Basic pressures on vertical face (p lb. per sq. ft.)

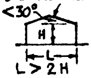
Height (ft.)	A	B*	C	D
≯10	4	6	8	10
20	5	7	9	12
30	5	8	11	14
40	6	9	12	16
50	7	10	14	18
60	8	11	15	20
80	9	12	17	22
100	9	13	18	24
120	10	14	19	25
140	11	15	21	27
160	11	16	22	28
180	12	17	23	30
>200	12	17	24	31

Roofs

Angle of windward slope (degs.)	General pressures: Windward slope	General pressures: Leeward slope	Local pressures: Windward slope	Local pressures: Leeward slope
0	−1·0p	−0·75p	−1·3p	−1·05p
10	−0·7p	−0·5p	−1·1p	−0·8p
20	−0·4p	−0·45p	−0·7p	−0·75p
30	−0·1p	−0·45p	−0·4p	−0·75p
40	+0·1p	−0·45p	+0·4p	−0·75p
50	+0·3p	−0·45p	+0·6p	−0·75p
60	+0·4p	−0·45p	+0·7p	−0·75p
70	+0·5p	−0·45p	+0·8p	−0·75p
80	+0·5p	−0·45p	+0·8p	−0·75p
90	+0·5p	−0·5p	+0·8p	−0·8p

Walls

General (stability)	±0·5p
Design of walls as unit:— Ordinary openings	±0·7p
Large openings	±1·0p
Special case <30° (L > 2H)	±0·8p
Design of wall panels	±0·8p



p = basic pressure (lb. per square foot) on vertical face
Data comply with B. S. Code No. 3, Chap. V
* Denotes requirements of London By-laws.

BRIDGES

Horizontal pressure = 30 lb. per square foot of exposed area (per B. S. No. 153)

Exposed areas.

For unloaded bridge $= A\left[1+\frac{\ell}{16d}\right]$

Loaded bridge $= A + LH$

ℓ = distance between windward and leeward beams.
d = depth or height of beam or parapet.
L = length of bridge

A = net horizontally-projected area of windward side of bridge
H = additional height of "area" subjected to wind due to live load. For calculating H the top of the vehicles, etc., forming the live load is considered to be at a uniform level thus:—
Road bridge: 8 ft above road
Railway bridge: 12 ft. above rail
Footpath: 5 ft. above footpath

Vertical pressure (for stability calculations) = 5 lb. per sq ft. of plan area (acting upwards)

CHIMNEYS TOWERS ETC.

Horizontal pressure (lb./sq.ft.) on any part of structure at height h ft. above ground. Adjustment to be made for shape. (Per B. S. Code No. 3, Chap V, 1958 revision)

Velocity of wind (miles/hr) at height of 40 ft. (mean velocity during 1 min.) \ Height h =	20	40	60	80	100	150	200	300	400	500 or greater
50	8	10	11	12	16			SEE	NOTE	BELOW
55	8	12	13	15	16					
60	12	14	16	17	18	20	22			
65	14	17	19	21	22	24	26	28	36	38
70	16	20	22	24	25	27	30	33	36	38
75	18	23	25	27	29	32	34	38	41	43
80	21	26	28	31	33	36	39	43	47	49

Structures more than 100 ft. high to be designed for pressures due to velocities not less than those below the thick line.

POLES (B.S.)

Transmission-line poles	8 lb. per sq. ft.	On projected area of ice-covered conductors
	15 " " " "	On flat windward surface of solid pole
Lamp posts: heights 20 ft.	12 lb. per sq. ft.	Subject to reduction for shape. Are to include lantern, etc.
30 "	14 " " " "	
40 "	15 " " " "	

TRANSMISSION-LINE POLES.

If the spans of the conductors on either side of an intermediate pole are unequal the pole is subject to an unbalanced pull in the direction of the line, as is also a pole at the end of a line, and a strut or guy is generally provided at a pole at the end of a line. In *Table* 9 are given formulæ for the forces in the direction of the line on an intermediate pole and on an end pole, with expressions for the forces in a strut or guy in the same plane as the line, and the resultant vertical thrust on the pole. Normally the vertical load on a pole is the sum of the weights of the conductors, of the ice on the conductors, of the brackets, insulators and other fixings, and of the pole itself. The net weight, diameter, and strength of typical conductors are given in *Table* 9, with an expression for the weight of the conductor and the ice thereon.

A pole at a point where a change in the direction of a transmission line is made is subject to an additional transverse force which is usually resisted by a guy or strut. When the change in direction is small a guy or strut is not always provided, in which case the additional transverse force (see *Table* 9) has to be taken into account.

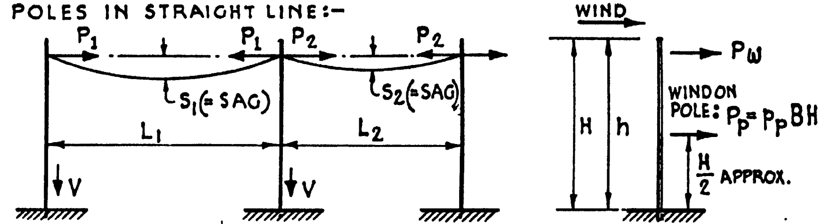
If a conductor breaks, an intermediate pole in a line of equal or unequal spans is subject to an unbalanced pull in the direction of the line. If the pole carries several conductors and only one breaks the unbalanced force may be comparatively small, but in a line of only one conductor the unbalanced force on the poles, especially those at either end of the broken span, may have serious effects if the pole has not been designed for this condition. The maximum unbalanced load to which a pole can be subjected when all the conductors in one span break is the pull corresponding to the breaking load of the conductors in the adjacent unbroken span. It is, however, unlikely that all the conductors in one span will break simultaneously, and this extreme case is not usually worthy of consideration except for a pole carrying a single conductor. For small ratios of sag to span the maximum horizontal force that can act on a stiff pole in the direction of the line is about equal to the maximum tension in the conductor, which in the limit equals the tensile strength, some values of which are given in *Table* 9. If a pole is designed for this extreme load the factor of safety can be less than that for normal working conditions. If a pole carries two or more conductors, the breaking of one causes, in addition to an unbalanced force in the direction of the line, a twisting moment on the pole. The effect of the breaking of one or more conductors can be analysed mathematically having regard to the elastic deformation of the poles, but usually sufficient security against serious overstressing, due to this cause, is obtained by compliance with the regulation of British authorities that the strength of the pole in the direction of the line shall be at least one-quarter of that in the direction normal to the line.

Transmission-line poles are classified in the British Standard according to the minimum ultimate transverse loads they are designed to resist. The working load is the ultimate load divided by a suitable factor of safety. Values of the ultimate loads are given in *Table* 9.

Light transmission-line poles to carry, say, four low-tension conductors in a vertical plane and spanning 150 ft., and telegraph poles, are usually satisfactory if designed for a working load of 500 lb. applied at the top of the pole.

TYPICAL CONDUCTORS	COPPER						STEEL-CORED ALUMINIUM (S = STEEL; A = ALUMINIUM)						ALUMINIUM					
No. OF STRANDS.	3	3	7	7	7	19	7	7	6A 7S	37	37	37	3	7	7	19	19	19
DIAMETER OF WIRE.	·104	·108	·162	·180	·204	·144	·094	·161	·208A ·069S	·110	·125	·146	·132	·149	·193	·139	·157	·183
OVERALL DIAMETER OF CONDUCTOR (d_c IN.)	0·22	0·39	0·46	0·54	0·61	0·72	0·28	0·48	0·62	0·77	0·88	1·0	0·29	0·45	0·58	0·70	0·79	0·92
STRENGTH (T_U LB.)	1,550	4,250	7,240	9,920	12,510	17,380	2,115	5,690	9,120	17,350	22,100	29,270	975	2,900	4,570	6,630	8,310	10,920
WEIGHT PER FT. (W_c LB.)	0·1	0·3	0·5	0·7	0·9	1·2	0·07	0·21	0·33	0·56	0·73	0·98	0·05	0·15	0·24	0·34	0·44	0·60

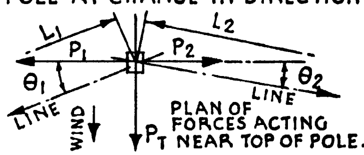
POLES IN STRAIGHT LINE:—



POLE AT END OF LINE. INTERMEDIATE POLE. VIEW IN DIRECTION OF LINE.

POLE AT END OF LINE	INTERMEDIATE POLE	
$P_W = \frac{N p_c L_1 (d_c + d_i)}{24}$	$P_W = \frac{N p_c (L_1 + L_2)(d_c + d_i)}{24}$	TRANSVERSE FORCE DUE TO WIND ON CONDUCTORS.
$P_1 = \frac{N w_t L_1^2}{8 s_1}$	$P_1 = \frac{N w_t L_1^2}{8 s_1} \quad P_2 = \frac{N w_t L_2^2}{8 s_2}$	HORIZONTAL FORCES IN DIRECTION OF LINE.
$V = \frac{N w_t L_1}{2} + W_D$	$V = N w_t \frac{L_1 + L_2}{2} + W_D$	TOTAL VERTICAL LOAD.

POLE AT CHANGE IN DIRECTION OF LINE:—



$$P_1 = \frac{N w_t L_1^2}{8 s_1} \cos \theta_1 ; \qquad P_2 = \frac{N w_t L_2^2}{8 s_2} \cos \theta_2$$

$$P_T = \frac{N w_t}{8}\left(\frac{L_1^2}{s_1} \sin \theta_1 + \frac{L_2^2}{s_2} \sin \theta_2\right) + \frac{N p_c (d_c + d_i)}{24}(L_1 \cos \theta_1 + L_2 \cos \theta_2)$$

NOTATION: ALL UNITS ARE FEET AND LB. UNLESS OTHERWISE STATED.
- N = NUMBER OF CONDUCTORS. B = BREADTH OR DIAMETER OF POLE
- d_c = NORMAL DIAMETER OF CONDUCTOR (IN.). W_D = WT. OF POLE, ATTACHMENTS, ETC.
- d_i = INCREASE IN DIAMETER OF CONDUCTORS DUE TO ADHERING ICE.
- w_t = WEIGHT OF CONDUCTOR (w_c) AND ICE (LB. PER FT.) = $w_c + \frac{d_i}{3}(2 d_c + d_i)$.
- p_c = WIND PRESSURE ON CONDUCTORS; p_P = WIND PRESSURE ON POLE.

IF SAG IS LARGE COMPARED WITH L: REPLACE L_1 BY $\left(L_1 + \frac{8 s_1^2}{3 L_1}\right)$ AND REPLACE L_2 BY $\left(L_2 + \frac{8 s_2^2}{3 L_2}\right)$

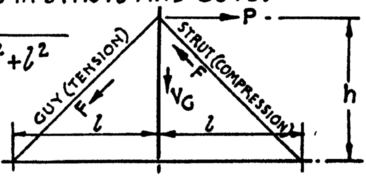
MIN^M. ULTIMATE TRANSVERSE LOADS (B.S.) APPLIED AT $h = H - 2$ FT.

CLASS	A	B	C	D	E	F
MAX. O/L LENGTH	34	38	42	50	50	50 FT.
LOAD (W_U LB.)	875	1,250	1,750	2,500	3,000	3,500
O/L LENGTH	28	34	38	42	46	50 FT.
NET h	21	26	29	33	36	40 FT.

FORCES IN STRUTS AND GUYS.

$$F = \frac{P}{l}\sqrt{h^2 + l^2}$$

$$V_G = \frac{h}{l} P$$



For wind pressure on transmission-line poles, see *Table* 8.

ACTIVE PRESSURES DUE TO RETAINED MATERIALS.

Granular Materials.

Examples of Pressure of Dry Materials.—(*a*) Find the total horizontal active pressure on the back of a vertical wall 15 ft. high retaining ordinary earth with a level surface subject to an imposed load of 2 cwt. per sq. ft. (= W).

Assume for earth: $w = 100$ lb. per cu. ft.; $\theta = 35$ deg. From *Table* 11, $k_2 = 0{\cdot}271$.

Pressure at base of wall due to retained earth and surcharge (*Table* 13):

$$p_2 = k_2(wH + W) = 0{\cdot}271[(100 \times 15) + 224] = 467 \text{ lb. per sq. ft.}$$

Pressure at top of wall due to surcharge: $p_2 = 0{\cdot}271 \times 224 = 60{\cdot}6$ lb. per sq. ft.

Total pressure $= \frac{1}{2}H(467 + 60{\cdot}6) = 0{\cdot}5 \times 15 \times 528 = 3960$ lb.

(*b*) A vertical wall 20 ft. high retains a heap of dry coal, the top surface of which is sloped downwards from the wall at the natural slope. Find the intensity of active horizontal pressure at the base of wall, neglecting friction on the back of wall.

From *Table* 11, $w = 58$ lb. per cu. ft. and $\theta = 40$ deg.;

$k_3 = 0{\cdot}16$;

$p_3 = k_3wh = 0{\cdot}16 \times 58 \times 20 = 186$ lb. per sq. ft.

Effect of Ground-water.—If ground-water occurs at a depth h_w below the top of the wall, the intensity of horizontal pressure is k_2wh when h does not exceed h_w; when h is greater than h_w the pressure is given by

$$p_a = k_2wh_w + (k_2w_B + w_w)(h - h_w),$$

where w_B is the buoyant density of the soil (about 60 per cent. of the drained density w), and w_w is the density of water (62·4 lb. per cu. ft.).

For a dry granular material with level fill, the passive resistance is $\left(\frac{1 + \sin\theta}{1 - \sin\theta}\right)wh$, that is $\frac{wh}{k_2}$ as given in *Table* 10; values of k_2 are given in *Table* 11. This expression also applies to drained soil above ground-water level; for saturated soil below this level the passive resistance is given by

$$p_p = \frac{wh_w}{k_2} + \left(\frac{w_B}{k_2} + w_w\right)\left(h - h_w\right).$$

The symbols have the same significance as in *Table* 11; h_w is the depth to ground-water.

Active Pressure Normal to Inclined Surfaces.—The intensity of pressure normal to the slope of an inclined surface, such as a hopper bottom loaded with coal, grain, sand, stone, or other granular material, at a depth h below the level surface of the filling is

$$wh(k_2 \sin^2\theta_1 + \cos^2\theta_1),$$

where θ_1 is the angle between the horizontal and the sloping surface. Values of k_2 and of $k_2 \sin^2\theta_1 + \cos^2\theta_1$ for various angles θ from 30 deg. to 45 deg. are given in *Table* 11.

Materials Immersed or Floating in Liquids.

Materials Heavier than the Liquid.—With granular material, of which the specific gravity exceeds that of the liquid in which it is just fully immersed, the intensity of horizontal active pressure on the vertical wall of the container is

$$p_a = w_Lh\left[1 + k\left(\frac{w_m}{w_L} - 1\right)(1 - V)\right],$$

where w_L is the weight per cu. ft. of the liquid (lb.), w_m is the weight per cu. ft. of the solid material (lb.), V is the ratio of the volume of the voids in a given volume of the dry material, k is the horizontal pressure factor depending on the slope of the surface of the material and the angle of repose of the dry material (that is, factors k_1, k_2, or k_3 in *Table* 11), and h is the depth (ft.) from the top of the submerged material to the level at which the pressure is being calculated. If the surface of the liquid is at a distance h_0 ft. above the top of the submerged material there is an additional pressure of w_Lh_0. The total intensity of pressure at any depth $h_0 + h$ below the surface of the liquid is $w_L(h_0 + hF)$ where

$$F = \left[1 + k\left(\frac{w_m}{w_L} - 1\right)(1 - V)\right].$$

If the material is immersed in water, w_L is 62·4 lb. per cu. ft. and the pressure is $62{\cdot}4(h_0 + hF)$ lb. per sq. ft.

(*Continued on page 162.*)

HORIZONTAL PRESSURES DUE TO RETAINED MATERIALS.—TABLE 10.

DRY GRANULAR MATERIALS

ACTIVE PRESSURES

INTENSITY OF PRESSURE NORMAL TO BACK OF WALL AT DEPTH h FT. $p = kwh\sin\beta$

TOTAL PRESSURE NORMAL TO BACK OF WALL OF HEIGHT H FT. $P = \tfrac{1}{2}kwH^2\sin\beta$

VALUES OF k FOR VARIOUS CONDITIONS ARE GIVEN BELOW.

TOTAL PRESSURE ON BACK OF WALL $P_1 = P\sec\mu$

ANGLE BETWEEN LINE OF ACTION OF P_1 AND BACK OF WALL $= (90 - \mu)$

FORCE PARALLEL TO BACK OF WALL $F = P\tan\mu = P_1\sin\mu$

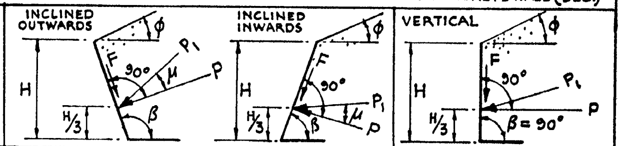
WHEN FRICTION ON BACK OF WALL IS NEGLECTED ($\mu = 0$): $P_1 = P$; $F = 0$; ANGLE $= 90°$

NOTATION. w = WEIGHT OF RETAINED MATERIAL (LB. PER CUBIC FOOT).

ϕ = ANGLE OF SLOPE OF BANK OF RETAINED MATERIAL (DEG.)

θ = ANGLE OF INTERNAL FRICTION OF MATERIAL RETAINED (DEG.)

μ = ANGLE OF FRICTION BETWEEN MATERIAL AND CONCRETE WALL (DEG.)



FRICTION ON BACK OF WALL	SLOPE OF BANK OF RETAINED MATERIAL	INCLINED OUTWARDS / INCLINED INWARDS	VERTICAL
$\mu = 0$	ANY SLOPE ϕ	$K = \left[\frac{\sin(\beta-\theta)}{(n+1)\sin\beta}\right]^2 \frac{1}{\sin^2\beta}$ $\quad n = \sqrt{\frac{\sin\theta\sin(\theta-\phi)}{\sin\beta\sin(\beta-\phi)}}$	$K = \left(\frac{\cos\theta}{n+1}\right)^2$ $n = \sqrt{\sin^2\theta - \tfrac{1}{2}\tan\phi\sin 2\theta}$
	$\phi = \theta$ (MAX)	$K_1 = \left[\frac{\sin(\beta-\theta)}{\sin\beta}\right]^2 \frac{1}{\sin^2\beta}$	$K_1 = \cos^2\theta$
	$\phi = 0$ (LEVEL)	$K_2 = \left[\frac{\sin(\beta-\theta)}{\sin\theta+\sin\beta}\right]^2 \frac{1}{\sin^2\beta}$	$K_2 = \frac{1-\sin\theta}{1+\sin\theta}$
	$\phi = -\theta$ (MIN)	$K_3 = \left[\frac{\sin(\beta-\theta)}{(n+1)\sin\beta}\right]^2 \frac{1}{\sin^2\beta}$ $\quad n = \sqrt{\frac{2\sin^2\theta\cos\theta}{\sin\beta\sin(\beta+\theta)}}$	$K_3 = \left[\frac{\cos\theta}{1+\sqrt{2}\sin\theta}\right]^2$
μ TAKEN INTO ACCOUNT	ALL CASES	$K = \left[\frac{\sin(\beta-\theta)}{(n+1)\sin\beta}\right]^2 \frac{\cos\mu}{\sin(\mu+\beta)\sin\beta}$	$K = \left(\frac{\cos\theta}{n+1}\right)^2$
	ANY SLOPE ϕ	$n = \sqrt{\frac{\sin(\theta+\mu)\sin(\theta-\phi)}{\sin(\mu+\beta)\sin(\beta-\phi)}}$	$n = \sqrt{\frac{\sin(\theta+\mu)\sin(\theta-\phi)}{\cos\mu\cos\phi}}$
	$\phi = \theta$ (MAX.)	$K_1 = K$ $\quad n = 0$	$K_1 = \cos^2\theta$ $\quad (n = 0)$
	$\phi = 0$ (LEVEL)	$K_2 = K$ $\quad n = \sqrt{\frac{\sin(\theta+\mu)\sin\theta}{\sin(\mu+\beta)\sin\beta}}$	$K_2 = K$ $n = \sqrt{\sin\theta(\sin\theta+\cos\theta\tan\mu)}$
	$\phi = -\theta$ (MIN.)	$K_3 = K$ $\quad n = \sqrt{\frac{\sin(\theta+\mu)\sin 2\theta}{\sin(\mu+\beta)\sin(\beta+\theta)}}$	$K_3 = \left[\frac{\cos\theta}{n\sqrt{2}+1}\right]^2$ $n = \sqrt{\sin\theta(\sin\theta+\cos\theta\tan\mu)}$

PASSIVE RESISTANCE

INTENSITY OF HORIZONTAL PASSIVE RESISTANCE AGAINST VERTICAL WALL AT DEPTH h FT. ($\phi = 0$)

$$p_p = \frac{wh}{k_2} = \left(\frac{1+\sin\theta}{1-\sin\theta}\right) wh$$

COHESIVE SOILS

ACTIVE PRESSURE

INTENSITY OF HORIZONTAL PRESSURE AT DEPTH OF h FT. ON VERTICAL WALL. ($\phi = 0$)

$$p_2 = \left(\frac{1-\sin\theta}{1+\sin\theta}\right) wh - 2c\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = k_2 wh - 2c\sqrt{k_2}$$

PASSIVE RESISTANCE

INTENSITY OF HORIZONTAL PASSIVE RESISTANCE AGAINST VERTICAL WALL AT DEPTH h FT. ($\phi = 0$)

RETAINING ORDINARY SATURATED CLAY: $p_p = \frac{wh}{k_2} + \frac{2c}{k_2}$

c = COHESION FACTOR = SHEARING STRENGTH OF UNLOADED SOIL.

OTHER NOTATION AS FOR GRANULAR MATERIALS.

TABLE 11.—PRESSURES DUE TO GRANULAR MATERIALS.

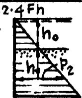
PROPERTIES OF GRANULAR MATERIALS

Retained cohesionless soils etc.	Angle of internal friction θ	Weight (lb. per cu. ft.) in bulk w	Weight (lb. per cu. ft.) moist (drained) w_c
Gravel: common	35° to 45°	110 to 140	-
Shingle loose	40°	115	-
Sandy compact	40° to 45°	130	-
Sandy loose	35° to 40°	120	-
Sand: fine dry	30° to 35°	100	-
Sand: fine wet	0° to 30°	115 to 120	-
Well graded compact	40° to 45°	110 to 120	120 to 140
Well graded loose	35° to 40°	100 to 110	110 to 120
Uniform fine or silty: compact	35° to 40°	100 to 110	110 to 135
Uniform fine or silty: loose	30° to 35°	90 to 100	100 to 110
Coarse or medium: compact	35° to 40°	100 to 110	120 to 130
Coarse or medium: loose	30° to 35°	90 to 100	105 to 120
Crushed rock: granite	35° to 45°	100 to 130	-
Basalt, dolerites		110 to 140	-
Limestone, sandstone		80 to 120	-
Broken chalk		60 to 80	-

Contained granular materials	Pressure calculations θ	Pressure calculations w lb.	Capacity calculations angle of repose	Capacity calculations w lb.
Coal: dry unwashed	40°	58	45°	45
Coal: dry washed	40°	56	45°	45
Wet (15% moisture)	25°	56	45°	45
Fine	20°	56	40°	45
Slurry	0° to 20°	62½	40°	50
Anthracite	27°	52	40°	45
Coke, breeze, etc.	40°	35	45°	30
Shale: broken	30°	130	35°	100
Colliery dirt	35°	70	45°	60
Grain: wheat	25°	48	-	45
Barley	25°	25	-	25
Cement: static fine	10°	90	-	84
Coarse	18°	90	-	90
Air-agitated	0°	75	-	75
Ashes	35°	60	45°	40
Broken brick	35°	100	45°	70

Horizontal pressure of granular materials submerged in water	θ approx. when submerged	Weight in solid (lb. per cu. ft.) w_M	Values of 62·4 F (lb. per sq. ft./ft.), percentage of voids (= 100V): 25	30	35	40	45	50%
Coal (crushed)	35°	80	66	66	65½	65	65	64½
Stone (crushed)	35°	160	82	81	79½	78½	77	76
Sand	0°	160	135	130	125	122	116	111

$p_2 = 62{\cdot}4h_o + 62{\cdot}4Fh$ lb. per sq. ft.



k_2 – see below

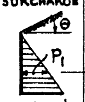
$F = 1 + k_2\left(\frac{w_M}{62{\cdot}4} - 1\right)(1 - V)$

ACTIVE PRESSURES (FRICTION BETWEEN MATERIAL AND FACE OF WALL OR SURFACE NEGLECTED)

VERTICAL WALLS

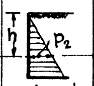
Angle of internal friction of contained or retained material (approximate angle of repose)

Maximum positive surcharge



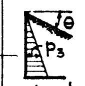
$p_1 = k_1wh$

Level fill



$p_2 = k_2wh$

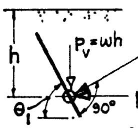
Minimum negative surcharge



$p_3 = k_3wh$

Angle θ	Equivalent slope	$k_1 = \cos^2\theta$	$k_2 = \frac{1-\sin\theta}{1+\sin\theta}$	$k_3 = \left[\frac{\cos\theta}{1+\sin\theta\sqrt{2}}\right]^2$
5°	1 in 11·4	0·99	0·84	0·79
10°	1 in 5·7	0·97	0·70	0·63
11° 20′	1 in 5	0·96	0·68	0·59
14°	1 in 4	0·94	0·61	0·52
15°	1 in 3·7	0·93	0·59	0·50
18° 30′	1 in 3	0·90	0·52	0·43
20°	1 in 2·7	0·88	0·49	0·40
25°	1 in 2·1	0·82	0·41	0·32
26° 35′	1 in 2	0·80	0·38	0·30
30°	1 in 1·7	0·75	0·33	0·26
33° 40′	1 in 1·5	0·69	0·29	0·22
35°	1 in 1·4	0·67	0·27	0·21
40°	1 in 1·2	0·59	0·22	0·16
45°	1 in 1·1	0·50	0·17	0·13
50°	1 in 0·84	0·41	0·13	0·10
55°	1 in 0·7	0·33	0·10	0·07
60°	1 in 0·58	0·25	0·07	0·05
65°	1 in 0·47	0·18	0·05	0·04
70°	1 in 0·36	0·12	0·03	0·02
80°	1 in 0·18	0·03	0·01	0·01

INCLINED SURFACES



Intensity of pressure normal to surface (lb. per sq. ft.)

$p_n = k_4wh + w_s\cos\theta_1$

w = wt. of material (lb. per cu. ft.)

$p_h = k_2wh$ w_s = wt. of inclined slab (lb. per sq. ft.)

$k_4 = k_2\sin^2\theta_1 + \cos^2\theta_1$ Values of k_4 tabulated below

Angle θ	Inclination of surface θ_1: 30°	35°	40°	45°	50°	55°	60°	65°
30°	0·83	0·78	0·72	0·67	0·61	0·55	0·50	0·45
32½°	0·83	0·77	0·71	0·65	0·59	0·53	0·48	0·42
35°	0·82	0·76	0·70	0·64	0·57	0·51	0·45	0·40
37½°	0·81	0·75	0·69	0·62	0·55	0·49	0·43	0·38
40°	0·81	0·74	0·68	0·61	0·54	0·47	0·42	0·36
42½°	0·80	0·73	0·67	0·60	0·52	0·46	0·40	0·33
45°	0·79	0·73	0·66	0·58	0·51	0·44	0·38	0·32

NOTES ON VERTICAL WALLS

Tabulated factors apply only to intensity of active horizontal pressures of granular (cohesionless) material retained or contained by reinforced concrete walls, sheet piles, and the like (not heavy gravity walls).

Weights (lb per cu. ft.)

w = weight (lb. per cu. ft.) of retained or contained material (= w_A, w_B or w_C).

w_A = weight of dry soil if no water in ground or of contained material.

w_C = weight of moist (drained) soil above level of ground water.

w_B = buoyant weight of soil below ground water level (= 0·6 w_A approx.)

PROPERTIES OF COHESIVE SOILS

TYPE OF SOIL	SATURATED DENSITY (LB. PER CU. FT.) w	ANGLE OF INTERNAL FRICTION θ	COHESION (LB. PER SQ. FT.) C	TYPE OF SOIL	BULK DENSITY (LB. PER CU. FT.) w	ANGLE OF INTERNAL FRICTION θ	COHESION
CLAYS:-							
VERY STIFF BOULDER		16°	3580	CLAY: DRY	110	30°	DETERMINE BY TEST
HARD SHALEY	120 TO 140	–	>3000	DAMP (WELL DRAINED)	115	45°	
STIFF		7°	1500 TO 3000	WET	120	15°	
FIRM	110 TO 130	6°	750 TO 1500	GRAVELLY	125	35°	
MODERATELY FIRM	110 TO 120	5°	1120	EARTH: TOP SOIL	85	(35°)	IN ABSENCE OF TEST RESULTS, APPLY FORMULÆ FOR SATURATED CLAY WITH $\theta = 0°$
SOFT		4°	375 TO 750	COMMON	100	(35°)	
VERY SOFT	100 TO 120	3°	<375	DRY	90	30°	
PUDDLE CLAY: SOFT		3°	675	MOIST	100	45° TO 50°	
VERY SOFT		0°	450	VERY WET	105	17°	
SANDY CLAY: STIFF	100		1500 TO 3000	PUNNED	100	65° TO 75°	
FIRM			750 TO 1500	PEAT: DRY	30	15° TO 45°	
SILT	120		<750	WET	60		

ACTIVE PRESSURES ON VERTICAL WALL

TYPE OF SOIL	FORMULÆ FOR INTENSITY (LB PER SQ. FT.) OF HORIZONTAL PRESSURE AT DEPTH h FT.
CLAY (PARTIALLY SATURATED) AND SILT $\theta > 0°$	$h_o = \frac{2C}{w}\sqrt{\frac{1}{k_2}}$ ABOVE GROUND WATER LEVEL. $h \not> h_o$:- $p_2 = w_w h$ $h > h_o \not> h_w$:- $p_2 = k_2 w h - 2C\sqrt{k_2}$ BELL'S FORMULA BELOW GROUND WATER LEVEL. $h > h_o > h_w$:- $p_2 = k_2 w h_w + (k_2 w_B + w_w)(h - h_w) - 2C\sqrt{k_2}$
FISSURED CLAY	$h_o = \frac{2C_s}{w}\sqrt{1 + \frac{C_w}{C_s}}$ $h \not> h_o \not> \frac{H}{2}$ (5 FT. MIN.):- $p_2 = w_w h$ $h > h_o$:- $p_2 = wh - 2C_s\sqrt{1 + \frac{C_w}{C_s}}$
NON-FISSURED CLAY $\theta = 0°$	$h_o = \frac{2C_H}{w}\sqrt{1 + \frac{C_w}{C_H}}$ $h \not> h_o \not> \frac{H}{2}$ (5 FT. MIN.):- $p_2 = w_w h$ $h > h_o$:- $p_2 = wh - 2C_H\sqrt{1 + \frac{C_w}{C_H}}$

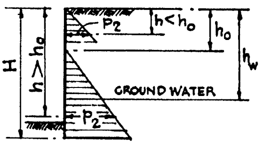
FORMULÆ APPLY TO R.C. WALLS, SHEET PILES, ETC. (NOT HEAVY GRAVITY WALLS)

PASSIVE RESISTANCE

TYPE OF SOIL	FORMULÆ
CLAY (PARTIALLY SATURATED) AND SILT $\theta > 0°$	WATER LEVEL BELOW GROUND LEVEL IN FRONT OF WALL $h_p \not> h_w$:- $p_p = \frac{wh_p}{k_2} + \frac{2C}{\sqrt{k_2}}$ $h_p > h_w$:- $p_p = \frac{wh_w}{k_2} + \left[\frac{w_B}{k_2} + w_w\right](h_p - h_w) + \frac{2C}{\sqrt{k_2}}$ WATER LEVEL ABOVE GROUND LEVEL IN FRONT OF WALL $h_p \not> h_w$:- $p_p = w_w h_p$ $h_p > h_w$:- $p_p = \frac{w_B}{k_2}(h_p - h_w) + w_w h_p + \frac{2C}{\sqrt{k_2}}$
NON-FISSURED CLAY $\theta = 0°$	$p_p = wh_p + \frac{2C_H}{\sqrt{k_2}}$

NOTATION

ACTIVE PRESSURES.



WEIGHTS (LB. PER CU. FT.)

w = WEIGHT OF SATURATED CLAY OR BULK WEIGHT OF EARTH.

w_w = WEIGHT OF WATER = 62·4 LB. PER CU. FT.

w_B = BOUYANT WEIGHT = 0·6w APPROX.

COHESION (LB. PER SQ. FT.)

C = COHESION AT NO LOAD ON CLAY

C_s = COHESION OF SOFTENED CLAY

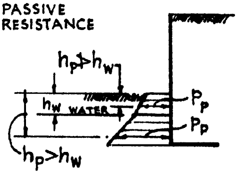
C_H = COHESION AT DEPTH h

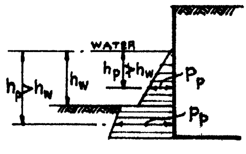
C_w = COHESION BETWEEN CLAY AND WALL = C_H BUT $\not>$ 1000 LB. PER SQ. FT.

$k_2 = \frac{1 - \sin\theta}{1 + \sin\theta}$ (SEE TABLE Nº 5)

θ = ANGLE OF INTERNAL FRICTION

PASSIVE RESISTANCE





ACTIVE PRESSURES DUE TO RETAINED MATERIALS

(*continued from page* 158).

The foregoing expressions apply to materials such as coal or broken stone, the angle of repose of which is not materially affected by submergence. It is advisable to reduce the value of the angle of repose to about 5 deg. below that for the dry material when determining the value of k for the submerged material. In the calculation of k for such material it can be assumed that the angle of repose and angle of internal friction are identical.

For a material such as sand, which has a definite angle of repose when dry but none when saturated, k is unity when submerged.

The values of Fw_L (lb. per sq. ft.) for unit h for crushed coal, broken stone, and sand, when immersed in water, and with level surface are given in *Table* 11.

Materials Lighter than the Liquid.—If the specific gravity of the material is less than that of the liquid, the intensity of horizontal pressure, at any depth h below the surface of the liquid in which the material floats, is $w_L h$. Therefore, if stored in water, the horizontal pressure on the walls of the container is equal to the simple hydrostatic pressure of $62{\cdot}4h$ lb. per sq. ft.

Cohesive Materials.

Find the intensity of horizontal active pressure at a depth of 25 ft. below the top of wall supporting moderately firm clay assumed to be partially saturated.

From *Table* 12, $\theta = 5$ deg. and $C = 1120$ lb. per sq. ft.

From *Table* 11, for $\theta = 5$ deg., $k_2 = 0{\cdot}84$; assume $w = 110$ lb. per cu. ft.

Substituting in Bell's formula, $p_2 = (0{\cdot}84 \times 110 \times 25) - [2 \times 1120 \times \sqrt{0{\cdot}84}] = 260$ lb. per sq. ft.

(This method of calculation should not be adopted unless the values of θ and C have been well proved by test, and factors that may affect these values in the future have been fully considered.)

SURCHARGE ON GRANULAR MATERIAL.

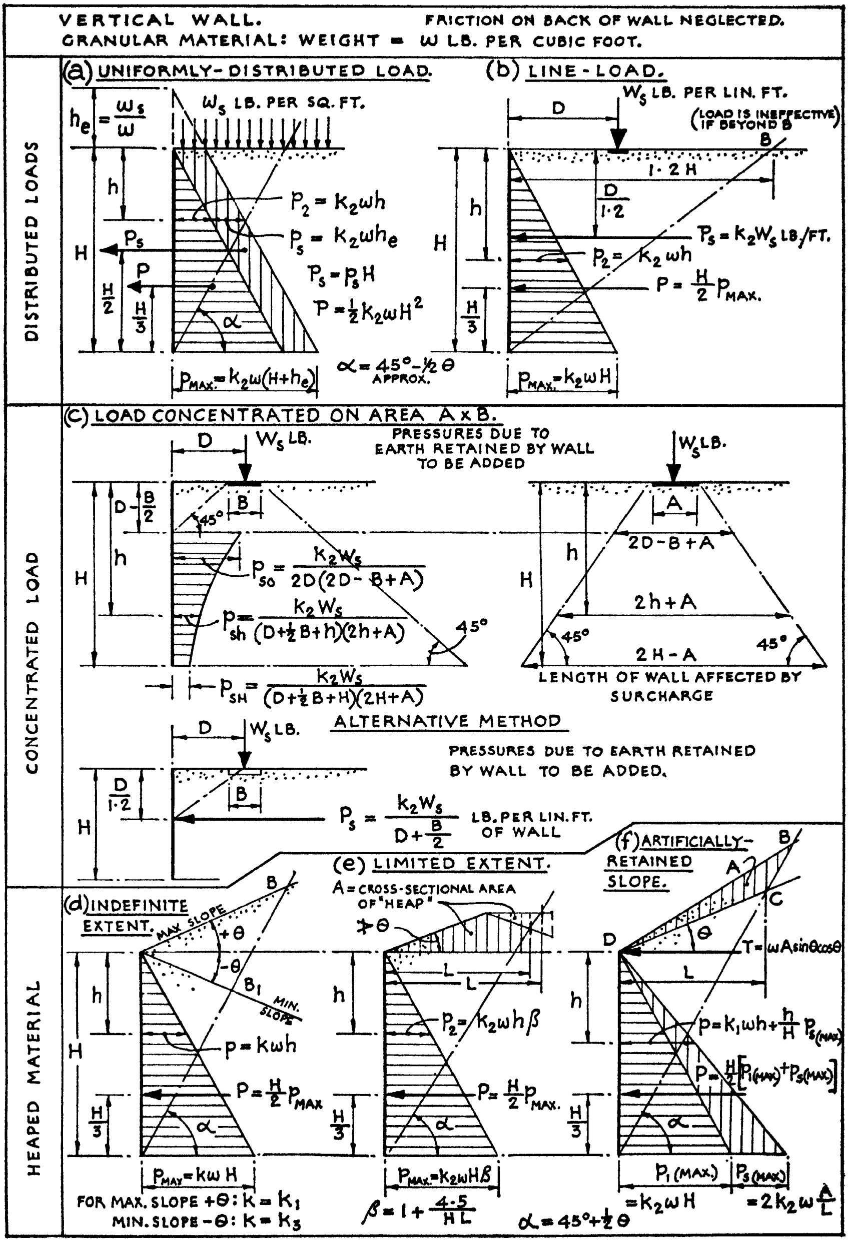
Non-level Filling.—As is seen from the factors given in *Table* 11, the slope of the surface of the filling behind the wall has a marked effect on the theoretical pressures. For practical purposes any alteration of the shape of the surface beyond the point B in the diagrams in *Table* 13, or any additional loading beyond this point, has little or no effect upon the pressure on the wall. The general formulæ in *Table* 11 allows for any slope between the limits DB and DB_1.

When the surface of the filling is not at a uniform slope, special consideration is required. In the common cases depicted in *Table* 13 the magnitude of the active horizontal pressure on the wall is between that when the surface is level and when the surface slopes upwards at the maximum angle. On the diagrams are given empirical expressions for the increase of pressure due to these intermediate conditions. The total pressure on the wall is increased above that for a level surface by an amount proportional to the mean increase in the head of the material. These surcharges give no pressure at the top of the wall, and the centre of total pressure is assumed to be one-third of the way up the wall.

Load Imposed on Filling.—When the filling behind a wall is level but liable to self-retaining loads, such as stacked materials, traffic, or buildings, the total imposed load should be converted into an equivalent head of the same material as that retained by the wall, and the pressure intensity on the back of the wall increased uniformly throughout the depth of the wall. In this case there is a positive intensity of pressure at the top of the wall; these conditions are illustrated in *Table* 11.

Slope Beyond the Natural Angle of Repose.—A type of surcharge not dealt with in the foregoing is shown in *Table* 13 for the case where the angle of the slope of the surface exceeds the natural angle of repose, as occurs by protecting a bank of earth with turf or stone pitching. For such a case it is suggested that the weight of earth W above the angle of repose, represented by the triangle BCD, should be considered as a load operating at the top of the wall. The magnitude of the resultant horizontal thrust T and increase in pressure would be as shown in *Table* 13, and each section of the wall should be designed for the extra moment due to T.

Concentrated Load.—A single concentrated load imposed on the filling behind a wall can be dealt with approximately by dispersion. With 45 deg. as the angle of dispersion, as indicated in *Table* 13, the intensity of active horizontal pressure additional to the pressure due to the filling is given by the appropriate formulæ in *Table* 13.



For values of k_1, k_2 and k_3, see *Table* 11.

Applications of Janssen's Formula.

Grain Silo.—Consider the special case of grain for which tan $\mu = 0{\cdot}444$ and $k = 0{\cdot}5$. Substitution in Janssen's formula (*Table* 14) gives an expression for the horizontal pressure p_h at any depth h in terms of the hydraulic radius R, the depth h, and the density w of the grain. From this expression values of $\dfrac{p_h}{w}$ can be derived for definite values of h and R as in the tabulation.

VALUES OF $\frac{p_h}{w}$ FOR GRANULAR MATERIALS (TAN $\mu = 0{\cdot}444$; $k = 0{\cdot}5$)

VALUES OF h FEET	VALUES OF R								
	10	7·5	5·0	4·0	3·0	2·5	2·0	1·5	1·0
10	4·5	4·0	4·0	3·9	3·5	3·3	3·1	2·5	2·2
15	6·5	5·9	5·5	5·1	4·5	4·1	3·6	3·0	2·3
20	8·1	7·5	6·7	6·0	5·2	4·7	4·0	3·2	2·4
25	9·5	8·8	7·7	6·7	5·7	5·0	4·2	3·3	-
30	10·9	9·9	8·4	7·1	6·0	5·3	4·4	3·4	-
35	12·2	10·9	8·9	7·7	6·3	5·4	4·4	-	-
40	13·2	11·8	9·4	8·1	6·5	5·5	4·5	-	-
45	14·2	12·3	9·8	8·3	6·6	5·6	-	-	-
50	15·1	13·0	10·1	8·6	6·7	5·6	-	-	-
55	16·0	13·5	10·3	8·7	6·7	-	-	-	-
60	16·5	14·0	10·6	8·8	6·7	-	-	-	-
65	17·2	14·3	10·7	8·9					
70	17·7	14·7	10·9	9·0					
75	18·2	15·0	11·0						
80	18·7	15·3	11·1						
90	19·4	15·7	11·3						
100	20·0	16·0	11·4						

INCREASE IN LATERAL PRESSURE IS NEGLIGIBLE BELOW THE DEPTHS TABULATED

EXAMPLE.—Calculate (i) the horizontal pressures at various depths in a 20-ft.-diameter, silo 60 ft. deep containing grain; also (ii) the total load carried on the wall due to the filling and (iii) the intensity of pressure on the bottom.

For grain assume $w = 48$ lb. per cu. ft., $k = 0{\cdot}5$, and tan $\mu = 0{\cdot}444$.

(i) Therefore, $p_h = \left(\dfrac{p_h}{w}\right)w = 48\left(\dfrac{p_h}{w}\right)$.

The following values of $\dfrac{p_h}{w}$ are obtained from the tabulation above for

$$R = 0{\cdot}25 \times 20 = 5{\cdot}0.$$

At 10 ft. depth: $p_h = 4{\cdot}0 \times 48 = 192$ lb. per sq. ft.
,, 20 ft. ,, $= 6{\cdot}7 \times 48 = 322$,, ,,
,, 30 ft. ,, $= 8{\cdot}4 \times 48 = 403$,, ,,
,, 40 ft. ,, $= 9{\cdot}4 \times 48 = 452$,, ,,
,, 60 ft. ,, $= 10{\cdot}6 \times 48 = 498$,, ,,

(ii) The load transferred to the wall of the silo by friction is

$$\left(wh - \frac{p_h}{k}\right)R = [(48 \times 60) - (2{\cdot}0 \times 498)]5$$

$= 9420$ lb. per ft. of circumference at the bottom of the wall. (This calculation assumes $k = 0{\cdot}5$; if $k = 0{\cdot}33$ for walls the weight per foot carried on the wall will be greater.)

(iii) The intensity of vertical pressure on the bottom of the silo is

$$\frac{p_h}{k} = \frac{498}{0{\cdot}5} = 996 \text{ lb. per sq. ft.}$$

Cement Silo.—Calculate the intensity of horizontal pressure on the wall of a cylindrical cement silo at a depth of 56 ft. from the surface, assuming $D = 32$ ft., $\theta = 18$ deg., $w = 90$ lb. per cu. ft., and $\mu = 29$ deg. (Cement not fluidised.)

From *Table* 14, tan $\mu = 0{\cdot}5543$.
From *Table* 11, for $\theta = 18$ deg. $k = k_2 = 0{\cdot}53$ approximately.
From *Table* 14, $R = 0{\cdot}25 \times 32 = 8$ ft. Thus

$$\frac{hk \tan \mu}{2{\cdot}303R} = \frac{56 \times 0{\cdot}5543 \times 0{\cdot}53}{2{\cdot}303 \times 8} = 0{\cdot}842;$$

therefore $N = 6{\cdot}95$ and $p_h = \dfrac{90 \times 8}{0{\cdot}5543}\left(1 - \dfrac{1}{6{\cdot}95}\right) = 1130$ lb. per sq. ft.

D, H = WIDTH AND TOTAL DEPTH OF CONTAINER (FT.)

w = WEIGHT OF CONTAINED MATERIAL (LB. PER CU. FT.)

p_h = INTENSITY OF HORIZONTAL PRESSURE AT DEPTH h FT. $= kp_v$ (LB. PER SQ. FT.)

p_v = " " VERTICAL " " " " $= \dfrac{p_h}{k}$ (LB PER SQ. FT.)

$k = \dfrac{p_h}{p_v}$. LIMITING VALUE FOR UNCONFINED GRANULAR MATERIALS $= K_2 = \dfrac{1-\sin\theta}{1+\sin\theta}$

FOR CONFINED MATERIALS:—
APPROXIMATE VALUE FOR GRAIN = 0·5; FOR STATIC CEMENT = 0·53.

θ = ANGLE OF INTERNAL FRICTION OF CONTAINED MATERIAL (DEG.)

μ = ANGLE OF FRICTION BETWEEN CONTAINED MATERIAL AND CONCRETE WALL (DEG.)

N = NUMBER, THE COMMON LOG. OF WHICH $= \dfrac{hk\tan\mu}{2{\cdot}303\,R}$ (IN JANSSEN'S FORMULA)

R = HYDRAULIC RADIUS OF CONTAINER (FT.) $= \dfrac{\text{PLAN AREA OF CONTAINER (SQ.FT.)}}{\text{PERIMETER OF CONTAINER (FT.)}}$

SHAPE OF CONTAINER ON PLAN.

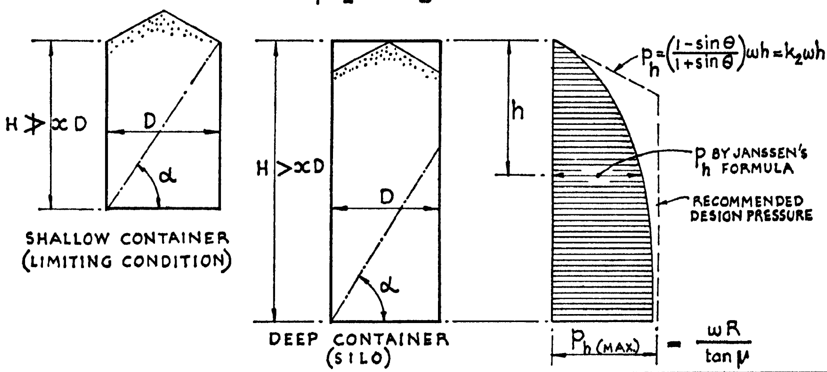
SQUARE:	$D \times D$	$R = \dfrac{D}{4}$
CIRCULAR:	DIAMETER $= D$	$R = \dfrac{D}{4}$
REGULAR POLYGON:	DISTANCE ACROSS FLATS $= D$	$R = \dfrac{D}{4}$
RECTANGLE:	$B \times D$	$R = \dfrac{BD}{2(B+D)}$

x = COEFFICIENT DEFINING "DEEP" CONTAINER $= \tan\theta + \sqrt{\tan\theta\,\dfrac{1+\tan^2\theta}{\tan\theta+\tan\mu}}$

OR $= \tan\alpha = \tan(45^\circ + \tfrac{1}{2}\theta)$.

$p_{h\,(max.)}$ = LIMITING VALUE OF HORIZONTAL PRESSURE $= \dfrac{wR}{\tan\mu}$ (PER JANSSEN'S FORMULA)

JANSSEN'S FORMULA: $p_h = \dfrac{wR}{\tan\mu}\left[1 - \dfrac{1}{N}\right]$



TYPICAL VALUES x AND $p_{h\,(MAX)}$

CONTAINED MATERIALS	WEIGHT LB./CU.FT. w	FRICTION ON WALL μ	FRICTION ON WALL $\tan\mu$	INTERNAL FRICTION θ	INTERNAL FRICTION $\tan\theta$	x $\tan\alpha$ (μ)	$p_{h\,(MAX.)}$ LB. PER SQ. FT. $R = 10$	5	3	2	1
GRAIN: WHEAT	48	24	0·444	25	0·466	1·6(1·3)	1030	540	324	216	108
BARLEY	25	24	0·444	25	0·466	1·6(1·3)	560	280	168	112	56
SAND, GRAVEL	100	30	0·577	35	0·700	1·9(1·6)	1730	865	519	346	173
CEMENT (STATIC) FINE	90	30	0·577	10	0·176	1·4(0 8)	1560	780	468	312	156
COARSE	90	30	0·577	18	0·325	1·4(1·0)	560	780	468	312	156
COAL (CRUSHED)	56	35	0·700	40	0·839	2·1(1·8)	800	400	240	160	80
ASHES	60 AV.	40	0·839	35	0·700	1·9(1·5)	715	355	205	145	72

TABLE 15.—BENDING MOMENTS AND SHEARING FORCES: BASIC DATA.

BASIC RELATION

AT ANY SECTION:—

SHEARING FORCE. $Q = \Sigma \left[\text{LOADS AND REACTIONS ON ONE SIDE OF SECTION}\right]$ = RATE OF CHANGE OF M.

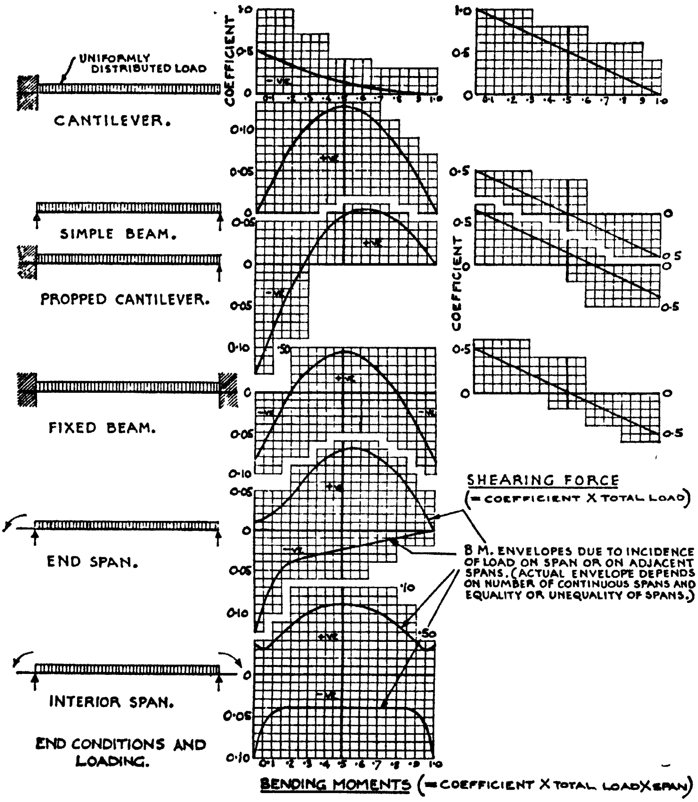
BENDING MOMENT. $M = \Sigma \left[\text{MOMENTS OF LOADS AND REACTIONS ON ONE SIDE OF SECTION}\right]$ = RATE OF CHANGE OF $EI\theta$

SLOPE. $\theta = \int \frac{M}{EI}$ = RATE OF CHANGE OF Δ

DEFLECTION (ELASTIC). $\Delta = \int \theta$.

I = MOMENT OF INERTIA OF MEMBER AT SECTION
E = ELASTIC MODULUS OF MATERIAL.

TYPICAL DIAGRAMS OF BENDING MOMENT AND SHEARING FORCE



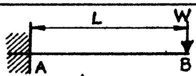
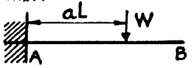
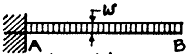
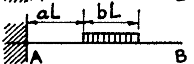
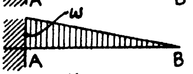
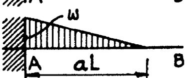
MAXIMUM RATIOS OF SPAN/DEPTH

CONDITION	BEAMS	SLABS* SPANNING IN ONE DIRECTION	SLABS* SPANNING IN TWO DIRECTIONS	
CANTILEVERED	10	12	-	PER B. S. CODE Nº 114
FREE	20	30	35	
CONTINUOUS	25	35	40	

* For limiting span-thickness ratios for flat slabs, see page facing *Table* 45.

CANTILEVERS

TOTAL LOAD ON CANTILEVER = W. LENGTH OF CANTILEVER = L.
REACTION AT A = MAX. SHEARING FORCE = W.
B. M. AT A = (TABULATED OR CALCULATED COEFFICIENT k_A) WL.
B. M. AT B = ZERO
MAX. DEFLECTION AT B = (TABULATED OR CALCULATED COEFFICIENT Δ_B) $\frac{WL^3}{EI}$

LOADING	TOTAL LOAD W	BENDING MOMENT COEFFICIENT k_A	DEFLECTION COEFFICIENT Δ_B
	W	$-1{\cdot}0$	$\frac{1}{3}$
	W	$-a$	$\frac{a^2}{6}(3-a)$
	wL	$-\frac{1}{2}$	$\frac{1}{8}$
	wbL	$-(a+\frac{1}{2}b)$	$X+Y(1-a-b)$ WHERE $X=\frac{1}{24}[8a^3+18a^2b+12ab^2+3b^2]$ AND $Y=\frac{1}{6b}[(a+b)^3-a^3]$
	$\frac{wL}{2}$	$-\frac{1}{3}$	$\frac{1}{15}$
	$\frac{waL}{2}$	$-\frac{a}{3}$	$\frac{a^2}{60}(5-a)$
	$\frac{wL}{2}$	$-\frac{2}{3}$	$\frac{11}{60}$
	$\frac{waL}{2}$	$-(1-\frac{a}{3})$	$\frac{1}{60}[20-10a+a^3]$
 PARABOLIC LOADING	$\frac{2}{3}wL$	$-\frac{1}{2}$	$\frac{7}{60}$
	$\frac{wL}{2}$	$-\frac{1}{2}$	$\frac{11}{96}$

FREELY-SUPPORTED BEAMS

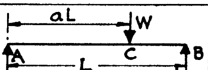
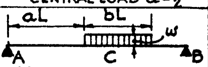
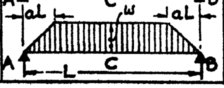
LOADING	TOTAL LOAD W	REACTION COEFFICIENTS r_A	r_B	COEFFICIENT FOR MAX. POS. B. M. K_C	COEFFICIENT FOR MAX. DEFLECTION $\Delta_{MAX.}$	NOTES
	W	$1-a$	a	$+a(1-a)$	$\frac{a^2}{3}(1-a)^2$	TOTAL LOAD ON BEAM = W
CENTRAL LOAD $a=\frac{1}{2}$	W	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{4}$	$\frac{1}{48}$	SPAN OF BEAM = L
	wbL	$1-(a+\frac{b}{2})$	$a+\frac{b}{2}$	$+\frac{(a+\frac{b}{2})(1-a-\frac{b}{2})(2-b)}{2}$	—	REACTION = rW
SPAN FULLY LOADED $a=0$ $b=1{\cdot}0$	wL	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{8}$	$\frac{5}{384}$	MAX. POS. B.M. = K_cWL
	$\frac{wL}{2}$	$\frac{2}{3}$	$\frac{1}{3}$	$+\frac{1}{7{\cdot}81}$	$\frac{1}{76{\cdot}75}$	MAX. DEFLECTION
	$\frac{wL}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{6}$	$\frac{1}{60}$	$=\Delta_{MAX}\left(\frac{WL^3}{EI}\right)$
	$wL(1-a)$	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{3-4a^2}{24(1-a)}$	$\frac{25-40a^2+16a^4}{1920(1-a)}$	B. M. AT A&B = 0.

TABLE 17.—BEAMS FIXED AT ONE OR BOTH ENDS.
GENERAL FORMULÆ.

W = TOTAL LOAD ON BEAM L = SPAN

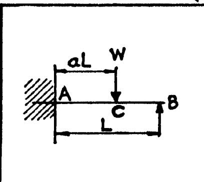
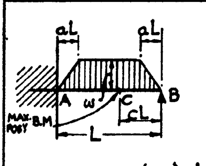
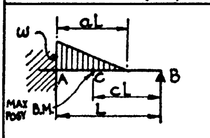
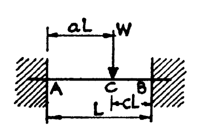
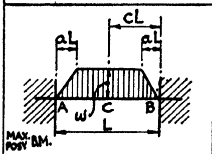
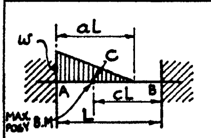
REACTION AT B = $r_B W$ REACTION AT A = $W - r_B W$

BENDING MOMENTS :—

NEG. AT SUPPORT A = $K_A WL$ AT SUPPORT B = $K_B WL$

MAX. POS^Y (AT C) = $K_C WL$ DISTANCE OF C FROM B = cL

MAX. DEFLECTION = $(\Delta_{MAX.})\left(\frac{WL^3}{EI}\right)$

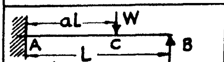
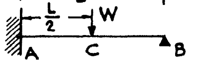
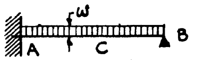
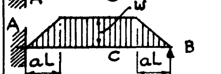
BEAMS FIXED AT SUPPORT A FREELY SUPPORTED AT B ($K_B=0$)		REACTION AT B B. MS. DEFLECTION	$r_B = \frac{1}{2}a^2(3-a)$ $c = 1-a$ $K_A = -\frac{1}{2}a(1-a)(2-a)$ $K_C = \frac{1}{2}a^2(1-a)(3-a)$ WHEN $a < 0{\cdot}586$ $\Delta_{MAX.} = \frac{1}{12}(1-a)x^2\left[2x - a(2-a)(3-x)\right]$ WHERE $x = 1-\sqrt{\frac{1-a}{3-a}}$ WHEN $a > 0{\cdot}586$ $\Delta_{MAX.} = \frac{1}{12}(1-x)a^2\left[2a - x(2-x)(3-a)\right]$ WHERE $x = \frac{a(2-a)}{1-a-\frac{1}{2}a^2}$
	 TOTAL LOAD = W = $(1-a)wL$	REACTION AT B B. MS. DEFLECTION	$r_B = \frac{1}{8}(3-a+a^2)$ $K_A = -\frac{1}{8}(1+a-a^2)$ $K_C = r_B c - \frac{1}{2(1-a)}\left[c(1+a) - a(1+\frac{2}{3}a)\right]$ WHERE $c = \frac{1}{8}(3+2a^2-a^3)$ WHEN $a = \frac{1}{2}$, $\Delta_{MAX.} = \frac{1}{139{\cdot}5}$ WHEN $a = 0$, $\Delta_{MAX.} = \frac{1}{185}$ INTERMEDIATE VALUES CAN BE INTERPOLATED.
	 TOTAL LOAD = W = $\frac{1}{2}waL$	REACTION AT B B. MS.	$r_B = \frac{1}{20}a^2(5-a)$ $K_A = -\frac{a}{3} + \frac{a^2}{4} - \frac{a^3}{20}$ $K_C = r_B c - \frac{1}{3a^2}(a+c-1)^3$ WHERE $c = 1 - a + a^2\sqrt{\frac{5-a}{20}}$
BEAMS FIXED AT BOTH SUPPORTS		REACTION AT B B. MS. DEFLECTION	$r_B = a^2(3-2a)$ $c = 1-a$ $K_A = -a(1-a)^2$ $K_B = -a^2(1-a)$ $K_C = 2a^2(1-a)^2$ WHEN $a > \frac{1}{2}L$. $\Delta_{MAX.} = \frac{2a^3(L-a)^2}{3(L+2a)^2}$ WHEN $a < \frac{1}{2}L$. $\Delta_{MAX.} = \frac{2a^2(L-a)^3}{3(L-2a)^2}$
	 TOTAL LOAD = W = $(1-a)wL$	REACTION AT B B. MS. DEFLECTION	$r_B = \frac{1}{2}$ $c = \frac{1}{2}$ $K_A = K_B = -\frac{1}{12(1-a)}(1-2a^2+a^3)$ $K_C = \frac{1-2a^3}{24(1-a)}$ WHEN $a = \frac{1}{2}$, $\Delta_{MAX.} = \frac{7}{1920}$ WHEN $a = 0$, $\Delta_{MAX.} = \frac{1}{384}$ INTERMEDIATE VALUES CAN BE INTERPOLATED
	 TOTAL LOAD = W = $\frac{1}{2}waL$	REACTION AT B B. MS.	$r_B = \frac{a^2}{10}(5-2a)$ $K_A = -\frac{a}{3} + \frac{a^2}{3} - \frac{a^3}{10}$ $K_B = -\frac{a^2}{6} + \frac{a^3}{10}$ WHERE $c = 1 - a + a^2\sqrt{\frac{5-2a}{10}}$ $K_C = r_B c - \frac{1}{3a^2}(a+c-1)^3 + K_B$

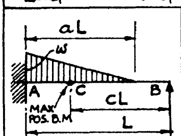
NOTE.—Values of coefficients for special cases of loading are given in *Table* 17A.

TOTAL LOAD ON SPAN $L = W$.
REACTION AT B = (TABULATED OR CALCULATED COEFFICIENT r_B)W; REACTION AT $A = W(1-r_B)$.
NEGATIVE B.M. AT A = (TABULATED OR CALCULATED COEFFICIENT K_A) WL.
MAX. POSITIVE B.M. AT C = (TABULATED OR CALCULATED COEFFICIENT K_C)WL.
POSITION OF PLANE OF MAX. B.M. = cL = DISTANCE CB.
MAX. DEFLECTION = (TABULATED OR CALCULATED COEFFICIENT $\Delta_{MAX.}$) $\frac{WL^3}{EI}$.

BEAMS FIXED AT ONE SUPPORT FREELY-SUPPORTED AT OTHER SUPPORT.

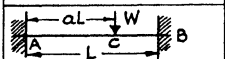
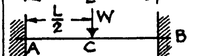
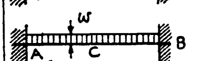
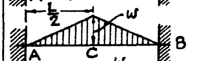
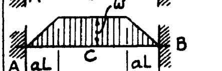
B.M. AT $B = 0$

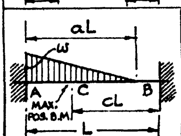
LOADING	TOTAL LOAD W	REACTION COEFFICIENT r_B	BENDING MOMENT COEFFICIENTS K_A (AT SUPPORT A)	K_C	c	DEFLECTION COEFFICIENT $\Delta_{MAX.}$
	W	$\frac{a^2}{2}(3-a)$	$-\frac{a}{2}(1-a)(2-a)$	$\frac{a^2}{2}(1-a)\times(3-a)$	$1-a$	SEE FORMULA ON FACING PAGE
	W	$\frac{5}{16}$	$-\frac{3}{16}$	$+\frac{5}{32}$	$\frac{1}{2}$	$\frac{1}{107.3}$
	wL	$\frac{3}{8}$	$-\frac{1}{8}$	$+\frac{9}{128}$	$\frac{3}{8}$	$\frac{1}{185}$
	$\frac{wL}{2}$	$\frac{11}{32}$	$-\frac{5}{32}$	$+\frac{1}{9.5}$	$\frac{27}{64}$	$\frac{1}{139.5}$
	$(1-a)wL$	SEE FORMULA ON FACING PAGE	$-\frac{1}{8}(1+a-a^2)$	SEE FORMULA ON FACING PAGE	$\frac{3+2a^2-a^3}{8}$	SEE FACING PAGE

LOADING	a	$\frac{1}{2}waL$	r_B	K_A	K_C	c	$\Delta_{MAX.}$
 SEE FORMULÆ ON FACING PAGE	1·0	0·5 wL	0·20	− 0·133	+ 0·060	0·45	$\frac{1}{210}$
	0·9	0·45 wL	0·17	− 0·133	+ 0·057	0·47	—
	0·8	0·4 wL	0·13	− 0·132	+ 0·053	0·49	
	0·7	0·35 wL	0·10	− 0·128	+ 0·047	0·59	
	0·6	0·3 wL	0·08	− 0·121	+ 0·041	0·57	
	0·5	0·25 wL	0·06	− 0·111	+ 0·032	0·62	

BEAMS FIXED AT BOTH SUPPORTS.

NEGATIVE B.M. AT B = (TABULATED OR CALCULATED COEFFICIENT K_B) WL.

LOADING	TOTAL LOAD W	REACTION COEFFICIENT r_B	BENDING MOMENT COEFFICIENTS K_A	K_B	K_C	c	DEFLECTION COEFFICIENT $\Delta_{MAX.}$
	W	$a^2(3-2a)$	$-a(1-a)^2$	$-a^2(1-a)$	$+2a^2(1-a)^2$	$1-a$	SEE FORMULA ON FACING PAGE
	W	$\frac{1}{2}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$+\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{92}$
	wL	$\frac{1}{2}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$+\frac{1}{24}$	$\frac{1}{2}$	$\frac{1}{384}$
	$\frac{wL}{2}$	$\frac{1}{2}$	$-\frac{5}{48}$	$-\frac{5}{48}$	$+\frac{1}{16}$	$\frac{1}{2}$	$\frac{1}{274}$
	$(1-a)wL$	$\frac{1}{2}$	$K_A = K_B = -\frac{1-2a^2+a^3}{12(1-a)}$		$+\frac{1-2a^3}{24(1-a)}$	$\frac{1}{2}$	SEE FACING PAGE

LOADING	a	$\frac{1}{2}waL$	r_B	K_A	K_B	K_C	c	$\Delta_{MAX.}$
 SEE FORMULÆ ON FACING PAGE	1·0	0·5 wL	0·30	− 0·100	− 0·067	+ 0·043	0·55	$\frac{1}{382}$
	0·9	0·45 wL	0·26	− 0·103	− 0·062	+ 0·042	0·56	—
	0·8	0·4 wL	0·22	− 0·105	− 0·056	+ 0·041	0·57	
	0·7	0·35 wL	0·18	− 0·104	− 0·047	+ 0·040	0·59	
	0·6	0·3 wL	0·14	− 0·102	− 0·038	+ 0·036	0·62	
	0·5	0·25 wL	0·10	− 0·096	− 0·029	+ 0·032	0·66	

NOTE.—See also *Table* 18 for factors for beams fixed at both ends.

NOTES ON CANTILEVERS AND BEAMS OF ONE SPAN.

Propped Cantilevers.

A propped cantilever, that is, a beam fixed rigidly at one support and freely supported at the other, can be considered as being subject to two systems of loading: (i) the imposed load; and (ii) the reaction of the prop, the magnitude of which being that which gives the same upward deflection at the end of a cantilever as the downward deflection produced by the imposed load. The coefficients of the maximum deflection given in *Table* 16 can be used for this purpose if the type of load is not one of those given for the beams fixed at one end and simply supported at the other in *Tables* 17 and 17A. As comparative deflections only are required the numerical values of I and E are not required. When the reaction of the prop has been calculated, the shearing forces and bending moments can be computed from the basic definitions.

Example.—A beam AB is rigidly fixed at A and freely supported at B. The span AB is 10 ft. and a concentrated load of 10 tons ($= W$) acts at a point 7 ft. from A. Find the load on the support at B and the bending moment at the support A.

The beam is a propped cantilever and the reaction at B can be found by equating the deflection at B of a cantilever loaded with W, as in the second case in *Table* 16, to the deflection at B of a cantilever loaded at its extremity (the first case in *Table* 16). The deflection due to W with $a = \frac{7}{10} = 0{\cdot}7$, is $\frac{0{\cdot}7^2(3 - 0{\cdot}7)10 \times 10^3}{6EI} = \frac{1880}{EI}$. The load at B is the unknown reaction R_B at this point, and the deflection of a cantilever is

$$\frac{R_B \times 10^3}{3EI} = \frac{333R_B}{EI}.$$

As one deflection counteracts the other, $\frac{1880}{EI} = \frac{333R_B}{EI}$, therefore $R_B = 5{\cdot}64$ tons. The bending moment at A is $(5{\cdot}64 \times 10) - (10 \times 7) = -13{\cdot}6$ ft.-tons, and the shearing force at A is $10 - 5{\cdot}64 = 4{\cdot}36$ tons. The shearing force at B is $5{\cdot}64$ tons.

Beams Fixed at Both Ends.

The bending moment on a beam fixed at both ends is derived from the principle that the area of the bending-moment diagram due to the same load imposed on a freely-supported beam of equal span (the "free-moment" diagram) is equal to the area of the restraint-moment diagram; the centroids of the two diagrams are in the same vertical line. The shape of the free-moment diagram depends upon the characteristics of the imposed load, but the restraint-moment diagram is a trapezium. For loads symmetrically disposed on the beam the centroid of the free-moment diagram is above the mid-point of the span, and thus the restraint-moment diagram is a rectangle, giving a restraint bending moment at each support equal to the mean height of the free-moment diagram.

The amount of the shearing force in a beam with one or both ends fixed is calculated from the variation of the bending moment. The shearing force resulting from the restraint moment alone is constant throughout the beam, and equals the difference between the two end moments divided by the span, that is, the rate of change of the restraint moment. This shearing force is algebraically added to the shearing force due to the imposed load considering the beam to be freely supported; that is, the reaction at a support is the sum (or difference) of the restraint-moment shearing force and the free-moment shearing force. For a symmetrically-loaded beam with both ends fixed the restraint moment at each end is the same, and the shearing force due to this moment considered alone is zero; therefore the resultant shearing forces are identical with those for the same beam freely supported, and the reactions both equal half the total load on the beam.

THE LOAD FACTORS C_{AB} AND C_{BA} CAN BE USED THUS:-

(i) B.M. AT SUPPORTS OF SINGLE SPAN BEAMS FIXED BOTH ENDS [SEE NOTE (1).]

$M_{AB} = -C_{AB}L_{AB}$ $\quad M_{BA} = -C_{BA}L_{AB}$ $\quad$ WITH SYMMETRICAL LOAD $M_{AB} = M_{BA}$

(ii) CONTINUOUS BEAMS – MOMENT DISTRIBUTION METHOD [SEE NOTE (2).]

SPAN-LOAD FACTORS ARE $F_{AB} = C_{AB}L_{AB}$ $\quad F_{BA} = C_{BA}L_{AB}$

WITH SYMMETRICAL LOAD $F_{AB} = F_{BA}$

(iii) FRAMED STRUCTURES [SEE NOTE (3).]

LOADING FACTORS P AND Q HAVE FOLLOWING VALUES:-

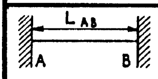
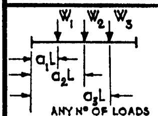
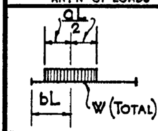
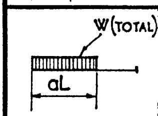
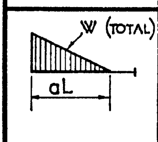
$P_{AB} = C_{AB}L_{AB}$ $\quad Q_{BA} = C_{BA}L_{AB}$ $\quad$ WITH SYMMETRICAL LOAD $P_{AB} = Q_{BA} = \dfrac{A_{AB}}{L_{AB}} = C_{AB}L_{AB}$

(iv) PORTAL FRAMES [SEE NOTE (4).]

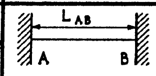
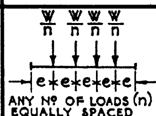
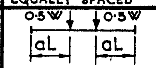
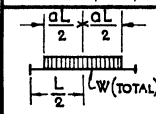
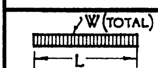
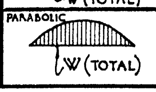
$$N = \frac{\text{AREA OF FREE B.M. DIAGRAM}}{\text{LOADED SPAN}}\left[=\frac{A}{L} \text{ OR } \frac{A}{0{\cdot}5L} \text{ OR } \frac{A}{H} \text{ OR } \frac{A}{fH}\right] = \frac{C_{AB}+C_{BA}}{2}\cdot L_{AB}; \quad Z = \frac{C_{AB}+2C_{BA}}{3(C_{AB}+C_{BA})}$$

DISTANCE FROM LEFT HAND OR LOWER SUPPORT TO CENTROID OF FREE B.M. DIAG. = Z × LOADED SPAN

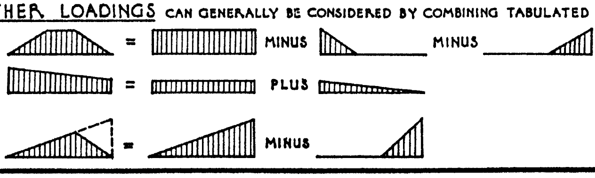
UNSYMMETRICAL LOADING

	LOAD FACTORS C_{AB}	LOAD FACTORS C_{BA}
	$\Sigma a(1-a)^2 W$	$\Sigma a^2(1-a)W$
	$\left[b(1-b)^2 - a^2\left(\frac{1}{6}-\frac{b}{4}\right)\right]W$	$\left[b^2(1-b) - a^2\left(\frac{b}{4}-\frac{1}{12}\right)\right]W$
	$a(3a^2-8a+6)\frac{W}{12}$	$a^2(4-3a)\frac{W}{12}$
 a = 1·0	0·100 W	0·067 W
= 0·8	0·105 W	0·055 W
= 0·6	0·102 W	0·039 W
= 0·5	0·096 W	0·029 W
= 0·4	0·086 W	0·020 W

SYMMETRICAL LOADING

		LOAD FACTOR $C_{AB} = C_{BA}$
	n = 1	0·125 W
	= 2	0·111 W
	= 3	0·104 W
	= 4	0·100 W
		$\frac{a}{2}(1-a)W$
		$0{\cdot}125\left(1-\frac{a^2}{3}\right)W$
		0·083 W
		0·104 W
		0·100 W

OTHER LOADINGS CAN GENERALLY BE CONSIDERED BY COMBINING TABULATED CASES, THUS:-



NOTE 1.—See also *Tables* 17 and 17A.
„ 2.—Factors for use with *Table* 26.
„ 3.—Factors for use with *Table* 46.
„ 4.—Factors for use with *Tables* 49 and 50; also for continuous beams (*Table* 24).

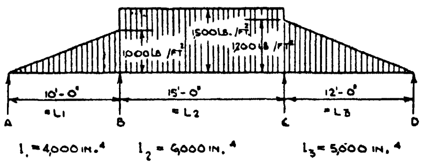
CONTINUOUS BEAMS.

Three-moment Theorem.—Formulæ in general and special cases are given in *Table* 19; the values of the factors $\frac{A}{L}$ and z for use in these formulæ are given in *Table* 18.

When known factors relating to the load, span, moment of inertia and relative levels of the supports are substituted in the general formula, an equation with three unknown support bending moments is obtained for each pair of spans; that is, for n spans, $n - 1$ equations are obtained containing $n + 1$ unknowns (the moments at $n + 1$ supports). The two excess unknowns represent the bending moments at the end supports and, if these bending moments are known or can be assumed, the bending moments at the intermediate supports can be determined. At a freely-supported end the bending moment is zero. For a perfectly fixed end the support bending moments can be determined if an additional span is considered continuous at the fixed end; this additional span must be identical in all respects with the original end span except that the load should produce symmetry about the original end support with the load on the original end span. The bending moment at the new end support is considered to be equal to that at the original penultimate support, and thus an additional equation is obtained without introducing another unknown.

When the bending moments at the supports have been calculated, the diagram of the support bending moments is combined with the diagram of the "free moments" and the resulting bending moments are obtained.

Example.—Determine by the Theorem of Three Moments the bending moments at the supports of the beam in the diagram, assuming level supports.



From *Table* 19 the appropriate formula modified for spans AB and BC (span $L_1 = L_{AB}$, and $L_2 = L_{BC}$) is

$$M_A\frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_C L_2}{I_2} = -\left[\frac{A_1 z_1}{I_1 L_1} + \frac{A_2(L_2 - z_2)}{I_2 L_2}\right]6 \quad . \quad . \quad (1)$$

and for spans BC and CD (span $L_2 = L_{BC}$ and $L_3 = L_{CD}$)

$$M_B\frac{L_2}{I_2} + 2M_C\left(\frac{L_2}{I_2} + \frac{L_3}{I_3}\right) + \frac{M_D L_3}{I_3} = -\left[\frac{A_2 z_2}{I_2 L_2} + \frac{A_3(L_3 - z_3)}{I_3 L_3}\right]6 \quad . \quad . \quad (2)$$

For span AB, $\frac{L_1}{I_1} = \frac{10}{4000} = 0{\cdot}0025 \left(\frac{\text{ft.}}{\text{in.}^4}\right)$ and $\frac{A_1}{L_1} = \frac{w_1 L_1^2}{24} = \frac{1000 \times 10^2}{24} = 4170$ ft.-lb.

$z_1 = \frac{8}{15}L_1$; thus $\frac{z_1}{I_1} = \frac{8}{15} \times 0{\cdot}0025 = 0{\cdot}00133$. Hence $\frac{A_1 z_1}{I_1 L_1} = 4170 \times 0{\cdot}00133 = 5{\cdot}55$.

For span BC, $\frac{L_2}{I_2} = \frac{15}{6000} = 0{\cdot}0025$, and $\frac{A_2}{L_2} = \frac{w_2 L_2^2}{12} = \frac{1500 \times 15^2}{12} = 28{,}125$ ft.-lb ;

$$\frac{z_2}{I_2} = \frac{L_2}{2I_2} = \frac{L_2 - z_2}{I_2} = 0{\cdot}5 \times 0{\cdot}0025 = 0{\cdot}00125.$$

Hence $\quad \frac{A_2 z_2}{I_2 L_2} = \frac{A_2(L_2 - z_2)}{I_2 L_2} = 28{,}125 \times 0{\cdot}00125 = 35{\cdot}156.$

For span CD, $\frac{L_3}{I_3} = \frac{12}{5000} = 0{\cdot}0024$, and $\frac{A_3}{L_3} = \frac{w_3 L_3^2}{24} = \frac{1200 \times 12^2}{24} = 7200$ ft.-lb.; $z_3 = \frac{7}{15}L_3$: thus $\frac{L_3 - z_3}{I_3} = \frac{8}{15} \times 0{\cdot}0024 = 0{\cdot}00128$. Hence $\frac{A_3(L_3 - z_3)}{I_3 L_3} = 7200 \times 0{\cdot}00128 = 9{\cdot}21$.

If the beam is simply supported at A and D, $M_A = M_D =$ zero, and substituting known values in (1) and (2),

$$2M_B(0{\cdot}0025 + 0{\cdot}0025) + 0{\cdot}0025M_C = -(5{\cdot}55 + 35{\cdot}16)6 \quad . \quad (3)$$
$$0{\cdot}0025M_B + 2M_C(0{\cdot}0025 + 0{\cdot}0024) = -(35{\cdot}16 + 9{\cdot}21)6 \quad . \quad (4)$$

Thus

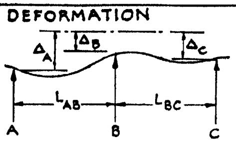
$$0{\cdot}0100M_B + 0{\cdot}0025M_C = -244 \quad . \quad (5)$$
$$0{\cdot}0025M_B + 0{\cdot}0098M_C = -266 \quad . \quad (6)$$

Multiplying (6) by $\frac{0{\cdot}0100M_B}{0{\cdot}0025M_B} = 4$; $0{\cdot}0100M_B + 0{\cdot}0392M_C = -1064 \quad . \quad (7)$

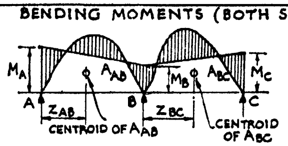
(*Continued on page* 174.)

GENERAL EQUATION

DEFORMATION



BENDING MOMENTS (BOTH SPANS LOADED)



$$M_A\frac{L_{AB}}{I_{AB}} + 2M_B\left(\frac{L_{AB}}{I_{AB}} + \frac{L_{BC}}{I_{BC}}\right) + M_C\frac{L_{BC}}{I_{BC}} = 6E\left[\frac{\Delta_B-\Delta_A}{L_{AB}} - \frac{\Delta_C-\Delta_B}{L_{BC}}\right] - 6\left[\frac{A_{AB}Z_{AB}}{I_{AB}L_{AB}} + \frac{A_{BC}(L_{BC}-Z_{BC})}{I_{BC}L_{BC}}\right]$$

SPECIAL CASES

RIGID SUPPORTS.

$$M_A\frac{L_{AB}}{I_{AB}} + 2M_B\left(\frac{L_{AB}}{I_{AB}} + \frac{L_{BC}}{I_{BC}}\right) + M_C\frac{L_{BC}}{I_{BC}} = -6\left[\frac{A_{AB}Z_{AB}}{I_{AB}L_{AB}} + \frac{A_{BC}(L_{BC}-Z_{BC})}{I_{BC}L_{BC}}\right]$$

RIGID SUPPORTS, UNIFORM MOMENT OF INERTIA.

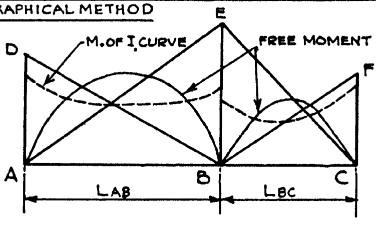
$$M_A L_{AB} + 2M_B(L_{AB}+L_{BC}) + M_C L_{BC} = -6\left[\frac{A_{AB}Z_{AB}}{L_{AB}} + \frac{A_{BC}(L_{BC}-Z_{BC})}{L_{BC}}\right]$$

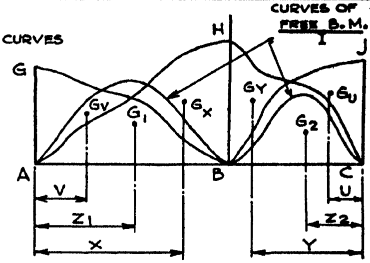
UNIFORMLY-DISTRIBUTED LOAD (w LB. PER FOOT) ON BOTH SPANS
RIGID SUPPORTS, UNIFORM MOMENT OF INERTIA.

$$M_A L_{AB} + 2M_B(L_{AB}+L_{BC}) + M_C L_{BC} = -\frac{w}{4}\left(L_{AB}^3 + L_{BC}^3\right)$$

NON-UNIFORM MOMENT OF INERTIA

GRAPHICAL METHOD





TREAT EACH CONSECUTIVE PAIR OF SPANS THUS:—

ON THE SPANS DRAWN TO SCALE, CONSTRUCT THE FREE-MOMENT CURVES AND THE MOMENT OF INERTIA CURVES. DIVIDE THE ORDINATES OF THE FORMER BY THOSE OF THE LATTER TO ENABLE THE CURVES OF $\frac{\text{FREE B.M.}}{I}$ TO BE DRAWN.

FIND A_{AB} = AREA UNDER $\frac{\text{FREE B.M.}}{I}$ CURVE FOR SPAN L_{AB} AND POSITION OF CENTROID G_1
AND A_{BC} = " " " " " " L_{BC} " " " " G_2

SET UP AD, BE, AND CF TO A SUITABLE SCALE TO REPRESENT ANY ASSUMED VALUES OF THE MOMENTS AT A, B, & C RESPECTIVELY. CONNECT DB, AE, EC, & BF.

LET $M_A = k_A AD$ $M_B = k_B BE$ $M_C = k_C CF$.

DIVIDE THE ORDINATES OF DBF AND AEC BY THE ORDINATES OF THE M. OF I. CURVE TO GIVE THE CURVES AHC AND GBJ.

FIND A_V = AREA UNDER CURVE GB AND POSITION OF CENTROID G_V
A_X = " " " AH " " " " G_X
A_Y = " " " HC " " " " G_Y
AND A_U = " " " BJ " " " " G_U

SUBSTITUTE IN $$\left(A_{AB}Z_1 + k_A V A_V + k_B X A_X\right)\frac{1}{L_{AB}} = -\left(A_{BC}Z_2 + k_C A_U W + k_B A_Y Y\right)\frac{1}{L_{BC}}$$

UNKNOWNS ARE k_A, k_B, & k_C AND REQUISITE NUMBER OF EQUATIONS FOLLOWS FROM CONSECUTIVE PAIRS OF SPANS AND END SUPPORT CONDITIONS.

APPROXIMATE METHOD.

CALCULATE B.M.S. FOR UNIFORM MOMENT OF INERTIA AND INCREASE OR DECREASE BY THE APPROPRIATE PERCENTAGE TABULATED.
I_S = M. OF I. AT SUPPORT
I_C = " " MIDSPAN

RATIO OF $I_S : I_C$		·25	·50	·75	1·25	1·50
BOTH ENDS FIXED	MIDSPAN	+55%	+26%	+7%	−6%	−13%
	SUPPORT	−28%	−15%	−5%	+3%	+4%
ONE FIXED ONE FREE	MIDSPAN	+27%	+13%	+6%	−2½%	−10%
	SUPPORT	−40%	−20%	−8%	+5%	+13%

CONTINUOUS BEAMS (*continued from page 172*).

Subtracting (5) from (7): $0{\cdot}0367M_C = -820$; hence $M_C = -22{,}400$ ft.-lb.
By substituting in (5): $0{\cdot}0100M_B = -(244 - 56) = -190$; hence $M_B = -19{,}800$ ft.-lb.

Non-uniform Moment of Inertia.—When the moment of interia is practically uniform throughout each span of a series of continuous spans, but differs in one span relative to another, the general expressions for the Theorem of Three Moments, the formulæ for which are given in *Table* 19, are applicable as in the example above. When the moment of inertia varies irregularly within the length of each span the semi-graphical method given in *Table* 19 can be used. The moments of inertia of common reinforced concrete sections are given in *Tables* 64, 65 and 65A.

If the moment of inertia varies in such a way that it can be represented by an equation, the Theorem of Three Moments can be used if for M is substituted $\frac{M}{I}$ and if the area of the $\frac{M}{I}$ diagram is used instead of the area of the free-moment diagram. The solution of the derived simultaneous equations then gives values of the support bending moment $\div I$, enabling a complete $\frac{M}{I}$ diagram for the beam to be constructed from which the bending moment at any section is readily obtained by multiplying the appropriate ordinate of the $\frac{M}{I}$ diagram by the moment of inertia at the section.

When circumstances do not permit the foregoing methods to be used, the bending moments can be calculated on the assumption of uniform moment of inertia, and an approximate adjustment can be made for the effect of the neglected variation. An increase in the moment of inertia near a support causes an increase in the negative bending moment at that support and a consequent decrease in the positive bending moments in the adjacent spans, and vice versa. As a guide in making the adjustment the approximate factors given in *Table* 19 represent a percentage addition to, or deduction from, the calculated bending moments.

Incidence of Live Load to Produce Maximum Bending Moments.—The values of the bending moments at the support and in the span depend upon the incidence of the live load, and for equal spans or with spans approximately equal the dispositions of live load illustrated in *Table* 20 give the maximum positive bending moment at midspan, and the maximum negative bending moment at a support. The B.S. Code recommends the incidence of live load to be less severe, since when calculating the maximum negative bending moments at any support only the spans immediately on either side of the support under consideration need be loaded. This affects only the coefficients for four or more continuous spans and the reduction is commonly much less than 5 per cent.

Positive and Negative Bending Moments in the Span.—When the negative bending moments at the supports of a continuous beam have been determined, the positive bending moments on a loaded span can be determined graphically or, in the case of a uniformly-distributed load, by means of the expressions in *Table* 20.

Beams and slabs, such as those in bridge decks, where the ratio of live load to dead load is high, should be designed for a possible negative bending moment occurring at midspan. Formulæ for the approximate evaluation of this bending moment, which apply if the lengths of adjacent spans do not differ by more than 20 per cent. of the shorter span, are given in *Table* 20. These formulæ make some allowance for the torsional restraint of the supports.

Shearing Forces.—The variation of shearing force on a continuous beam is determined by first considering each span as freely supported and algebraically adding the rate of change of restraint moment for the span considered. Formulæ for calculating the component and resultant shearing forces are given in *Table* 20. The shearing force due to the load can be determined from statics. The shearing force due to the restraint moments is constant throughout the span.

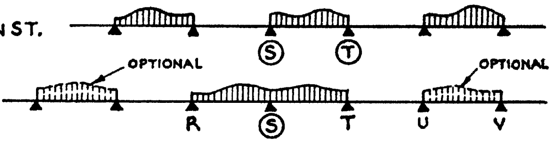
Approximate Bending-moment Coefficients.—The bending-moment coefficients in *Table* 20 apply to beams or slabs (spanning in one direction) continuous over three or more spans. The coefficients given for end spans and penultimate supports assume that the beam or slab is nominally freely supported on the end support. The coefficients given for the dead load and live load separately on beams and slabs conform to the recommendations of B.S. Code No. 114; the corresponding coefficients for total load are calculated for various ratios of live to dead load. The coefficients given for slabs only, without splays, are values commonly assumed and apply to the total load on a slab spanning in one direction; they take into account the fact that the slab is partially restrained at the end supports because of monolithic construction. If the slab is provided with splays, of sizes not less than indicated in the diagram, the positive bending moments are decreased and the negative bending moments increased; suitable coefficients for this condition are also given in *Table* 20.

CRITICAL LOADING

INCIDENCE OF LIVE LOAD

TO PRODUCE MAXIMUM POSITIVE B.M. ON SPAN ST.

TO PRODUCE MAXIMUM NEGATIVE B.M. AT SUPPORT S.



B. S. CODE.— LOADS ON SPANS RS AND ST ONLY NEED BE TAKEN INTO ACCOUNT.

MAX. POSITIVE B.M.

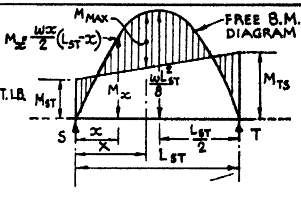
UNIFORMLY-DISTRIBUTED LOAD w ON SPAN ST.

MAX. POSITIVE B.M. ON SPAN ST.

$$M_{MAX.} = \frac{w}{2}\left[\frac{M_{ST} - M_{TS}}{wL_{ST}} + \frac{L_{ST}}{2}\right]^2 - M_{ST} \text{ FT.LB.}$$

IF w IS IN LB./FT., B.Ms. MUST BE IN FT.-LB., AND L_{ST} MUST BE IN FT.

$$X = \frac{L_{ST}}{2} + \frac{M_{ST} - M_{TS}}{wL_{ST}} \text{ (FT.)}$$



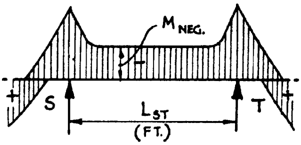
NEGATIVE B.M. IN SPAN (APPROXIMATE)

NEGATIVE B.M. ON UNLOADED SPAN ST. (BETWEEN TWO LOADED SPANS)

$$M_{NEG.} = (kw_L - w_D)\frac{L_{ST}}{24} \text{ FT. LB.}$$

w_L = LIVE LOAD, w_D = DEAD LOAD } LB./FT. FOR BEAMS. LB./SQ. FT. FOR SLABS.

FOR BEAMS: $k = \frac{2}{3}$ FOR SLABS: $k = \frac{1}{2}$



SHEARING FORCES

DUE TO LOAD.

$$Q'_T = \frac{Z_o}{L_{ST}} W$$

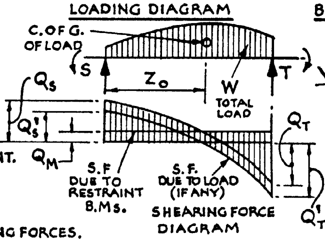
$$Q'_S = -\frac{L_{ST} - Z_o}{L_{ST}} W = -(W - Q'_T)$$

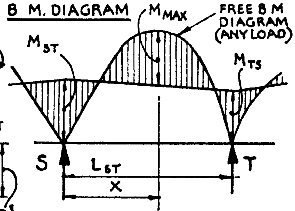
DUE TO END RESTRAINT.

$$Q_M = \frac{M_{ST} - M_{TS}}{L_{ST}}$$

RESULTANT SHEARING FORCES.

$$Q_S = Q'_S + Q_M \qquad Q_T = -(Q'_T - Q_M)$$



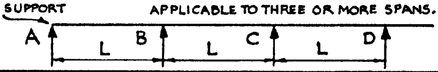


NOTE.— IF $M_{TS} > M_{ST}$, Q_M IS NEG.

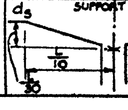
APPROXIMATE BENDING MOMENT COEFFICIENTS

UNIFORMLY-DISTRIBUTED LOAD ON EQUAL SPANS

B.M. = (COEFF.) × (TOTAL LOAD) × L



COEFFICIENTS APPLY ALSO TO UNEQUAL SPANS IF INEQUALITY DOES NOT EXCEED 15 PERCENT. OF THE SHORTEST SPAN.

	BEAMS AND SLABS (B. S. CODE)						SLABS ONLY MONOLITHIC WITH END SUPPORT A (NOMINALLY FREELY SUPPORTED) COEFFICIENTS FOR TOTAL LOAD		
	DEAD LOAD	LIVE LOAD	TOTAL LOAD. R = $\frac{\text{LIVE LOAD}}{\text{DEAD LOAD}}$				WITHOUT SPLAYS	WITH SPLAYS	
			R = $\frac{1}{2}$	1	2	3			
END SPAN AB ABOUT MIDSPAN	$+\frac{1}{12}$	$+\frac{1}{10}$	$+\frac{1}{11{\cdot}2}$	$+\frac{1}{10{\cdot}9}$	$+\frac{1}{10{\cdot}6}$	$+\frac{1}{10{\cdot}4}$	$+\frac{1}{10}$	$+\frac{1}{12}$	MINIMUM PROPORTIONS OF SPLAYS 
PENULTIMATE SUPPORT B	$-\frac{1}{10}$	$-\frac{1}{9}$	$-\frac{1}{9{\cdot}6}$	$-\frac{1}{9{\cdot}5}$	$-\frac{1}{9{\cdot}3}$	$-\frac{1}{9{\cdot}2}$	$-\frac{1}{10}$	$-\frac{1}{9}$	
INTERIOR SPAN BC, ETC. ABOUT MIDSPAN	$+\frac{1}{24}$	$+\frac{1}{12}$	$+\frac{1}{17{\cdot}8}$	$+\frac{1}{16}$	$+\frac{1}{14{\cdot}4}$	$+\frac{1}{13{\cdot}7}$	$+\frac{1}{12}$	$-\frac{1}{16}$	
INTERIOR SUPPORTS C, ETC.	$-\frac{1}{12}$	$-\frac{1}{9}$	$-\frac{1}{10{\cdot}8}$	$-\frac{1}{10{\cdot}3}$	$-\frac{1}{9{\cdot}8}$	$-\frac{1}{9{\cdot}6}$	$-\frac{1}{12}$	$+\frac{1}{10}$	
END SUPPORT A.	—	—	—	—	—	—	$-\frac{1}{24}$	$-\frac{1}{16}$	

CONTINUOUS BEAMS: EQUAL SPANS.

Note on Table 21.—The coefficients in *Table* 21 for bending moments at supports due to incidental live load are for alternate spans loaded including the two spans immediately adjacent to the support. The corresponding coefficients given in brackets are for the two spans immediately adjacent to the support only loaded.

Examples of Use of Tables 20 and 21.

(*a*) Calculate the maximum bending moments at the middles of the end and central spans and at the penultimate and interior supports of a beam continuous over five equal spans of 15 ft. each if the dead load is 600 lb. per ft. and the live load is 1200 lb. per ft.

(i) From *Table* 21 (using coefficients for all alternate spans being loaded):

Penultimate support: Dead load: $0{\cdot}105 \times 600 \times 15^2 = 14{,}200$ ft.-lb.
Live load: $0{\cdot}120 \times 1200 \times 15^2 = 32{,}300$,,
Total (negative) $= 46{,}500$

Interior support: Dead load: $0{\cdot}080 \times 600 \times 15^2 = 10{,}400$ ft.-lb.
Live load: $0{\cdot}111 \times 1200 \times 15^2 = 29{,}900$,,
Total (negative) $= 40{,}300$,,

Middle of end span: Dead load: $0{\cdot}078 \times 600 \times 15^2 = 10{,}250$ ft.-lb.
Live load: $0{\cdot}100 \times 1200 \times 15^2 = 27{,}000$,,
Total (positive) $= 37{,}250$

Middle of interior span: Dead load: $0{\cdot}046 \times 600 \times 15^2 = 6{,}200$ ft.-lb.
Live load: $0{\cdot}086 \times 1200 \times 15^2 = 23{,}200$,,
Total (positive) $= 29{,}400$,,

(ii) By means of *Table* 20, using the coefficients recommended in the B.S. Code.

Ratio of live to dead load, $\frac{1200}{600} = 2$. Total load $= 1200 + 600 = 1800$ lb. per ft.

Penultimate support: $-\frac{1}{9{\cdot}3} \times 1800 \times 15^2 = 43{,}500$ ft.-lb. (neg.)

Interior support: $-\frac{1}{9{\cdot}8} \times 1800 \times 15^2 = 41{,}300$,, (neg.)

Middle of end span: $+\frac{1}{10{\cdot}6} \times 1800 \times 15^2 = 38{,}200$,, (pos.)

Middle of central span: $+\frac{1}{14{\cdot}4} \times 1800 \times 15^2 = 28{,}100$,, (pos.)

(*b*) Solve, by means of *Table* 21, Example (*a*) opposite *Table* 25.

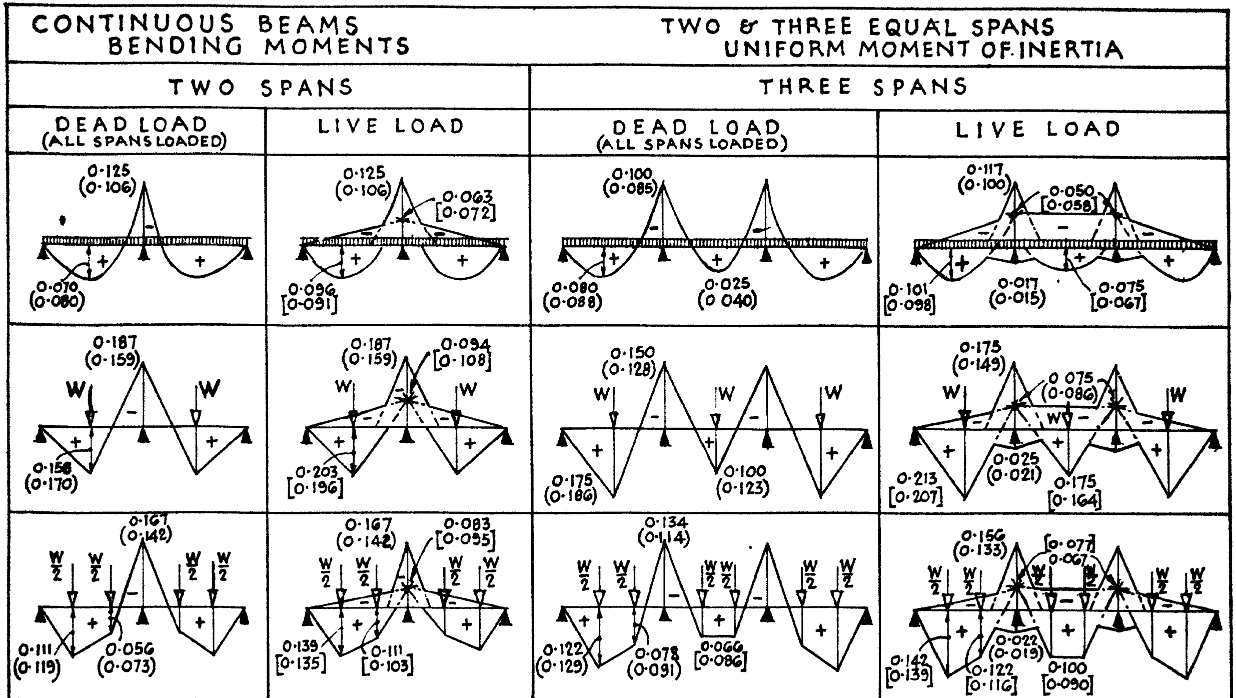
Bending moment at penultimate support of a beam continuous over four spans:

Dead load: $0{\cdot}107 \times 1000 \times 15^2 = 24{,}100$ ft.-lb.
Live load: $0{\cdot}181 \times 10{,}000 \times 15 = 27{,}200$,,
Total (negative) $= 51{,}300$,,

-	LOAD	ALL SPANS LOADED (e.g. DEAD LOAD)	LIVE LOAD (SEQUENCE OF LOADED SPANS TO GIVE MAX. B.M. OR S.F.)
COEFFICIENTS FOR MAXIMUM BENDING MOMENTS	UNIFORMLY-DISTRIBUTED	0·125 0·071 0·071 0·100 0·100 0·080 0·025 0·080 0·107 0·072 0·107 0·077 0·036 0·036 0·077 0·105 0·080 0·080 0·105 0·078 0·033 0·046 0·033 0·078	0·125 0·096 0·096 0·117 0·117 0·101 0·075 0·101 (0·116) (0·107) (0·116) 0·121 0·107 0·121 0·099 0·081 0·081 0·099 (0·116) (0·107) (0·107) (0·116) 0·120 0·111 0·111 0·120 0·100 0·080 0·086 0·080 0·100
	CONCENTRATED AT MID-SPAN	0·188 0·156 0·156 0·150 0·150 0·175 0·100 0·175 0·161 0·107 0·161 0·169 0·116 0·116 0·169 0·158 0·119 0·119 0·158 0·171 0·110 0·130 0·110 0·171	0·188 0·203 0·203 0·175 0·175 0·213 0·175 0·213 (0·174) (0·160) (0·174) 0·181 0·160 0·180 0·210 0·183 0·183 0·210 (0·174) (0·160) (0·160) (0·174) 0·179 0·167 0·167 0·179 0·211 0·181 0·191 0·181 0·211
	CONCENTRATED AT THREE POINTS	0·167 0·111 0·111 0·133 0·133 0·123 0·034 0·123 0·143 0·095 0·143 0·119 0·056 0·056 0·119 0·141 0·106 0·106 0·141 0·120 0·050 0·061 0·050 0·120	0·167 0·139 0·139 0·157 0·157 0·145 0·100 0·145 (0·155) (0·143) (0·155) 0·160 0·144 0·160 0·143 0·111 0·111 0·143 (0·155) (0·142) (0·142) (0·155) 0·159 0·148 0·148 0·159 0·144 0·108 0·115 0·108 0·144
COEFFICIENTS FOR MAX. SHEARING FORCE	UNIFORMLY-DISTRIBUTED	0·38 0·62 0·62 0·38 0·40 0·50 0·60 0·60 0·50 0·40 0·39 0·54 0·46 0·61 0·61 0·46 0·54 0·39 0·40 0·53 0·50 0·47 0·60 0·60 0·47 0·50 0·53 0·40	0·44 0·62 0·62 0·44 0·45 0·58 0·62 0·62 0·58 0·45 0·45 0·60 0·57 0·62 0·62 0·57 0·60 0·45 0·45 0·60 0·59 0·58 0·62 0·62 0·58 0·59 0·60 0·45

BENDING MOMENT = (COEFFICIENT) x (TOTAL LOAD ON ONE SPAN) x (SPAN).
SHEARING FORCE = (COEFFICIENT) x (TOTAL LOAD ON ONE SPAN).
B. M. COEFFICIENTS ABOVE LINE APPLY TO NEGATIVE B.M. AT SUPPORTS.
BELOW " " " POSITIVE B.M. IN SPAN.
S. F. COEFFICIENTS ABOVE " " " S. F. AT RIGHT-HAND SIDE OF SUPPORT.
BELOW " " " S. F. AT LEFT-HAND " " "
COEFFICIENTS APPLY WHEN ALL SPANS ARE EQUAL (OR LONGEST ≯ 15% MORE THAN SHORTEST)
LOADS ON EACH LOADED SPAN ARE EQUAL.
MOMENT OF INERTIA SAME THROUGHOUT ALL SPANS.
B. M. COEFFICIENTS (LIVE LOAD) IN BRACKETS APPLY IF TWO SPANS ONLY ARE LOADED.

TABLE 22.—CONTINUOUS BEAMS: BENDING-MOMENT DIAGRAMS. TWO AND THREE EQUAL SPANS.



EQUAL LOAD W (TOTAL LOAD) ON EACH LOADED SPAN. DIAGRAMS ARE SYMMETRICAL BUT NOT DRAWN TO SCALE.

BENDING MOMENT = (COEFFICIENT) x (W) x (SPAN)

COEFFICIENTS THUS: 0·125 = THEORETICAL BENDING MOMENTS

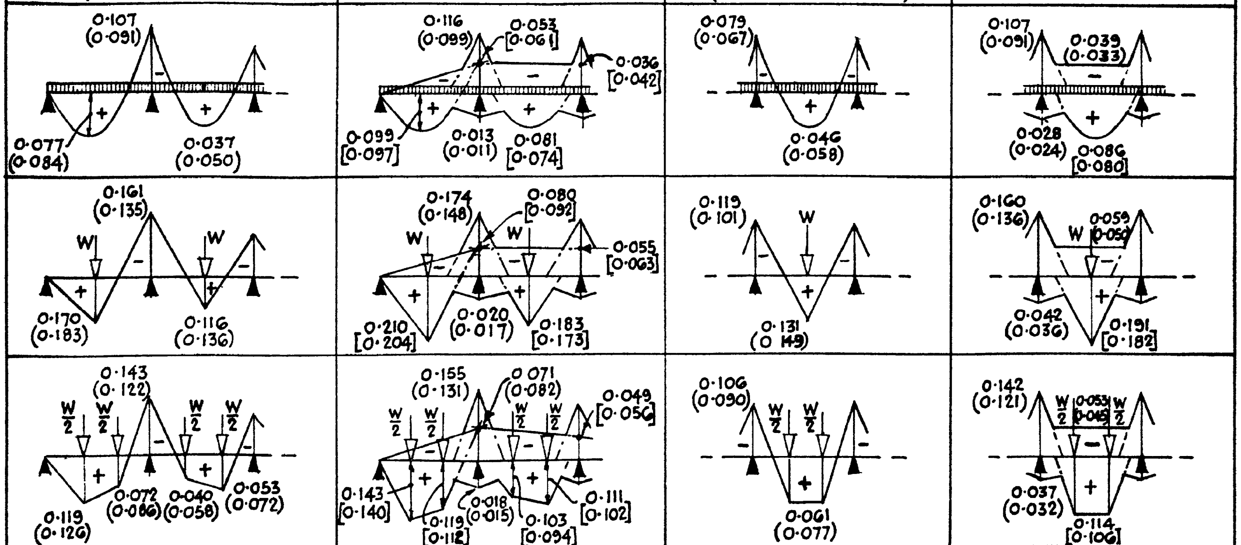
(0·106) = BENDING MOMENTS WITH B.Ms. AT SUPPORTS REDUCED BY 15 PER CENT. (GIVE ADJUSTED B.Ms. FOR DEAD LOAD AND REDUCED SUPPORT B.Ms. FOR LIVE LOAD)

[0·191] = BENDING MOMENTS WITH B.Ms AT SUPPORTS INCREASED BY 15 PER CENT. [GIVE REDUCED MID-SPAN B.Ms. FOR LIVE LOAD]

CONTINUOUS BEAMS
BENDING MOMENTS

FOUR OR MORE EQUAL SPANS
UNIFORM MOMENT OF INERTIA

END SPAN AND PENULTIMATE SPAN & SUPPORT (BASED ON FOUR CONTINUOUS SPANS)		OTHER INTERIOR SPANS & SUPPORTS (BASED ON CENTRAL SPAN OF FIVE CONTINUOUS SPANS)	
DEAD LOAD (ALL SPANS LOADED)	LIVE LOAD	DEAD LOAD (ALL SPANS LOADED)	LIVE LOAD



EQUAL LOAD W (= TOTAL LOAD) ON EACH LOADED SPAN.
BENDING MOMENT = (COEFFICIENT) X (W) X (SPAN)
COEFFICIENTS THUS: 0·116 = THEORETICAL BENDING MOMENTS.
(0·099) = BENDING MOMENTS WITH B.Ms. AT SUPPORTS REDUCED BY 15 PER CENT. (GIVE ADJUSTED B.Ms. FOR DEAD LOAD AND REDUCED SUPPORT B.Ms. FOR LIVE LOAD)
[0·097] = BENDING MOMENTS WITH B.Ms. AT SUPPORTS INCREASED BY 15 PER CENT. [GIVE REDUCED MID-SPAN B.Ms. FOR LIVE LOAD]

DIAGRAMS ARE NOT DRAWN TO SCALE.
DIAGRAMS FOR "OTHER INTERIOR SPANS & SUPPORTS" ARE SYMMETRICAL.

NOTES ON TABLES 24 AND 25.

Graphical Determination of Bending Moments by Fixed Points.—A graphical method of determining the bending moments on a continuous beam is given in *Table* 24. The basis of the method is that there is a point (termed the "fixed point") adjacent to the left-hand support of any span of a continuous system at which the bending moment is unaffected by any alteration in the bending moment at the right-hand support. A similar point occurs near the right-hand support, the bending moment at this point being unaffected by alteration in the bending moment at the left-hand support. When a beam is rigidly fixed at a support the "fixed point" is one-third of the span from that support; when freely supported the fixed point coincides with the support. For intermediate conditions of fixity the "fixed point" is between these extremes. In two continuous spans, L_1 and L_2, if the distance from the left-hand (or right-hand) support to the adjacent fixed point is l_1, then the distance l_2 from the left-hand (or right-hand) support of span L_2 to the adjacent fixed point is

$$l_2 = \frac{L_2^2(L_1 - l_1)}{3(L_1 + L_2)(L_1 - l_1) - L^2}.$$

Alternatively l_2 can be found from l_1 by the graphical construction shown in *Table* 24. Upon combining the free-moment diagram with the position of the fixed points for a span, as described in *Table* 24, the resultant negative and positive bending moments throughout the system, due to the load on this span, can be determined. By treating each span separately the envelopes of the maximum possible bending moments throughout the system can be drawn.

Bending Moments on Equal and Unequal Spans.—The factors in *Table* 25 apply to the calculation of the support moments for beams with uniform moment of inertia and continuous over two, three, or four equal or unequal spans, and carrying almost any type or incidence of live and dead loads. The basis of the method is that the load on any span of a given system can be divided into one or more of the types shown in *Table* 25. Other types of loading can be allowed for thus: partially-distributed load at one end of span gives coefficients between those for (*a*) and (*c*); three or more equal concentrated loads give coefficients between those for (*a*) and (*e*).

[NOTE.—The references to the types of load are: Type *a*.—Uniformly-distributed load. Type *b*.—Triangular load with apex at centre. Type *c*.—Triangular load with apex at left-hand support. Type *d*.—Central concentrated load. Type *e*.—Concentrated loads at each of one-third points.]

A given beam system can be divided into a series of identical systems each with only one span loaded with one type of load. Each of these loads produces a bending moment at each support, the total bending moment at any support being the algebraic sum of the moments due to each type of load. When the support moments are known, the mid-span moments can be calculated or determined graphically therefrom.

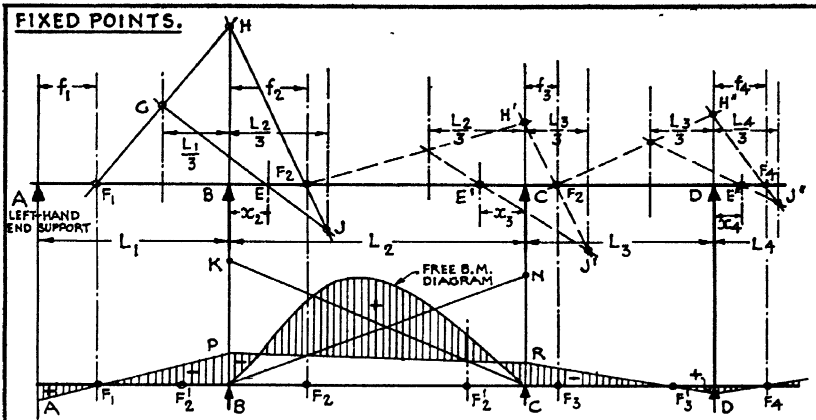
For equal spans the bending moment at any support due to any one span loaded with any one type of load is given by M_S = load factor (F) $\times$ support moment coefficient (Q) $\times$ total load (W) $\times$ span (L). The total moment at any support is ΣM_S.

For unequal spans the method is similar to that for equal spans, except for the introduction of the moment-multiplier, U, which varies with the support being treated and for the particular span loaded, and depends on the ratio that each span bears to the basic span. Thus for unequal spans $M_S = UFQ.WL$.

If one type of load extends equally over all spans of a system of equal spans, the beam need not be divided into separately loaded spans, as the values of Q tabulated for this condition apply.

Examples of the use of *Table* 25 are given on the page facing the table.

FIXED POINTS.



(a) TO DETERMINE THE POSITIONS OF THE FIXED POINTS.

(i) UNEQUAL SPANS (GRAPHICAL METHOD)

DRAW THE BEAM ABCD ... TO SCALE.

PLOT THE POSITION OF THE FIXED POINT F_1 IN LEFT-HAND PART OF SPAN AB.

IF THE BEAM IS FREELY SUPPORTED AT A, F_1 IS AT A.

IF THE BEAM IS FIXED AT A, $f_1 = 0{\cdot}333\,L_1$.

SET UP VERTICALS THROUGH THE THIRD POINTS OF EACH SPAN.

SET OFF $BE = x_2 = 0{\cdot}333(L_2 - L_1)$; $CE' = x_3 = 0{\cdot}333(L_3 - L_2)$; $DE'' = x_4 = 0{\cdot}333(L_4 - L_3)$, ETC. AND SET UP VERTICALS THROUGH E, E', E" ETC.

(IF $L_{n+1} > L_n$, x IS NEGATIVE AND SHOULD BE SET OFF ON LEFT-HAND SIDE OF SUPPORT, FOR EXAMPLE AS x_3)

DRAW GEJ AT ANY CONVENIENT ANGLE THROUGH E. JOIN F_1 G AND PRODUCE TO INTERSECT VERTICAL THROUGH B AT H. JOIN HJ, INTERSECTING BC AT F_2 WHICH IS THE FIXED POINT IN THE LEFT-HAND PART OF SPAN BC.

REPEAT THE CONSTRUCTION IN SPANS CD, DE, ETC., WORKING TO THE RIGHT-HAND END OF THE BEAM THEREBY ESTABLISHING THE LEFT-HAND FIXED POINTS F_3, F_4, ETC.

COMMENCE AT THE EXTREME RIGHT-HAND SUPPORT, AND REPEAT THE CONSTRUCTION WORKING TO THE LEFT-HAND END OF THE BEAM, THUS ESTABLISHING THE POSITION OF THE RIGHT-HAND FIXED POINTS, F_1', F_2', F_3', ETC.

(ii) UNEQUAL SPANS (ALGEBRAICAL METHOD).

IF THE DISTANCE OF THE LEFT-HAND FIXED POINT IN THE END SPAN AB IS KNOWN ($= f_1$), THE POSITION OF THE LEFT-HAND FIXED POINT IN SPAN BC IS

$$f_2 = \frac{L_2(L_1 - f_1)}{3(L_1 + L_2)(L_1 - f_1) - L_1^2} \quad \text{OR GENERALLY,} \quad f_{n+1} = \frac{L_{n+1}(L_n - f_n)}{3(L_n + L_{n+1})(L_n - f_n) - L_n^2}$$

(iii) EQUAL SPANS. ($L_1 = L_2 = L_3$, ETC. $= L$)

DISTANCE OF FIXED POINTS:	f_1	f_2	f_3	f_4, ETC.
BEAM FREELY SUPPORTED AT A:	0	0·200L	0·211L	0·211L
BEAM FIXED AT A:	0·333L	0·200L	0·212L	0·211L

(b) TO DETERMINE THE BENDING MOMENTS.

DRAW THE BEAM ABCD.... TO SCALE AND MARK THE POSITIONS OF THE FIXED POINTS F_1, F_1', F_2, F_2' ETC.

SET UP BK AND CN AT THE SUPPORTS OF THE LOADED SPAN.

$BK = \frac{6A_2 z_2}{L_2^2}$ AND $CN = \frac{6A_2(L_2 - z_2)}{L_2^2}$. FOR SYMMETRICAL LOAD $BK = CN = \frac{3A_2}{L_2}$

A_2 = AREA OF FREE-MOMENT DIAGRAM; z_2 = DISTANCE OF CENTROID OF A_2 FROM B.

JOIN KC AND BN; DRAW LINE PR THROUGH INTERSECTIONS WITH VERTICALS THROUGH F_2 AND F_2'.

COMPLETE B.M. DIAGRAM (SHOWN SHADED) BY DRAWING PF_1 PRODUCED, AND RF_3', ETC.

REPEAT FOR OTHER LOADED SPANS AND COMBINE DIAGRAMS TO GIVE TOTAL B.Ms.

For values of $\frac{A}{L}$ ($= C_{AB}$ for BK and C_{BA} for CN) and z ($= z_1 L$), see *Table* 18.

EXAMPLES OF THE USE OF TABLE 25.

(*a*) Find the maximum bending moment at the penultimate support of a beam continuous over four equal spans of 15 ft. carrying a uniformly-distributed load of 1000 lb. per lin.-ft.; a live load of 10,000 lb. can occur at the centre of one or more of the spans simultaneously.

The maximum bending moment occurs when the first, second, and fourth spans are loaded with the concentrated load. Dividing the total load into single-span loads:

Load = W.	*Span.*	*Load factor* (F).	*Support moment coefficient* (Q).	*Product* (FQW).
Uniformly distributed $W = 1000 \times 15 = 15{,}000$ lb.	All spans	1·00	− 0·107	− 1605
Concentrated load $W = 10{,}000$ lb. . .	1st span	1·50	− 0·067	− 1000
Ditto	2nd span	1·50	− 0·049	− 740
Ditto	4th span	1·50	− 0·004	− 60
			$\Sigma FQW =$	− 3405

Bending moment $= L \times \Sigma FQW = -3405 \times 15 = -51{,}075$ ft.-lb.

It is seen from the table that these results are the maximum, because if the third span is loaded a positive bending moment is introduced that reduces the total negative bending moment.

(*b*) Find the bending moment at the centre of the central span of three continuous spans of 10 ft., 15 ft. and 10 ft. respectively. The load on the central span is a uniform dead load of 7000 lb. per lin.-ft. and on each of the end spans the load is triangularly distributed, being 6000 lb. per ft. at the inner supports and zero at the outer supports at which the beam is freely supported. (This type of load occurs in a rectangular tank when the counterforts, tied in at the top, are continuous with the beams of the suspended bottom.)

First determine the bending moment at the inner supports by dividing the load into a series of loads on one span at a time and evaluating the corresponding products WL. Owing to symmetry the bending moments at each of the interior supports are equal.

$$k = \frac{\text{length of span}}{\text{base span}}, \qquad k_1 = k_2 = \frac{\text{end span}}{\text{middle span}} = \frac{10}{15} = 0{\cdot}67$$

$$x = y = 0{\cdot}67 + 1 = 1{\cdot}67$$

$$H = \frac{5}{(4 \times 1{\cdot}67 \times 1{\cdot}67) - 1} = 0{\cdot}493.$$

For 1st span loaded: $U_C = 3 \times 0{\cdot}67^2 \times 0{\cdot}493 = 0{\cdot}663$
$U_B = 0{\cdot}5 \times 1{\cdot}67 \times 0{\cdot}663 = 0{\cdot}555$

For 2nd span loaded: $U_C (= U_B$ owing to symmetry$) = 0{\cdot}493(1{\cdot}67 + 0{\cdot}67) = 1{\cdot}15$

For 3rd span loaded: Owing to symmetry $U_C = 0{\cdot}555$; $U_B = 0{\cdot}663$.

Load (W)	*Span.*	*Load factor* × *Support moment coefficient* (FQ).	*Moment multiplier* (U).	*Product* ($FQUWL$).
Triangularly distributed (apex at left-hand support) $W = 6000 \times 10 \times 0{\cdot}5 = 30{,}000$ lb.	3rd	+ 0·018	$U_B = 0{\cdot}663$	+ 5370 ft.-lb.
Uniformly distributed $W = 7000 \times 15 = 105{,}000$ lb.	2nd $L = 15$ ft.	1·00 × (− 0·050)	$U_B = 1{\cdot}15$	− 90,500 ,,
Triangularly distributed (apex at right-hand support: therefore reverse system) $W = 30{,}000$ lb.	1st	− 0·071	U_C for 3rd span loaded = 0·555	− 17,740

Net bending moment at inner support = − 102,870 ft.-lb.

Free bending moment at mid-span $= \dfrac{7000 \times 15^2}{8} = 197{,}000$ ft.-lb.

Less negative bending moment at mid-span = 102,870 ,,

Net positive bending moment at mid-span = 94,130 ,,

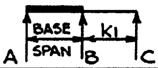
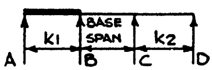
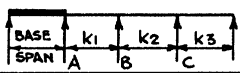
NOTES: DIVIDE GIVEN BEAM SYSTEM INTO A NUMBER OF SIMILAR SYSTEMS EACH HAVING ONE SPAN LOADED WITH A PARTICULAR TYPE OF LOAD. TO FIND THE B.M. AT ANY SUPPORT DUE TO ANY ONE OF THESE LOADS. EVALUATE THE FOLLOWING FACTORS FOR THE PARTICULAR SUPPORT AND TYPE OF LOAD:–

LOAD FACTOR = F (= UNITY FOR DISTRIBUTED LOAD).
SUPPORT MOMENT COEFFICIENT = Q.
MOMENT MULTIPLIER = U (= UNITY FOR EQUAL SPANS).

B.M. AT SUPPORT = FQU × W × BASE SPAN.

NOTE ON TRIANGULARLY DISTRIBUTED LOADS: FOR FQ USE THE VALUE GIVEN IN BRACKETS IN COLUMN HEADED "SUPPORT MOMENT COEFFICIENTS" WHEN APEX IS AT L.H. SUPPORT. REVERSE SYSTEM WHEN APEX IS AT R.H.S.

TYPE OF LOAD (W = TOTAL LOAD ON SPAN)	LOAD FACTOR = F	MAX. FREE B.M.
	1·00	·125 WL
	1·25	·107 WL
	SEE NOTE	·128 WL
	1·50	·250 WL
	1·33	·167 WL

Nº OF SPANS	LOADED SPAN	EQUAL SPANS SUPPᵀ. Mᵀ. COEFFᵀˢ Q_A	Q_B	Q_C	UNEQUAL SPANS. MOMENT MULTIPLIERS = U	
2.		-	–·063 (–·058)	-	$U_B = \frac{2}{1+k_1}$	
		-	–·063 (–·067)	-	$U_B = \frac{2k_1^2}{1+k_1}$	
	BOTH SPANS LOADED WITH IDENTICAL LOAD.	-	–·125 (–·125)	-	SEE NOTE BELOW FOR UNEQUAL SPANS SIMULTANEOUSLY LOADED.	
3.		-	–·067 (–·056)	+·017 (+·016)	$U_B = \cdot 5yU_C$ $U_C = 3k_1^2 H$	$x = k_1+1$ $y = k_2+1$ $H = \frac{5}{4xy-1}$
		-	–·050 (–·056)	–·050 (–·044)	$U_B = H(y+k_2)$ $U_C = H(x+k_1)$	
		-	+·017 (+·018)	–·067 (–·071)	$U_B = 3k_2^2 H$ $U_C = \cdot 5 x U_B$	
	ALL SPANS LOADED WITH IDENTICAL LOAD.	-	–·100 (–·092)	–·100 (–·101)	FOR TWO, THREE, OR FOUR UNEQUAL SPANS LOADED SIMULTANEOUSLY, DETERMINE B.Ms. FOR EACH SPAN LOADED SEPARATELY AND ADD.	
4.		–·067 (–·063)	+·018 (+·017)	–·004 (–·004)	$U_A = \frac{\cdot 133}{x}(14 + k_1H_1z)$ $U_B = zH_1$ $U_C = 2k_2H_1$	$x = k_1+1$ $y = k_1+k_2$ $z = k_2+k_3$ $H_1 = 14k_1Y$ $H_2 = 4\cdot 67k_1^2Y(x+1)$ $H_3 = 4\cdot 67k_2^2Y(k_3+z)$ $H_4 = 14k_3^2Y$
		–·049 (–·055)	–·054 (–·048)	+·013 (+·012)	$U_A = \frac{\cdot 543}{x}k_1(4\cdot 67k_1 - U_B)$ $U_B = zH_2$ $U_C = 2k_2H_2$	
		+·013 (+·015)	–·054 (–·060)	–·049 (–·043)	$U_A = 2k_1H_3$ $U_B = xH_3$ $U_C = \frac{\cdot 543}{z}k_2(4\cdot 67k_2 - U_B)$	
		–·004 (–·005)	+·018 (+·019)	–·067 (–·072)	$U_A = 2k_1H_4k_2$ $U_B = xH_4k_2$ $U_C = \cdot 133H_4(4xy - k_1^2)$	
	ALL SPANS LOADED WITH IDENTICAL LOAD.	–·107 (–·107)	–·071 (–·072)	–·107 (–·107)	SEE NOTE ABOVE	$Y = \frac{1}{4xyz - k_1^2z - k_3^2x}$

MOMENT DISTRIBUTION APPLIED TO CONTINUOUS BEAMS.

The notes which follow explain the basis of the formulæ and other data given in *Tables* 26 to 30.

The symbols used are given in *Table* 26. The span-load-factor F is the numerical value of the bending moment produced at the end support of a span L by the load on that span, assuming the beam to be perfectly fixed at both ends. For equal spans, arithmetical work is saved by considering only the load-factor C, $\left(=\frac{F}{L}\right)$, values of which are given in *Table* 18 for common cases of unsymmetrical and symmetrical loads; note that $F = CL$.

The distribution factor D is derived from the relative stiffness of two adjacent spans meeting at a support; for prismatic beams the stiffness of each span is proportional to the moment of inertia of the beam divided by the span. Consider any two continuous spans ST and TU having span lengths of L_{ST} and L_{TU} respectively. If the moment of inertia of span ST is I_{ST} and of span TU is I_{TU}, the stiffness factor of span ST is $\frac{I_{ST}}{L_{ST}}$ and of span TU is $\frac{I_{TU}}{L_{TU}}$. If a knife-edge support is assumed at support T, the sum of the distribution factors at this support is unity. The distribution factor of span ST at support T is

$$D_{TS} = \frac{1}{1 + \frac{I_{TU}L_{ST}}{L_{TU}I_{ST}}},$$

and the distribution factor of span TU at support T is

$$D_{TU} = \frac{1}{1 + \frac{I_{ST}L_{TU}}{L_{ST}I_{TU}}}, \text{ that is, } D_{TS} = 1 - D_{TU}.$$

Since only the ratio of the stiffness factors is involved, it is not necessary for I and L to be in the same units, and it is usually more convenient for I to be in in.^{4} units and L to be in feet. Values of I for various sections, but omitting the effect of the reinforcement, are given in *Table* 64.

At the end support A of an end span AB the distribution factor D_{AB} is zero when the beam is fixed at A and unity when freely supported at A. A cantilever beyond an end support, or a bending moment applied externally at an end support, is considered in *Table* 31, but for assessing the distribution factors in such cases the end span should be treated as freely supported at the end support.

Basic Formulæ.—By moment-distribution operations to the extent of two distributions and one carry-over, the bending moment M_T at the support T of the series of continuous spans . . . RS, ST, TU, UV . . . is given by the expression at the bottom of *Table* 27.

Contrary to the usual signs for moment distribution, the signs in this expression are such that a bending moment producing "concave downwards" deformation is considered as negative and the reverse as positive; as these signs conform to the normally-accepted convention for continuous beams and there is therefore no need to adopt a new conception for negative and positive bending moments. The substitution of the numerical values of the load-factors (or span-load-factors) in the formulæ gives the correct sign to the bending moment at the support.

From the fundamental formula for M_T, the basic formulæ for end supports (when fixed), penultimate supports, and any interior supports in systems of two, three, four, five, or more spans, are derived by considering which spans are eliminated and what effect the conditions have on the distribution factors. The basic formulæ given in *Table* 26 are the foundation of all the expressions in *Tables* 27 (two spans), 28 (three spans), 29 (four spans), and 30 (five or more spans).

An example of the direct use of the formulæ in *Table* 26 is given on page 188.

SUPPORT	APPLICATION	FORMULA FOR B.M. AT SPECIFIED SUPPORT.		REF.
END SUPPORT A	TWO OR MORE SPANS	FIXED AT A	$-F_{AB} - \frac{D_{BA}}{2}(F_{BA} - F_{BC})$	M_A
END SUPPORT C	TWO SPANS ONLY	FIXED AT C	$-F_{CB} - \frac{D_{BC}}{2}(F_{BC} - F_{BA})$	M_{C4}
CENTRAL SUPPORT B	TWO SPANS ONLY	FREE AT A & C	$-\frac{D_{BC}}{2}(2F_{BA} + F_{AB}) - \frac{D_{BA}}{2}(2F_{BC} + F_{CB})$	M_{B3}
		FIXED AT A FREE AT C	$-D_{BC}F_{BA} - \frac{D_{BA}}{2}(2F_{BC} + F_{CB})$	M_{B4}
		FIXED AT A & C	$-D_{BC}F_{BA} - D_{BA}F_{BC}$	M_{B5}
PENULTIMATE SUPPORT B.	THREE OR MORE SPANS	FREE AT A	$-\frac{D_{BC}}{2}(2F_{BA} + F_{AB}) - \frac{D_{BA}}{2}\left[2F_{BC} - D_{CB}(F_{CD} - F_{CB})\right]$	M_{B2}
		FIXED AT A	$-D_{BC}F_{BA} - \frac{D_{BA}}{2}\left[2F_{BC} - D_{CB}(F_{CD} - F_{CB})\right]$	M_{B1}
PENULTIMATE SUPPORT C.	THREE SPANS ONLY	FREE AT D	$-\frac{D_{CB}}{2}(2F_{CD} + F_{DC}) - \frac{D_{CD}}{2}\left[2F_{CB} - D_{BC}(F_{BA} - F_{BC})\right]$	M_{C2}
		FIXED AT D	$-D_{CB}F_{CD} - \frac{D_{CD}}{2}\left[2F_{CB} - D_{BC}(F_{BA} - F_{BC})\right]$	M_{C1}
END SUPPORT D.	THREE SPANS ONLY	FIXED AT D	$-F_{DC} - \frac{D_{CD}}{2}(F_{CD} - F_{CB})$	M_{D3}
PENULTIMATE SUPPORT D.	FOUR SPANS ONLY	FREE AT E	$-\frac{D_{DC}}{2}(2F_{DC} + F_{ED}) - \frac{D_{DE}}{2}\left[2F_{DC} - D_{CD}(F_{CB} - F_{CD})\right]$	M_{D2}
		FIXED AT E	$-D_{DC}F_{DE} - \frac{D_{DE}}{2}\left[2F_{DC} - D_{CD}(F_{CB} - F_{CD})\right]$	M_{D1}
END SUPPORT E.	FOUR SPANS ONLY	FIXED AT E	$-F_{ED} - \frac{D_{DE}}{2}(F_{DE} - F_{DC})$	M_E
INTERIOR SUPPORT C.	FOUR OR MORE SPANS	-	$-\frac{D_{CD}}{2}\left[2F_{CB} - D_{BC}(F_{BA} - F_{BC})\right] - \frac{D_{CB}}{2}\left[2F_{CD} - D_{DC}(F_{DE} - F_{DC})\right]$	M_{C3}
PENULTIMATE SUPPORT B.	TWO OR MORE SPANS	WITH EXTERNALLY APPLIED NEG. B.M. M_P AT A.	$+M_P \cdot \frac{D_{BC}}{2}$ NOTE:- IF M_P IS POSITIVE, B.M. AT B = $-M_P \cdot \frac{D_{BC}}{2}$	M_{B6}

SUPPORT REFERENCES:- A B C A B C D A B C D E

SYMBOLS:- L_{AB} = SPAN A-B, ETC. I_{AB}, ETC. = MOMENT OF INERTIA OF SECTION IN SPAN A·B, ETC.

D_{BA}, D_{BC}, ETC. = DISTRIBUTION FACTOR AT SUPPORT B FOR SPANS AB AND BC, ETC. $D_{BA} + D_{BC} = 1$

F_{AB}, F_{BA}, ETC. = SPAN-LOAD FACTOR AT A & B FOR LOAD ON SPAN A·B ETC.
= NUMERICAL VALUE OF B.Ms. AT A & B DUE TO LOAD ON A·B ONLY ASSUMING FIXITY AT BOTH ENDS OF SPAN.
= $L_{AB} \cdot C_{AB}$, $L_{AB} \cdot C_{BA}$ WHERE C_{AB}, ETC. = LOAD FACTOR (SEE NOTE BELOW)

$$D_{BA} = \frac{1}{1 + \frac{I_{AB}L_{BC}}{I_{BC}L_{AB}}}$$

GENERAL FORMULA FOR ANY SUPPORT T R S T U V

$$M_T = -\frac{D_{TU}}{2}\left[2F_{TS} - D_{TS}(F_{SR} - F_{ST})\right] - \frac{D_{TS}}{2}\left[2F_{TU} - D_{UT}(F_{UV} - F_{UT})\right] \qquad M_T$$

NOTE.—For values of factors C_{AB}, etc., and F_{AB}, etc., see *Table* 18.
For modified formulæ for special conditions, see the following tables.
Two spans: *Table* 27. Three spans: *Table* 28.
Four spans: *Table* 29. Five and more spans: *Table* 30.

MOMENT DISTRIBUTION APPLIED TO CONTINUOUS BEAMS.

Formulæ for Special Cases.—In practical design conditions are often such that one or more of the variants in the general case may be a constant resulting in simplification of the formulæ for bending moments at supports. If the load on every span is symmetrically disposed the span-load-factors for each support of any one span are equal. The condition of equal spans has little effect on the formulæ unless accompanied by some other constant factor. Thus the common case of equal spans with uniform section throughout all spans means that the stiffness factors for each span are identical at any interior support; the distribution factors are therefore equal on either side of the support, and each distribution factor is 0·5. Also, since the spans are equal, the load-factor can be used in place of the span-load-factor, and the numerical value of the length of the span is an overall multiplier. The common combination of symmetrical loading, equal spans, and uniform section throughout all spans combines the foregoing simplifications, and the modified expressions for these special cases are tabulated in *Tables* 27, 28, 29 and 30.

A condition that frequently occurs, for example in bridge girders, is that of symmetrical inequality of spans accompanied by symmetrical inequality of section, in which case there is equality of some of the distribution factors; the modified expressions for this condition in some systems are given in *Tables* 27 to 29. Symmetrical inequality of spans and section may be accompanied by symmetrical load, with further simplification of the formulæ also as given in the tables.

Incidental Loading.—The basic formulæ and the derivatives therefrom for special cases assume that all spans are loaded. Should any span not be loaded, zero must be substituted for the span-load-factors or load-factors for the unloaded span, and from this operation the formulæ for incidental loading are evolved to determine maximum bending moments at supports and in the spans due to various incidences of live loading. The sequence of spans which should be loaded to produce the maximum negative or positive bending moments is indicated in *Table* 20.

Condition at End Supports.—The special formulæ apply to the three conditions at end supports, namely, free support at both ends of the system of spans, free support at one end and fixity at the other, and fixity at both ends. The condition of partial restraint at one or both supports is dealt with in *Table* 31 and the page facing that table.

Application of Tables.—A problem is dealt with in the following stages.

(1) Select the table which applies to the given case as regards number of spans and support conditions at the ends.

(2) Select the group of formulæ applying to the problem as determined by any condition such as symmetrical loading, equal spans and uniform section, symmetrical inequality of spans, etc.

(3) Except in the case of uniform section throughout all spans, determine the moment of inertia of the section in each span and therefrom the stiffness factors for each span. Calculate only those distribution factors required for the group of formulæ applicable.

(4) Calculate the span-load-factors (or the load-factors if all spans are equal), using *Table* 18, for dead and live loads separately. Only those factors required for the group of formulæ which are applicable need be calculated.

(5) By direct substitution in the "all-spans-loaded" formulæ determine the bending moments at the supports for dead load only.

(6) By direct substitution in the appropriate "incidental-load" formulæ determine the bending moments at supports for live load only.

(7) If there is an externally-applied bending moment at one or both end supports, calculate the bending moments at the supports due thereto separately for dead and live load from *Table* 31.

(8) Algebraically add the results from (5), (6) and (7) to give the net bending moments at the supports; combine them with the free bending moments if the bending moments in the spans are required.

An example is given on pages 188, 190 and 192 [Example (*b*)].

Conditions	Support	Both spans loaded	Incidental loading	
CASE 2A (B.M. at end supports A and C = zero)		Max. neg. B.M. at support B	For max. pos. B.M. in span AB. For max. neg. B.M. in span BC.	For max. pos. B.M. in span BC. For max. neg. B.M. in span AB.
General case	B	$-\frac{D_{BC}}{2}(2F_{BA}+F_{AB})-\frac{D_{BA}}{2}(2F_{BC}+F_{CB})$	$-\frac{D_{BC}}{2}(2F_{BA}+F_{AB})$	$-\frac{D_{BA}}{2}(2F_{BC}+F_{CB})$
Symmetrical loading	B	$-\frac{3}{2}(D_{BC}F_{AB}+D_{BA}F_{BC})$	$-\frac{3}{2}D_{BC}F_{AB}$	$-\frac{3}{2}D_{BA}F_{BC}$
Equal spans and uniform section	B	$-\frac{L}{4}[2(C_{BA}+C_{BC})+C_{AB}+C_{CB}]$	$-\frac{L}{4}(2C_{BA}+C_{AB})$	$-\frac{L}{4}(2C_{BC}+C_{CB})$
Equal spans uniform section and symmetrical loading	B	$-\frac{3L}{4}(C_{AB}+C_{BC})$	$-\frac{3}{4}F_{BA}$	$-\frac{3}{4}F_{BC}$
CASE 2B (B.M. at end support C = zero)		Max. neg. B.M. at support B	Max. neg. B.M. at support A. For max. pos. B.M. in span AB. For max. neg. B.M. in span BC.	Max. pos. B.M. at support A. For max. pos. B.M. in span BC. For max. neg. B.M. in span AB.
General case	A	$-F_{AB}-\frac{D_{BA}}{2}(F_{BA}-F_{BC})$	$-F_{AB}-\frac{D_{BA}}{2}F_{BA}$	$+\frac{D_{BA}}{2}F_{BC}$
	B	$-D_{BC}F_{BA}-\frac{D_{BA}}{2}(2F_{BC}+F_{CB})$	$-D_{BC}F_{BA}$	$-\frac{D_{BA}}{2}(2F_{BC}+F_{CB})$
Symmetrical loading	A	$-\left(\frac{D_{BA}}{2}+1\right)F_{AB}+\frac{D_{BA}}{2}F_{BC}$	$-\left(\frac{D_{BA}}{2}+1\right)F_{AB}$	$+\frac{D_{BA}}{2}F_{BC}$
	B	$-D_{BC}F_{BA}-\frac{3}{2}D_{BA}F_{BC}$	$-D_{BC}F_{BA}$	$-\frac{3}{2}D_{BA}F_{BC}$
Equal spans and uniform section	A	$-\frac{L}{4}(4C_{AB}+C_{BA}-C_{BC})$	$-\frac{L}{4}(4C_{AB}+C_{BA})$	$+\frac{F_{BC}}{4}$
	B	$-\frac{L}{4}[2(C_{BA}+C_{BC})+C_{CB}]$	$-\frac{F_{AB}}{2}$	$-\frac{L}{4}(2C_{BC}+C_{CB})$
Equal spans uniform section and symmetrical loading	A	$-\frac{L}{4}(5C_{AB}-C_{BC})$	$-\frac{5}{4}F_{AB}$	$+\frac{F_{BC}}{4}$
	B	$-\frac{L}{4}(2C_{AB}+3C_{BC})$	$-\frac{F_{AB}}{2}$	$-\frac{3}{4}F_{BC}$
CASE 2C (B.M. at support A as case 2B)		Max. neg. B.M. at supports B&C	Max. neg. B.M. at support A. Max. pos. B.M. at support C. For max. pos. B.M. in span AB. For max. neg. B.M. in span BC.	Max. neg. B.M. at support C. Max. pos. B.M. at support A. For max. pos. B.M. in span BC. For max. neg. B.M. in span AB.
General case	B	$-D_{BC}F_{BA}-D_{BA}F_{BC}$	$-D_{BC}F_{BA}$	$-D_{BA}F_{BC}$
	C	$-F_{CB}-\frac{D_{BC}}{2}(F_{BC}-F_{BA})$	$+\frac{D_{BC}}{2}F_{BA}$	$-\frac{D_{BC}}{2}F_{BC}-F_{CB}$
Symmetrical loading	B	$-D_{BC}F_{AB}-D_{BA}F_{BC}$	$-D_{BC}F_{AB}$	$-D_{BA}F_{BC}$
	C	$-\left(\frac{D_{BC}}{2}+1\right)F_{BC}+\frac{D_{BC}}{2}F_{AB}$	$+\frac{D_{BC}}{2}F_{AB}$	$-\left(\frac{D_{BC}}{2}+1\right)F_{BC}$
Equal spans and uniform section	B	$-\frac{L}{2}(C_{BA}+C_{BC})$	$-\frac{F_{BA}}{2}$	$-\frac{F_{BC}}{2}$
	C	$-\frac{L}{4}(4C_{CB}+C_{BC}-C_{BA})$	$+\frac{F_{BA}}{4}$	$-\frac{F_{BC}}{4}-F_{CB}$
Equal spans uniform section and symmetrical loading	B	$-\frac{L}{2}(C_{AB}+C_{BC})$	$-\frac{F_{AB}}{2}$	$-\frac{F_{BC}}{2}$
	C	$-\frac{L}{4}(5C_{CB}-C_{BA})$	$+\frac{F_{AB}}{4}$	$-\frac{5}{4}F_{BC}$

Note.—For notation, see *Table* 26.

MOMENT DISTRIBUTION APPLIED TO CONTINUOUS BEAMS (*continued*).

Examples.—(*a*) Determine the bending moments at the interior supports of the beam on page 172, by means of the formulæ in *Table* 26.

$$D_{BC} = \frac{1}{1 + \frac{4000 \times 15}{6000 \times 10}} = 0{\cdot}5; \text{ therefore } D_{BA} = 1 - 0{\cdot}5 = 0{\cdot}5.$$

$$D_{CB} = \frac{1}{1 + \frac{5000 \times 15}{6000 \times 12}} = 0{\cdot}49; \text{ therefore } D_{CD} = 0{\cdot}51.$$

From *Table* 18,

$C_{BA} = 0{\cdot}100 \times 1000 \times 10 \times 0{\cdot}5 = 500$; $F_{BA} = 500 \times 10 = 5000$ ft.-lb.
$C_{AB} = 0{\cdot}067 \times 1000 \times 10 \times 0{\cdot}5 = 333$; $F_{AB} = 333 \times 10 = 3330$ ft.-lb.
$C_{BC} = C_{CB} = 0{\cdot}083 \times 1500 \times 15 = 1875$; $F_{BC} = F_{CB} = 1875 \times 15 = 28{,}125$ ft.-lb.
$C_{CD} = 0{\cdot}100 \times 1200 \times 12 \times 0{\cdot}5 = 720$; $F_{CD} = 720 \times 12 = 8640$ ft.-lb.
$C_{DC} = 0{\cdot}067 \times 1200 \times 12 \times 0{\cdot}5 = 480$; $F_{DC} = 480 \times 12 = 5760$ ft.-lb.

From *Table* 26, at interior support B (three spans, free at A):

Bending moment $= -0{\cdot}25[(2 \times 5000) + 3330] - 0{\cdot}25[(2 \times 28{,}125) - 0{\cdot}49(8640 - 28{,}125)]$
$= -3333 - 16{,}450 = -19{,}783$ ft.-lb.

At penultimate support C (three spans only, free at D):

Bending moment $= -0{\cdot}245[(2 \times 8640) + 5760] - 0{\cdot}255[(2 \times 28{,}125) - 0{\cdot}5(5000 - 28{,}125)]$
$= -5650 - 17{,}282 = -22{,}932$ ft.-lb.

(*b*). Determine the maximum negative bending moment at support B and the maximum positive bending moment in span BC for the beam shown on the Calculation Sheet (on the page facing *Table* 29) on which are given the spans, loads, and sections from which are calculated the moments of inertia, the distribution factors, and the span-load-factors for the dead and live loads separately, using the data in *Tables* 18 and 26. The moments of inertia of the sections are calculated from *Table* 64; for the tee-beam

$$\frac{b_r}{b} = \frac{6}{60} = 0{\cdot}1; \qquad \frac{d_s}{d} = \frac{5}{17} = 0{\cdot}3;$$

and the factor 0·19 is obtained by extrapolation.

It is unnecessary in this case to draw bending-moment diagrams, since the bending moments for combined dead and live load at each support can be obtained by direct summation of the bending moments calculated from the formulæ in *Table* 28, Case 3A.

(i) *Maximum negative bending moment at support B.*—Substituting the appropriate particulars of dead load from the Calculation Sheet in the formula for support B in *Table* 28 for conditions of symmetrical load (all spans loaded) and from *Table* 31 for dead load (general case, three or more spans), the bending moment due to dead load alone is

$$-\left(\frac{3}{2} \times 0{\cdot}58 \times 140{,}000\right) - \frac{0{\cdot}42}{2}[(2{\cdot}6 \times 312{,}500) - (0{\cdot}6 \times 146{,}250)] + \frac{0{\cdot}58 \times 52{,}500}{2}$$
$$= -259{,}000 \text{ in.-lb.}$$

By substitution in the formula for support B from *Table* 28 for the general case of maximum negative bending moment at supports due to incidental load (in this instance the basic formula M_{B2} in *Table* 26 with $F_{CD} = 0$), the bending moment due to the live load alone is

$$-\frac{0{\cdot}58}{2}[(2 \times 300{,}000) + 300{,}000] - \frac{0{\cdot}42}{2}[(2 \times 115{,}200) + (0{\cdot}6 \times 172{,}800)] = -331{,}150 \text{ in.-lb.}$$

From *Table* 31 it is seen that, for the maximum negative bending moment at support B, the effect of the live load on the cantilever should be omitted. Thus the bending moment at support B is $-(259{,}000 + 331{,}150) = -590{,}150$ in.-lb.

(ii) *Maximum positive bending moment in span BC.*—Calculate the minimum bending moments at supports B and C and combine the resulting negative bending-moment diagram with the positive freely-supported-span diagram. For dead load alone the negative bending moment at support B is, as already calculated, 259,000 in.-lb. For support C the substitution in the formulæ for dead load in *Tables* 28 and 31 gives

$$-\left(\frac{3}{2} \times 0{\cdot}60 \times 146{,}250\right) - \frac{0{\cdot}40}{2}[(2{\cdot}58 \times 312{,}500) - (0{\cdot}58 \times 140{,}000)]$$
$$- \frac{0{\cdot}40 \times 0{\cdot}58 \times 52{,}500}{4} = -280{,}000 \text{ in.-lb.}$$

(*Continued on page* 190.)

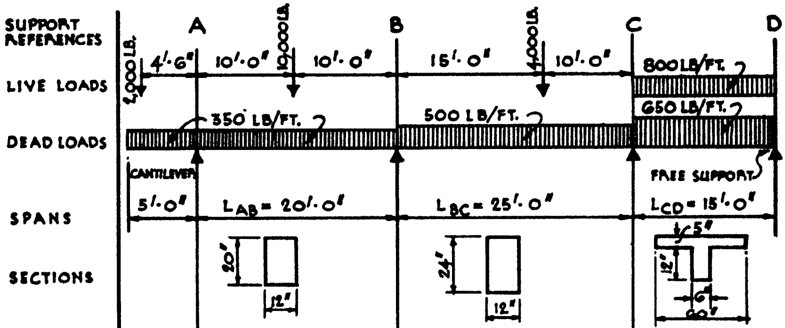
CONTINUOUS BEAMS: MOMENT-DISTRIBUTION METHOD.—TABLE 28.
THREE SPANS.

CONDITIONS	SUPPORT	ALL SPANS LOADED	INCIDENTAL LOADING			
CASE 3A (B M AT END SUPPORTS A AND D = ZERO)		B M AT SUPPORTS B & C	MAX NEG B M AT SUPPORTS. NOTE: THESE B Ms DO NOT OCCUR SIMULTANEOUSLY.	MAX POS B.M. AT SUPPORTS. NOTE: THESE B Ms DO NOT OCCUR SIMULTANEOUSLY.	FOR MAX POS. B.M IN SPANS AB & CD. FOR MAX. NEG. B.M. IN SPAN BC	FOR MAX. POS. B.M. IN SPAN BC. FOR MAX. NEG. B.M. IN SPANS AB & CD
GENERAL	B C	BASIC FORMULA M_{B2} BASIC FORMULA M_{C2}	M_{B2} WITH $F_{CD}=0$ M_{C2} WITH $F_{BA}=0$	$+\frac{D_{BA}D_{CB}}{2}F_{CD}$ $+\frac{D_{CD}D_{BC}}{2}F_{BA}$	$\left.\begin{matrix}M_{B2}\\M_{C2}\end{matrix}\right\}$ WITH $F_{BC}=F_{CB}=0$	$-\frac{D_{BA}}{2}(2F_{BC}+D_{CB}F_{CB})$ $-\frac{D_{CD}}{2}(2F_{CB}+D_{BC}F_{BC})$
SYMMETRICAL LOADING	B C	$-\frac{3}{2}D_{BC}F_{AB}-\frac{D_{BA}}{2}[(2+D_{CB})F_{BC}-D_{CB}F_{CD}]$ $-\frac{3}{2}D_{CB}F_{CD}-\frac{D_{CD}}{2}[(2+D_{BC})F_{BC}-D_{BC}F_{AB}]$	$-\frac{3D_{BC}}{2}F_{AB}-\frac{D_{BA}}{2}(2+D_{CB})F_{BC}$ $-\frac{3D_{CB}}{2}F_{CB}-\frac{D_{CD}}{2}(2+D_{BC})F_{BC}$	$+\frac{D_{BA}D_{CB}}{2}F_{CD}$ $+\frac{D_{CD}D_{BC}}{2}F_{AB}$	$-\frac{3}{2}D_{BC}F_{AB}+\frac{D_{BA}D_{CB}}{2}F_{CD}$ $-\frac{3}{2}D_{CB}F_{CD}+\frac{D_{CD}D_{BC}}{2}F_{AB}$	$-\frac{D_{BA}}{2}(2+D_{CB})F_{BC}$ $-\frac{D_{CD}}{2}(2+D_{BC})F_{BC}$
EQUAL SPANS AND UNIFORM SECTION	B C	$-\frac{L}{8}[4(C_{BA}+C_{BC})+2C_{AB}+C_{CB}-C_{CD}]$ $-\frac{L}{8}[4(C_{CD}+C_{CB})+2C_{DC}+C_{BC}-C_{BA}]$	$-\frac{L}{8}[4(C_{BA}+C_{BC})+2C_{AB}+C_{CB}]$ $-\frac{L}{8}[4(C_{CD}+C_{CB})+2C_{DC}+C_{BC}]$	$+\frac{F_{CD}}{8}$ $+\frac{F_{BA}}{8}$	$-\frac{L}{8}(4C_{BA}+2C_{AB}-C_{CD})$ $-\frac{L}{8}(4C_{CD}+2C_{DC}-C_{BA})$	$-\frac{L}{8}(4C_{BC}+C_{CB})$ $-\frac{L}{8}(4C_{CB}+C_{BC})$
EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING	B C	$-\frac{L}{8}(6C_{AB}+5C_{BC}-C_{CD})$ $-\frac{L}{8}(6C_{CD}+5C_{BC}-C_{AB})$	$-\frac{L}{8}(6C_{AB}+5C_{BC})$ $-\frac{L}{8}(6C_{CD}+5C_{BC})$	$+\frac{F_{CD}}{8}$ $+\frac{F_{AB}}{8}$	$-\frac{L}{8}(6C_{AB}-C_{CD})$ $-\frac{L}{8}(6C_{CB}-C_{AB})$	$-\frac{5F_{BC}}{8}$ $-\frac{5F_{BC}}{8}$
SYMMETRICAL INEQUALITY OF SPANS AND SECTIONS	B C	$\left.\begin{matrix}M_{B2}\\M_{C2}\end{matrix}\right\}$ WITH $D_{CB}=D_{BC}$, $D_{CD}=D_{BA}$	$-\frac{D_{BC}}{2}(2F_{BA}+F_{AB})-\frac{D_{BA}}{2}(2F_{BC}+D_{BC}F_{CB})$ $-\frac{D_{BC}}{2}(2F_{CD}+F_{DC})-\frac{D_{BA}}{2}(2F_{CB}+D_{BC}F_{BC})$	$+\frac{D_{BA}D_{BC}}{2}F_{CD}$ $+\frac{D_{BA}D_{BC}}{2}F_{BA}$	$-\frac{D_{BC}}{2}(2F_{BA}+F_{AB}-D_{BA}F_{CD})$ $-\frac{D_{BC}}{2}(2F_{CD}+F_{DC}-D_{BA}F_{BA})$	$-\frac{D_{BA}}{2}(2F_{BC}+D_{BC}F_{CB})$ $-\frac{D_{BA}}{2}(2F_{CD}+D_{BC}F_{BC})$
SYMMETRICAL INEQUALITY OF SPANS AND SECTIONS AND SYMMETRICAL LOAD	B C	$-\frac{D_{BC}}{2}[3F_{AB}+D_{BA}(F_{BC}-F_{CD})]-D_{BA}F_{BC}$ $-\frac{D_{BC}}{2}[3F_{CD}+D_{CD}(F_{BC}-F_{AB})]-D_{BA}F_{BC}$	$-\frac{D_{BC}}{2}(3F_{AB}+D_{BA}F_{BC})-D_{BA}F_{BC}$ $-\frac{D_{BC}}{2}(3F_{CD}+D_{CD}F_{BC})-D_{BA}F_{BC}$	$+\frac{D_{BA}D_{BC}}{2}F_{CD}$ $+\frac{D_{BA}D_{BC}}{2}F_{AB}$	$-\frac{D_{BC}}{2}(3F_{AB}-D_{BA}F_{CD})$ $-\frac{D_{BC}}{2}(3F_{CD}-D_{BA}F_{AB})$	$-\frac{D_{BA}}{2}(2+D_{BC})F_{BC}$ $-\frac{D_{BA}}{2}(2+D_{BC})F_{BC}$
CASE 3B (B Ms AT A AS FOR CASE 2B; B.Ms AT C AS FOR CASE 3A; B Ms. AT END SUPPORT D = ZERO)		B.M AT PENULTIMATE SUPPORT B	MAX. NEG. B.M. AT SUPPORT B.	MAX POS B M. AT SUPPORT B	FOR MAX. POS. B.M. IN SPAN AB & CD. FOR MAX NEG. B M. IN SPAN BC.	FOR MAX POS B.M. IN SPAN BC. FOR MAX NEG. B.M. IN SPANS AB & CD
GENERAL	B	$-D_{BC}F_{BA}-\frac{D_{BA}}{2}[2F_{BC}-D_{CB}(F_{CD}-F_{CB})]$	$-D_{BC}F_{BA}-\frac{D_{BA}}{2}(2F_{BC}+D_{CB}F_{CB})$	$+\frac{D_{BA}D_{CB}}{2}F_{CD}$	$-D_{BC}F_{BA}+\frac{D_{BA}D_{CB}}{2}F_{CD}$	$-\frac{D_{BA}}{2}(2F_{BC}+D_{CB}F_{CB})$
SYMMETRICAL LOADING	B	$-D_{BC}F_{AB}-\frac{D_{BA}}{2}[(2+D_{CB})F_{BC}-D_{CB}F_{CD}]$	$-D_{BC}F_{AB}-\frac{D_{BA}}{2}(2+D_{CB})F_{BC}$	$+\frac{D_{BA}D_{CB}}{2}F_{CD}$	$-D_{BC}F_{AB}+\frac{D_{BA}D_{CB}}{2}F_{CD}$	$-\frac{D_{BA}}{2}(2+D_{CB})F_{BC}$
EQUAL SPANS AND UNIFORM SECTION	B	$-\frac{L}{8}[4(C_{BA}+C_{BC})+C_{CB}-C_{CD}]$	$-\frac{L}{8}[4(C_{BA}+C_{BC})+C_{CB}]$	$+\frac{F_{CD}}{8}$	$-\frac{L}{8}(4C_{BA}-C_{CD})$	$-\frac{L}{8}(4C_{BC}+C_{CB})$
EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING	B	$-\frac{L}{8}(4C_{AB}+5C_{BC}-C_{CD})$	$-\frac{L}{8}(4C_{AB}+5_{BC})$	$+\frac{F_{CD}}{8}$	$-\frac{L}{8}(4C_{AB}-C_{CD})$	$-\frac{5}{8}F_{BC}$
CASE 3C (B Ms AT SUPPORT A (WHERE EXCLUDED) AS CASE 2B; B. Ms. AT SUPPORT B (WHERE EXCLUDED) AS CASE 3B)		B M AT SUPPORTS C & D (AND A & B)	MAX NEG B M AT SUPPORTS. NOTE. THESE B Ms DO NOT OCCUR SIMULTANEOUSLY	MAX POS B M AT SUPPORTS NOTE THESE B Ms. DO NOT OCCUR SIMULTANEOUSLY	FOR MAX. POS. B.M. IN SPANS AB & CD. FOR MAX. NEG. B M IN SPAN BC	FOR MAX. POS B M IN SPAN BC. FOR MAX NEG B M IN SPANS AB & CD
GENERAL CASE	C D	$-D_{CB}F_{CD}-\frac{D_{CD}}{2}[2F_{CB}-D_{BC}(F_{BA}-F_{BC})]$ $-F_{DC}-\frac{D_{CD}}{2}(F_{CD}-F_{CB})$	$-D_{CB}F_{CD}-\frac{D_{CD}}{2}(2F_{CB}+D_{BC}F_{BC})$ $-F_{DC}-\frac{D_{CD}}{2}F_{CD}$	$+\frac{D_{CD}D_{BC}}{2}F_{BA}$ $+\frac{D_{CD}}{2}F_{CB}$	$-D_{CB}F_{CD}+\frac{D_{CD}D_{BC}}{2}F_{BA}$ $-F_{DC}-\frac{D_{CD}}{2}F_{CD}$	$-\frac{D_{CD}}{2}(2F_{CB}+D_{BC}F_{BC})$ $+\frac{D_{CD}}{2}F_{CB}$
SYMMETRICAL LOADING	C D	$-D_{CB}F_{CD}-\frac{D_{CD}}{2}[(2+D_{BC})F_{BC}-D_{BC}F_{AB}]$ $-\left(1+\frac{D_{CD}}{2}\right)F_{CD}+\frac{D_{CD}}{2}F_{BC}$	$-D_{CB}F_{CD}-\frac{D_{CD}}{2}(2+D_{BC})F_{BC}$ $-\left(1+\frac{D_{CD}}{2}\right)F_{CD}$	$+\frac{D_{CD}D_{BC}}{2}F_{AB}$ $+\frac{D_{CD}}{2}F_{BC}$	$-D_{CB}F_{CD}+\frac{D_{CD}D_{BC}}{2}F_{AB}$ $-F_{CD}-\frac{D_{CD}}{2}F_{CD}$	$-\frac{D_{CD}}{2}(2+D_{BC})F_{BC}$ $+\frac{D_{CD}}{2}F_{BC}$
EQUAL SPANS AND UNIFORM SECTION	C D	$-\frac{L}{8}[4(C_{CD}+C_{CB})+C_{BC}-C_{BA}]$ $-\frac{L}{4}(4C_{DC}+C_{CD}-C_{CB})$	$-\frac{L}{8}[4(C_{CD}+C_{CB})+C_{BC}]$ $-\frac{L}{4}(4C_{DC}+C_{CD})$	$+\frac{F_{BA}}{8}$ $+\frac{F_{CB}}{4}$	$-\frac{L}{8}(4C_{CD}-C_{BA})$ $-\frac{L}{4}(4C_{DC}+C_{CD})$	$-\frac{L}{8}(4C_{CB}+C_{BC})$ $+\frac{F_{BC}}{4}$
EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING	C D	$-\frac{L}{8}(4C_{CD}+5C_{BC}-C_{AB})$ $-\frac{L}{4}(5C_{DC}-C_{CB})$	$-\frac{L}{8}(4C_{CD}+5C_{BC})$ $-\frac{5}{4}F_{DC}$	$+\frac{F_{AB}}{8}$ $+\frac{F_{BC}}{4}$	$-\frac{L}{8}(4C_{CD}-C_{AB})$ $-\frac{5F_{CD}}{4}$	$-\frac{5F_{BC}}{8}$ $+\frac{F_{BC}}{4}$
SYMMETRICAL INEQUALITY OF SPANS AND SECTIONS	A B C D	$-F_{AB}-\frac{D_{BA}}{2}(F_{BA}-F_{BC})$ $-D_{BC}F_{BA}-\frac{D_{BA}}{2}[2F_{BC}-D_{BC}(F_{CD}-F_{CB})]$ $-D_{BC}F_{CD}-\frac{D_{BA}}{2}[2F_{CB}-D_{BC}(F_{BA}-F_{BC})]$ $-F_{DC}-\frac{D_{BA}}{2}(F_{CD}-F_{CB})$	$-F_{AB}-\frac{D_{BA}}{2}F_{BA}$ $-D_{BC}F_{BA}-\frac{D_{BA}}{2}(2F_{BC}+D_{BC}F_{CB})$ $-D_{BC}F_{CD}-\frac{D_{BA}}{2}(2F_{CB}+D_{BC}F_{BC})$ $-F_{DC}-\frac{D_{BA}}{2}F_{CD}$	$+\frac{D_{BA}}{2}F_{BC}$ $+\frac{D_{BA}D_{BC}}{2}F_{CD}$ $+\frac{D_{BA}D_{BC}}{2}F_{BA}$ $+\frac{D_{BA}}{2}F_{CB}$	$-F_{AB}-\frac{D_{BA}}{2}F_{BA}$ $-D_{BC}\left(F_{BA}+\frac{D_{BA}}{2}F_{CD}\right)$ $-D_{BC}\left(F_{CD}+\frac{D_{BA}}{2}F_{BA}\right]$ $-F_{DC}-\frac{D_{BA}}{2}F_{CD}$	$+\frac{D_{BA}}{2}F_{BC}$ $-\frac{D_{BA}}{2}(2F_{BC}+D_{BC}F_{CB})$ $-\frac{D_{BA}}{2}(2F_{CB}+D_{BC}F_{BC})$ $+\frac{D_{BA}}{2}F_{CB}$

NOTE.—For notation and basic formulæ M_{B_2} and M_{C_2}, see *Table* 26. For Case 2B, see *Table* 27.

MOMENT-DISTRIBUTION APPLIED TO CONTINUOUS BEAMS (*continued*).

Examples (*continued from page* 188).



MOMENTS OF INERTIA		$I_{AB}=\frac{12\times20^3}{12}=8{,}000$ in.^{4}	$I_{BC}=\frac{12\times24^3}{12}=13{,}824$ in.^{4}	$I_{CD}=0.19\times6\times17^3$ $=5{,}570$ in.^{4}
STIFFNESS FACTORS		$\frac{8{,}000}{20}=400$	$\frac{13{,}824}{25}=553$	$\frac{5{,}570}{15}=371$
DISTRIBUTION FACTORS		$D_{BA}=\frac{1}{1+\frac{553}{400}}=0.42$	$D_{BC}=1-0.42=0.58$; $D_{CB}=\frac{1}{1+\frac{371}{553}}=0.60$	$D_{CD}=1-0.60=0.40$
DEAD LOAD SPAN-LOAD FACTORS		$F_{AB}=F_{BA}=\frac{350\times20^2\times12}{12}=140{,}000$ in.lb.	$F_{BC}=F_{CB}=\frac{500\times25^2\times12}{12}=312{,}500$ in.lb.	$F_{CD}=F_{DC}=\frac{650\times15^2\times12}{12}=146{,}250$ in.lb.
LOAD ON CANTILEVER	$M_P=-$ $=-$	$\frac{350\times5\cdot0^2\times12}{2}$ $52{,}500$ in.lb.		
LIVE LOAD SPAN-LOAD FACTORS		F_{AB} (Symmetrical Loading) F_{BA} $F_{AB}=F_{BA}=\frac{10{,}000\times20\times12}{8}$ $=300{,}000$ in.lb.	F_{BC} (Unsymmetrical Loading) F_{CB} $a=\frac{15}{25}=0\cdot6$ $F_{BC}=\cdot6\times\cdot4^2\times4000\times25\times12$ $=115{,}200$ in.lb. $F_{CB}=\cdot6^2\times\cdot4\times4000\times25\times12$ $=172{,}800$ in.lb.	F_{CD} (Symmetrical Loading) F_{DC} $F_{CD}=F_{DC}$ $=\frac{800\times15^2\times12}{12}$ $=180{,}000$ in.lb.
LOAD ON CANTILEVER	$M_P=-$ $=-$	$2000\times4\cdot5\times12$ $108{,}000$ in.lb.		

For the live-load bending moments the formulæ in the column for maximum positive bending moment in span BC in *Table* 28 are used. From *Table* 31, the effect of the live load on the cantilever is calculated. Substitution gives

Support B: $-\frac{0\cdot42}{2}[(2\times115{,}200)+(0\cdot60\times172{,}800)]+\frac{0\cdot58\times108{,}000}{2}=-38{,}800$ in.-lb.

Support C: $-\frac{0\cdot40}{2}[(2\times172{,}800)+(0\cdot58\times115{,}200)]-\frac{0\cdot4\times0\cdot58\times108{,}000}{4}$

$=-88{,}800$ in.-lb.

(*Continued on page* 192.)

CONDITIONS	SUPPORT	ALL SPANS LOADED	INCIDENTAL LOADING			
CASE 4A (B.M. AT B AS CASE 3A (WHERE EXCLUDED)) (B.M. AT END SUPPORTS A AND E = ZERO)		B. Ms AT SUPPORTS C & D (AND B)	MAX. NEG. B.M. AT SUPPORTS. NOTE: THESE B.Ms DO NOT OCCUR SIMULTANEOUSLY.	MAX POS B.M. AT SUPPORTS. NOTE: THESE B.Ms DO NOT OCCUR SIMULTANEOUSLY	FOR MAX. POS. B.M. IN SPAN CD. FOR MAX. NEG. B.M. IN SPANS BC & DE	FOR MAX. POS. B.M. IN SPANS BC & DE. FOR MAX. NEG. B.M. IN SPAN CD.
GENERAL CASE	C	BASIC FORMULA M_{C3}	BASIC FORMULA M_{C3} WITH $F_{BA} = F_{DE} = 0$	$+\frac{D_{CD}D_{BC}}{2}F_{BA} + \frac{D_{CB}D_{DC}}{2}F_{DE}$	$+\frac{D_{CB}D_{BC}}{2}F_{BA} - \frac{D_{CB}}{2}(2F_{CB} + D_{DC}F_{DC})$	$-\frac{D_{CD}}{2}(2F_{CB} + D_{BC}F_{BC}) + \frac{D_{CB}D_{DC}}{2}F_{ED}$
	D	BASIC FORMULA M_{D2}	BASIC FORMULA M_{D2} WITH $F_{CB} = F_{BC} = 0$	$+\frac{D_{DE}D_{CD}}{2}F_{CB}$	$-\frac{D_{DE}}{2}(2F_{DC} + D_{CD}F_{CD})$	$-\frac{D_{DC}}{2}(2F_{DE} + F_{ED}) + \frac{D_{DE}D_{CD}}{2}F_{CB}$
SYMMETRICAL LOADING	C	BASIC FORMULA M_{C3} WITH $F_{BA} = F_{AB}$, $F_{CB} = F_{BC}$, $F_{DC} = F_{CD}$, $F_{ED} = F_{DE}$	$-\frac{D_{CD}}{2}(2+D_{BC})F_{BC} - \frac{D_{CB}}{2}(2+D_{DC})F_{CD}$	$+\frac{D_{CD}D_{BC}}{2}F_{AB} + \frac{D_{CB}D_{DC}}{2}F_{DE}$	$+\frac{D_{CD}D_{BC}}{2}F_{AB} - \frac{D_{CB}}{2}(2+D_{DC})F_{CD}$	$-\frac{D_{CD}}{2}(2+D_{BC})F_{BC} + \frac{D_{CB}}{2}F_{DE}$
	D	BASIC FORMULA M_{D2}	$-\frac{3D_{DC}}{2}F_{DE} - \frac{D_{DE}}{2}(2+D_{CD})F_{CD}$	$+\frac{D_{DE}D_{CD}}{2}F_{BC}$	$-\frac{D_{DE}}{2}(2+D_{CD})F_{CD}$	$-\frac{3D_{DC}}{2}F_{DE} + \frac{D_{DE}D_{CD}}{2}F_{BC}$
EQUAL SPANS AND UNIFORM SECTION	C	$-\frac{L}{8}[4(C_{CB}+C_{CD}) - C_{BA} + C_{BC} + C_{DC} - C_{DE}]$	$-\frac{L}{8}[4(C_{CB}+C_{CD}) + C_{BC} + C_{DC}]$	$+\frac{L}{8}(C_{BA}+C_{DE})$	$-\frac{L}{8}(4C_{CD} - C_{BA} + C_{DC})$	$-\frac{L}{8}(4C_{CB} + C_{BC} - C_{DE})$
	D	$-\frac{L}{8}[4(C_{DE}+C_{DC}) + 2_{ED} - C_{CB} + C_{CD}]$	$-\frac{L}{8}[4(C_{DE}+C_{DC}) + 2C_{ED} + C_{CB}]$	$+\frac{F_{CB}}{8}$	$-\frac{L}{8}(4C_{DC} + C_{CD})$	$-\frac{L}{8}(4C_{DE} + 2C_{ED} - C_{CB})$
EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING	C	$-\frac{L}{8}[5(C_{BC}+C_{CD}) - C_{AB} - C_{DE}]$	$-\frac{5L}{8}(C_{CB}+C_{CD})$	$+\frac{L}{8}(C_{AB}+C_{DE})$	$-\frac{L}{8}(5C_{CD} - C_{AB})$	$-\frac{L}{8}(5C_{BC} - C_{DE})$
	D	$-\frac{L}{8}(6C_{DE} + 5C_{CD} - C_{BC})$	$-\frac{L}{8}(6C_{DE} + 5C_{CD})$	$+\frac{F_{BC}}{8}$	$-\frac{5}{8}F_{CD}$	$-\frac{L}{8}(6C_{DE} - C_{CB})$
SYMMETRICAL INEQUALITY OF SPANS AND SECTIONS	B	BASIC FORMULA M_{B2}	$-\frac{D_{BC}}{2}(2F_{BA}+F_{AB}) - \frac{D_{BA}}{2}(2F_{BC}+D_{CB}F_{CB})$	$+\frac{D_{BA}D_{CB}}{2}F_{CD}$	$-\frac{D_{BA}}{2}(2F_{BA}+F_{AB}) + \frac{D_{BA}D_{CB}}{2}F_{CD}$	$-\frac{D_{BA}}{2}(2F_{BC} + D_{CB}F_{CB})$
	C	BASIC FORMULA M_{C3} WITH $D_{DC} = D_{BC}$, $D_{DE} = D_{BA}$, $D_{CD} = D_{CB}$	$-\frac{D_{CB}}{2}[D_{BC}(F_{BC}+F_{DC}) + 2(F_{CB}+F_{CD})]$	$+\frac{D_{CB}D_{BC}}{2}(F_{BA}+F_{DE})$	$-\frac{D_{CB}}{2}[D_{BC}(F_{DC}-F_{BA}) + 2F_{CD}]$	$-\frac{D_{CB}}{2}[D_{BC}(F_{BC}-F_{DE}) + 2F_{CB}]$
	D	BASIC FORMULA M_{D2}	$-\frac{D_{DC}}{2}(2F_{DE}+F_{ED}) - \frac{D_{BA}}{2}(2F_{DC}+D_{CB}F_{CB})$	$+\frac{D_{BA}D_{CB}}{2}F_{CB}$	$-\frac{D_{BA}}{2}(2F_{DC} + D_{CB}F_{CD})$	$-\frac{D_{DC}}{2}(2F_{DE}+F_{ED}) + \frac{D_{BA}D_{CB}}{2}F_{CB}$
CASE 4B (B.M. AT SUPPORT E = ZERO.)		B MS AT SUPPORTS A, B, C & D	MAX NEG B M AT SUPPORTS	MAX POS B.M AT SUPPORTS	FOR MAX. POS. B.M. IN SPANS AB & CD. FOR MAX. NEG. B.M. IN SPANS BC & DE.	FOR MAX. POS. B.M. IN SPANS BC & DE. FOR MAX. NEG. B.M. IN SPANS AB & CD.
GENERAL CASE	A	BASIC FORMULA M_A	UNDER THESE CONDITIONS FOR 'INCIDENTAL LOADING' THE FORMULÆ FOR THE B.M. AT EACH SUPPORT ARE AS FOLLOWS. SUPPORT A – CASE 2B; SUPPORT B – CASE 3B; SUPPORT C – CASE 4A; SUPPORT D – CASE 4A			
	B	BASIC FORMULA M_{B1}				
	C	BASIC FORMULA M_{C3}				
	D	BASIC FORMULA M_{D2}				
SYMMETRICAL LOADING	–	UNDER THESE CONDITIONS FOR 'ALL SPANS LOADED' THE FORMULÆ FOR B.M. AT EACH SUPPORT ARE AS GIVEN BY THE CORRESPONDING CASES REFERRED TO FOR 'INCIDENTAL LOADING.'				
EQUAL SPANS AND UNIFORM SECTION	–					
EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING	–					
CASE 4C (B.M. AT A AS CASE 2B) (B.M. AT B AS CASE 3B. B.M. AT C AS CASE 4A (WHERE EXCLUDED))		B. Ms AT SUPPORTS D & E (AND B & C)	MAX NEG. B.M AT SUPPORTS. NOTE: THESE B.Ms. DO NOT OCCUR SIMULTANEOUSLY.	MAX. POS B M AT SUPPORTS. NOTE: THESE B Ms DO NOT OCCUR SIMULTANEOUSLY	FOR MAX. POS. B.M. IN SPAN CD. FOR MAX. NEG. B.M. IN SPAN DE.	FOR MAX. POS. B.M. IN SPAN DE. FOR MAX. NEG. B.M. IN SPAN CD.
GENERAL CASE	D	BASIC FORMULA M_{D1}	$-D_{DC}F_{DE} - \frac{D_{DE}}{2}(2F_{DC} + D_{CD}F_{CD})$	$+\frac{D_{DE}D_{CD}}{2}F_{CB}$	$-\frac{D_{DE}}{2}(2F_{DC} + D_{CD}F_{CD})$	$-D_{DC}F_{DE} + \frac{D_{DE}D_{CD}}{2}F_{CB}$
	E	$-F_{ED} - \frac{D_{DE}}{2}(F_{DE} - F_{DC})$	$-F_{ED} - \frac{D_{DE}}{2}F_{DE}$	$+\frac{D_{DE}}{2}F_{DC}$	$+\frac{D_{DE}}{2}F_{DC}$	$-F_{ED} - \frac{D_{DE}}{2}F_{DE}$
SYMMETRICAL LOADING	D	$-D_{DC}F_{DE} - \frac{D_{DE}}{2}[(2+D_{DC})F_{DC} - D_{CD}F_{BC}]$	$-D_{DC}F_{DE} - \frac{D_{DE}}{2}(2+D_{DC})F_{CD}$	$+\frac{D_{DE}D_{CD}}{2}F_{BC}$	$-\frac{D_{DE}}{2}(2+D_{CD})F_{CD}$	$-D_{DC}F_{DE} + \frac{D_{DE}D_{CD}}{2}F_{BC}$
	E	$-(1+\frac{D_{DE}}{2})F_{DE} + \frac{D_{DE}}{2}F_{DC}$	$-(1+\frac{D_{DE}}{2})F_{DE}$	$+\frac{D_{DE}}{2}F_{CD}$	$+\frac{D_{DE}}{2}F_{CD}$	$-(1+\frac{D_{DE}}{2})F_{DE}$
EQUAL SPANS AND UNIFORM SECTION	D	$-\frac{L}{8}[4(C_{DE}+C_{DC}) - C_{CB} + C_{CD}]$	$-\frac{L}{8}[4(C_{DE}+C_{DC}) + C_{CD}]$	$+\frac{F_{CB}}{8}$	$-\frac{L}{8}(4C_{DC} + C_{CD})$	$-\frac{L}{8}(4C_{DE} - C_{CB})$
	E	$-\frac{L}{4}(4C_{ED} + C_{DE} - C_{DC})$	$-\frac{L}{4}(4C_{ED} + C_{DE})$	$+\frac{F_{DC}}{4}$	$+\frac{F_{DC}}{4}$	$-\frac{L}{4}(4C_{ED} + C_{DE})$
EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING	D	$-\frac{L}{8}(5C_{CD} + 4C_{DE} - C_{BC})$	$-\frac{L}{8}(5C_{CD} + 4C_{DE})$	$+\frac{F_{BC}}{8}$	$-\frac{5F_{DC}}{8}$	$-\frac{L}{8}(4C_{DE} - C_{BC})$
	E	$-\frac{L}{4}(5C_{ED} - C_{DC})$	$-\frac{5}{4}F_{DE}$	$+\frac{F_{CD}}{4}$	$+\frac{F_{CD}}{4}$	$-\frac{5}{4}F_{DE}$
SYMMETRICAL INEQUALITY OF SPANS AND SECTIONS	B	BASIC FORMULA M_{B1}	$-D_{BC}F_{BA} - \frac{D_{BA}}{2}(2F_{BC} + D_{CB}F_{CB})$	$+\frac{D_{BA}D_{CB}}{2}F_{CD}$	$-D_{BC}F_{BA} + \frac{D_{BA}D_{CB}}{2}F_{CD}$	$-\frac{D_{BA}}{2}(2F_{BC} + D_{CB}F_{CB})$
	C	BASIC FORMULA M_{C3} WITH $D_{DE} = D_{BA}$, $D_{DC} = D_{BC}$, $D_{CD} = D_{CB}$	$-\frac{D_{CB}}{2}[D_{BC}(F_{BC}+F_{DC}) + 2(F_{CB}+F_{CD})]$	$+\frac{D_{CB}D_{BC}}{2}(F_{BA}+F_{DE})$	$-\frac{D_{CB}}{2}[D_{BC}(F_{DC}+F_{BA}) + 2F_{CD}]$	$-\frac{D_{CB}}{2}[D_{BC}(F_{BC}-F_{DE}) + 2F_{CB}]$
	D	BASIC FORMULA M_{D1}	$-D_{BC}F_{DE} - \frac{D_{BA}}{2}(2F_{DC} + D_{CB}F_{CD})$	$+\frac{D_{BA}D_{CB}}{2}F_{CB}$	$-\frac{D_{BA}}{2}(2F_{DC} + D_{CB}F_{CD})$	$-D_{BC}F_{DE} + \frac{D_{BA}D_{CB}}{2}F_{CB}$
	E	$-F_{ED} - \frac{D_{BA}}{2}(F_{DE} - F_{DC})$	$-F_{ED} - \frac{D_{DE}}{2}F_{DE}$	$+\frac{D_{DE}}{2}F_{DC}$	$+\frac{D_{DE}}{2}F_{DC}$	$-F_{ED} - \frac{D_{DE}}{2}F_{DE}$

NOTE.—For notation and basic formulæ, see *Table* 26.
For Case 2B, see *Table* 27.
For Cases 3A and 3B, see *Table* 28.

MOMENT DISTRIBUTION APPLIED TO CONTINUOUS BEAMS (*continued*).

Examples (*continued from page* 190).

For maximum positive bending moment on span BC the support moments are therefore

Support B: $-(259{,}000 + 38{,}800) = -297{,}800$ in.-lb.
Support C: $-(280{,}000 + 88{,}800) = -368{,}800$ in.-lb.

From the loading diagram it is clear that the position of maximum positive bending moment on span BC is at the point of application of the live concentrated load, at which point the free bending moment due to the dead and live loads is

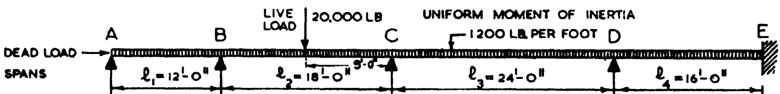
$$\frac{500 \times 15 \times 10 \times 12}{2} + \frac{4000 \times 15 \times 10 \times 12}{25} = 738{,}000 \text{ in.-lb.}$$

At this point the negative moment is

$$-\frac{1}{25}[(297{,}800 \times 10) + (368{,}800 \times 15)] = -340{,}400 \text{ in.-lb.}$$

Therefore the maximum positive bending moment is $738{,}000 - 340{,}400 = 397{,}600$ in.-lb.

(*c*).—The example in the following shows the method of calculating the bending moments on a continuous beam by direct application of the principles of moment distribution. The tables are used only to the extent of applying the expressions for fixed-end moments (*Table* 18) and distribution factors (*Table* 26).



	A	B	B	C	C	D	D	E
DISTRIBUTION FACTORS	1	3/5	2/5	4/7	3/7	2/5	3/5	0
DEAD LOAD F E M ÷ 100	−144	−144	−324	−324	−576	−576	−256	−256
1st DISTRIBUTION	+144	−108	+ 72	−144	+108	+128	−192	0
1st BALANCE	0	−252	−252	−468	−468	−448	−448	−256
1st CARRY-OVER	+ 54	− 72	+ 72	− 36	− 64	− 54	0	+ 96
2nd DISTRIBUTION	− 54	+ 86	− 56	− 16	+ 12	+ 22	− 32	0
2nd BALANCE	0	−238	−238	−520	−520	−480	−480	−160
2nd CARRY-OVER	− 43	+ 27	+ 8	+ 29	− 11	− 6	0	− 16
3rd DISTRIBUTION	+ 43	− 11	+ 8	− 23	+ 17	+ 2	− 4	0
3rd BALANCE	0	−222	−222	−514	−514	−484	−484	−176
LIVE LOAD F E M ÷ 100	0	0	−450	−450	0	0	0	0
1st DISTRIBUTION	0	−270	+180	+257	−193	0	0	0
1st BALANCE	0	−270	−270	−193	−193	0	0	0
1st CARRY-OVER	+135	0	−129	− 90	0	+ 97	0	0
2nd DISTRIBUTION	−135	− 77	+ 52	+ 51	− 39	− 39	+ 58	0
2nd BALANCE	0	−347	−347	−232	−232	+ 58	+ 58	0
2nd CARRY-OVER	+ 39	+ 68	− 26	− 26	+ 20	+ 20	0	− 29
3rd DISTRIBUTION	− 39	− 56	+ 38	+ 26	− 20	− 8	+ 12	0
3rd BALANCE	0	−335	−335	−232	−232	+ 70	+ 70	− 29

MAXIMUM COMBINED BENDING MOMENTS FOOT-POUND	A	B	C	D	E
	0	−55,700	−74,600	−48,400	−20,500

IF FIXED AT END SUPPORT: B.M. AT A AS CASE 2B. B.M. AT B AS CASE 3B.

IF FREE AT END SUPPORT: B.M. AT A = ZERO. B.M. AT B AS CASE 3A

CASE 5

CASE 5	ANY INTERNAL SUPPORT T (EXCEPT PENULTIMATE SUPPORT): ALL SPANS LOADED (e.g. DEAD LOAD)	INCIDENTAL LOADING: MAX. NEGATIVE B.M.	INCIDENTAL LOADING: MAX. POSITIVE B.M.
GENERAL	BASIC FORMULA M_T	$-\frac{D_{TU}}{2}(2F_{TS}+D_{ST}F_{ST})$ $-\frac{D_{TS}}{2}(2F_{TU}+D_{UT}F_{UT})$	$+\frac{D_{TU}D_{ST}}{2}F_{SR}+\frac{D_{TS}D_{UT}}{2}F_{UV}$
SYMMETRICAL LOADING	BASIC FORMULA M_T WITH $F_{TS}=F_{ST}$ AND $F_{UT}=F_{TU}$	$-\frac{D_{TU}}{2}(2+D_{ST})F_{TS}$ $-\frac{D_{TS}}{2}(2+D_{UT})F_{TU}$	$+\frac{D_{TU}D_{ST}}{2}F_{RS}+\frac{D_{TS}D_{UT}}{2}F_{UV}$
EQUAL SPANS AND UNIFORM SECTION	$-\frac{L}{8}\left[4(C_{TS}+C_{TU})-C_{SR}+C_{ST}-C_{UV}+C_{UT}\right]$	$-\frac{L}{8}\left[4(C_{TS}+C_{TU})+C_{ST}+C_{UT}\right]$	$+\frac{L}{8}(C_{SR}+C_{UV})$
EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING	$-\frac{L}{8}\left[5(C_{ST}+C_{TU})-C_{RS}-C_{UV}\right]$	$-\frac{5L}{8}(C_{ST}+C_{TU})$	$+\frac{L}{8}(C_{RS}+C_{UV})$

IF SPAN ST IS PENULTIMATE SPAN D_{RS} = UNITY IF FREE AT R = ZERO IF FIXED AT R

ANY INTERNAL SPAN ST	SUPPORT	INCIDENTAL LOADING: FOR MAX. POS. B.M. IN SPAN	INCIDENTAL LOADING: FOR MAX. NEG. B.M. IN SPAN
GENERAL	S	$+\frac{D_{ST}D_{RS}}{2}F_{RQ}-\frac{D_{SR}}{2}(2F_{ST}+D_{TS}F_{TS})$	$+\frac{D_{SR}D_{TS}}{2}F_{TU}-\frac{D_{ST}}{2}(2F_{SR}+D_{RS}F_{RS})$
	T	$+\frac{D_{TS}D_{UT}}{2}F_{UV}-\frac{D_{TU}}{2}(2F_{TS}+D_{ST}F_{ST})$	$+\frac{D_{TU}D_{ST}}{2}F_{SR}-\frac{D_{TS}}{2}(2F_{TU}+D_{UT}F_{UT})$
SYMMETRICAL LOADING	S	$+\frac{D_{ST}D_{RS}}{2}F_{QR}-\frac{D_{SR}}{2}(2+D_{TS})F_{ST}$	$+\frac{D_{SR}D_{TS}}{2}F_{TU}-\frac{D_{ST}}{2}(2+D_{RS})F_{RS}$
	T	$+\frac{D_{TS}D_{UT}}{2}F_{UV}-\frac{D_{TU}}{2}(2+D_{ST})F_{ST}$	$+\frac{D_{TU}D_{ST}}{2}F_{SR}-\frac{D_{TS}}{2}(2+D_{UT})F_{TU}$
EQUAL SPANS AND UNIFORM SECTION	S	$-\frac{L}{8}(4C_{ST}+C_{TS}-C_{RQ})$	$-\frac{L}{8}(4C_{SR}+C_{RS}-C_{TU})$
	T	$-\frac{L}{8}(4C_{TS}+C_{ST}-C_{UV})$	$-\frac{L}{8}(4C_{TU}+C_{UT}-C_{SR})$
EQUAL SPANS UNIFORM SECTION AND SYMMETRICAL LOADING	S	$-\frac{L}{8}(5C_{ST}-C_{QR})$	$-\frac{L}{8}(5C_{RS}-C_{TU})$
	T	$-\frac{L}{8}(5C_{ST}-C_{UV})$	$-\frac{L}{8}(5C_{TU}-C_{RS})$

BASIC FORMULÆ

$$M_T = -\frac{D_{TU}}{2}\left[2F_{TS}-D_{ST}(F_{SR}-F_{ST})\right]-\frac{D_{TS}}{2}\left[2F_{TU}-D_{UT}(F_{UV}-F_{UT})\right]$$

NOTE.—For notation, see *Table* 26.
For Case 2B, see *Table* 27.
For Cases 3A and 3B, see *Table* 28.

CONTINUOUS BEAMS.—END RESTRAINT.

General Case.—If a beam cantilevers beyond an end support, or is monolithic with an external column or other construction, the effect is to produce at the end support a bending moment equal to, and with the same sign as, the cantilever (or equivalent) bending moment. Ignoring any other load on the beam, the bending moments at the penultimate support due to either a negative or positive externally-applied bending moment are given in the upper part of *Table* 31 for sequence of unequal spans. The factors are based on moment distribution. To the extent of two distributions, an externally-applied bending moment affects only the end and penultimate supports, but with three distributions a small bending moment occurs at the next interior support. The bending moment at the penultimate support is unaffected whether two or three distributions are made. Thus the formulæ in *Table* 31 relate to three distributions for support C and either two or three distributions for support B.

Equal Spans.—The coefficients in the lower part of *Table* 31 apply to a series of equal spans at one or both end supports of which a bending moment is applied. The effect on the bending moments and shearing forces are taken into account. It must be noted that the units for this part of the table are as follows.

Bending moment.—The units of the bending moment at any support are the same as those in which the externally-applied bending moment is expressed.

Shearing force.—If the externally-applied bending moment is in *inch-pound* units and the span is in *feet*, the coefficients are such that the adjustments to the shearing forces are in *pounds*. If the externally-applied bending moment is in *foot-pound* units and the span is in *feet*, the coefficients must be multiplied by 12 to give the shearing forces in *pounds*.

Example.—If a negative bending moment of 200,000 in.-lb. is applied at one end support of the beam in Example (*a*) opposite *Table* 25, find the resultant maximum bending moment at the penultimate support.

Bending moment at penultimate support, − 51,075 ft.-lb.	= − 612,900 in.-lb.
Bending moment at support B (four equal spans) due to bending moment applied at one end only (*Table* 31, lower part), + 0·268 × 200,000	= + 53,600 ,,
Resultant net bending moment	= − 559,300 ,,

BENDING MOMENTS APPLIED AT END SUPPORTS.

APPROXIMATE METHOD

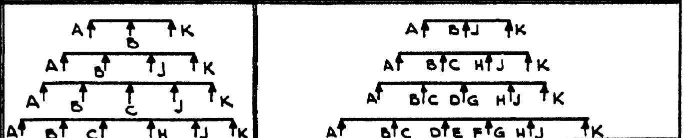
		Conditions	Support	Dead load (or similar): Three or more spans (A B C D E)	Dead load: Two spans (A B C)	Dead load: Two spans (A B C, C fixed)	Incidental loading: Max neg. B.M. at support B. Max pos. B.M. in span AB. Max neg. B.M. in span BC.	Incidental loading: Max pos. B.M. at support B. Max neg. B.M. in span AB. Max pos. B.M. in span BC.
Applied moments	$-M_p$	General case	A	$-M_p$	$-M_p$	$-M_p$	Omit effect of $-M_p$ due to incidental load	All support B.Ms. as for dead load
			B	$+\frac{D_{BC}M_p}{2}$	$+\frac{D_{BC}M_p}{2}$	$+\frac{D_{BC}M_p}{2}$		
			C	$-\frac{D_{CD}D_{BC}M_p}{4}$	Zero	$-\frac{D_{BC}M_p}{4}$		
		Equal spans and uniform section	A	$-M_p$	$-M_p$	$-M_p$		
			B	$+\frac{M_p}{4}$	$+\frac{M_p}{4}$	$+\frac{M_p}{4}$		
			C	$-\frac{M_p}{16}$	Zero	$-\frac{M_p}{8}$		
	$+M_p$	General case	A	$+M_p$	$+M_p$	$+M_p$	All support B.Ms. as for dead load	Omit effect of $+M_p$ due to incidental load
			B	$-\frac{D_{BC}M_p}{2}$	$-\frac{D_{BC}M_p}{2}$	$-\frac{D_{BC}M_p}{2}$		
			C	$+\frac{D_{CD}D_{BC}M_p}{4}$	Zero	$+\frac{D_{BC}M_p}{4}$		
		Equal spans and uniform section	A	$+M_p$	$+M_p$	$+M_p$		
			B	$-\frac{M_p}{4}$	$-\frac{M_p}{4}$	$-\frac{M_p}{4}$		
			C	$+\frac{M_p}{16}$	Zero	$+\frac{M_p}{8}$		

TABULATED DATA ARE RESULTS OF THREE DISTRIBUTIONS.
M_p = NUMERICAL VALUE OF EXTERNALLY-APPLIED BENDING MOMENT AT SUPPORT A.
D_{BC}, ETC., = DISTRIBUTION FACTOR FOR SPAN BC, ETC., AT SUPPORT B, ETC.
(SEE PRECEDING TABLES FOR EVALUATION)

EQUAL SPANS (MORE EXACT COEFFICIENTS)

–	No. of spans	B.M. A	B.M. B	B.M. C	B.M. H	B.M. J	B.M. K	S.F. A	S.F. B	S.F. C	S.F. D	S.F. E	S.F. F	S.F. G	S.F. H	S.F. J	S.F. K
Bending moment applied at A only.	2	−1·00	+·250	–	–	–	NIL	+·104	−·104	–	–	–	–	–	–	−·021	+·021
	3	−1·00	+·267	–	–	−·067	NIL	+·106	−·106	−·028	–	–	–	–	+·028	+·006	−·006
	4	−1·00	+·268	−·071	–	+·018	NIL	+·106	−·106	−·028	+·028	–	–	+·007	−·007	−·002	+·002
	5	−1·00	+·268	−·072	+·019	−·005	NIL	+·106	−·106	−·028	+·028	+·008	−·008	−·002	+·002	+·0004	−·0004
Equal B.M. applied at A and K	2	−1·00	+·500	–	–	–	−1·00	+·125	−·125	–	–	–	–	–	–	−·125	+·125
	3	−1·00	+·200	–	–	+·200	−1·00	+·100	−·100	NIL	–	–	–	–	NIL	−·100	+·100
	4	−1·00	+·286	−·142	–	+·286	−1·00	+·107	−·107	−·036	+·036	–	–	+·033	−·036	−·107	+·107
	5	−1·00	+·263	−·053	−·053	+·263	−1·00	+·105	−·105	−·026	+·026	NIL	NIL	+·026	−·026	−·105	+·105

KEY. 2, 3, 4, 5



NOTES

ADJUSTMENT TO B.M. = (COEFFICIENT) × (APPLIED BENDING MOMENT)

ADJUSTMENT TO SHEARING FORCE (LB.) = COEFFICIENT × $\left(\frac{\text{APPLIED B.M. IN INCH-LB.}}{\text{SPAN IN FEET}}\right)$

CONTINUOUS BEAMS AS MEMBERS OF A FRAME.

In ordinary buildings it is not necessary to consider the interaction of the columns and beams more accurately than by the application of the formulæ recommended in the B.S. Code for the bending moments on exterior and interior columns (see *Table* 46). The formulæ in *Table* 32 are suitable for more complex cases and are based on moment distribution. They are applicable only to vertical loading. (The effect of sway is excluded, so the method requires amplification if large out-of-balance loads act. The effects of wind and other lateral loads are dealt with as in *Table* 51.) The method is similar to that for continuous beams (on knife-edge supports) in *Tables* 26 to 31, but the stiffness of the supports is taken into account. The same convention of signs and notation is used. The B.S. Code recommends that the remote ends of members continuous with the span considered be assumed to be fixed, an assumption which simplifies the formulæ; therefore, as regards any interior span ST, the ends of the beams at R and U, the ends of the lower columns at O and X, and the ends of the upper columns at P and Y are assumed to be fixed.

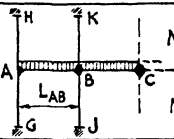
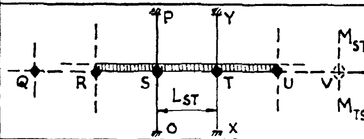
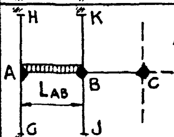
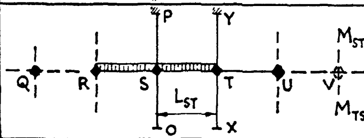
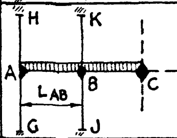
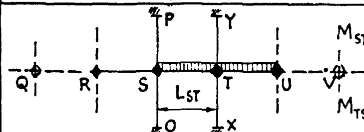
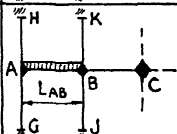
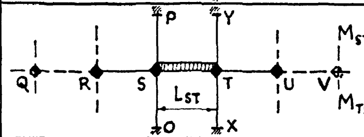
Interior Spans.—The stiffness factors and distribution factors are given in *Table* 32, in which the expressions for the bending moments are the result of two distribution operations and one intermediate carry-over. The bending moments at S and T in any interior span ST are due to vertical loads on interior spans RS, ST, and TU only, since loads on spans QR and UV and beyond do not affect S and T in this limited moment-distribution operation. The formulæ for the case of all spans being loaded apply to the dead load on an interior span. To produce the greatest negative bending moments at support S due to live load it is necessary to omit the load from span TU. Similarly, for support T, the live load on span RS is omitted. To produce the maximum positive bending moment on span ST due to live load, it is necessary to load span ST only. The formulæ for these conditions of live loading are also given in *Table* 32.

End Spans.—The formulæ for any interior span ST are re-written to apply to an end span AB by substituting A, B, C, etc., for S, T, U, etc.; A is the end support, and there is no span corresponding to RS; the modified stiffness and distribution factors are given in *Table* 32, together with the formulæ for all spans being loaded and also the formulæ for the maximum bending moments at the supports due to live load and the formulæ for the bending moments at the supports for the incidence of live load giving the maximum positive bending moment in end span AB.

When the bending moments at the supports are known, the positive and negative bending moments in the spans are obtained by combining the diagram of the free bending moments due to the load with the diagram of the corresponding support bending moments.

CONTINUOUS BEAMS
SUPPORT BENDING MOMENTS

BEAMS MONOLITHIC WITH SUPPORTS

	CONDITIONS	END SPAN AB		INTERIOR SPAN ST	
DEAD LOAD	ALL SPANS LOADED		$M_{AB}=-\left[F_{AB}-\frac{D_{BA}}{2}(F_{BC}-F_{BA})\right](1-D_{AB})$ $M_{BA}=-\left(F_{BA}+\frac{D_{AB}}{2}F_{AB}\right)(1-D_{BA})-D_{BA}F_{BC}$		$M_{ST}=-\left[F_{ST}-\frac{D_{TS}}{2}(F_{TU}-F_{TS})\right](1-D_{ST})-D_{ST}F_{SR}$ $M_{TS}=-\left[F_{TS}-\frac{D_{ST}}{2}(F_{SR}-F_{ST})\right](1-D_{TS})-D_{TS}F_{TU}$
LIVE LOAD	MAXIMUM BENDING MOMENT AT L.H. SUPPORT		$M_{AB}=-\left(F_{AB}+\frac{D_{BA}}{2}F_{BA}\right)(1-D_{AB})$ $M_{BA}=-\left(F_{BA}+\frac{D_{AB}}{2}F_{AB}\right)(1-D_{BA})$		$M_{ST}=-\left(F_{ST}+\frac{D_{TS}}{2}F_{TS}\right)(1-D_{ST})-D_{ST}F_{SR}$ $M_{TS}=-\left[F_{TS}-\frac{D_{ST}}{2}(F_{SR}-F_{ST})\right](1-D_{TS})$
	MAXIMUM BENDING MOMENT AT R.H. SUPPORT		$\left.\begin{matrix}M_{AB}\\M_{BA}\end{matrix}\right\}$ FORMULÆ AS FOR "ALL SPANS LOADED"		$M_{ST}=-\left[F_{ST}-\frac{D_{TS}}{2}(F_{TU}-F_{TS})\right](1-D_{ST})$ $M_{TS}=-\left(F_{TS}+\frac{D_{ST}}{2}F_{ST}\right)(1-D_{TS})-D_{TS}F_{TU}$
	B.Ms AT SUPPORTS TO PRODUCE MAXIMUM POSITIVE B.M. ON SPAN		$\left.\begin{matrix}M_{AB}\\M_{BA}\end{matrix}\right\}$ FORMULÆ AS FOR "MAX. B.M AT L.H. SUPPORT"		$M_{ST}=-\left(F_{ST}+\frac{D_{TS}}{2}F_{TS}\right)(1-D_{ST})$ $M_{TS}=-\left(F_{TS}+\frac{D_{ST}}{2}F_{ST}\right)(1-D_{TS})$
STIFFNESS FACTORS (K) AND DISTRIBUTION FACTORS (D)		$K_{AB}=\frac{I_{AB}}{L_{AB}}$ $K_{AG}=\frac{I_{AG}}{L_{AG}}$ $K_{AH}=\frac{I_{AH}}{L_{AH}}$ $K_{BC}=\frac{I_{BC}}{L_{BC}}$ $K_{BJ}=\frac{I_{BJ}}{L_{BJ}}$ $K_{BK}=\frac{I_{BK}}{L_{BK}}$	$D_{AB}=\frac{K_{AB}}{K_{AB}+K_{AG}+K_{AH}}$ $D_{BA}=\frac{K_{AB}}{K_{AB}+K_{BC}+K_{BJ}+K_{BK}}$	$K_{RS}=\frac{I_{RS}}{L_{RS}}$ $K_{ST}=\frac{I_{ST}}{L_{ST}}$ $K_{SO}=\frac{I_{SO}}{L_{SO}}$ $K_{SP}=\frac{I_{SP}}{L_{SP}}$ $K_{TU}=\frac{I_{TU}}{L_{TU}}$ $K_{TX}=\frac{I_{TX}}{L_{TX}}$ $K_{TY}=\frac{I_{TY}}{L_{TY}}$	$D_{ST}=\frac{K_{ST}}{K_{RS}+K_{ST}+K_{SO}+K_{SP}}$ $D_{TS}=\frac{K_{ST}}{K_{ST}+K_{TU}+K_{TX}+K_{TY}}$
NOTATION		F_{AB}, ETC. = NUMERICAL VALUE OF FIXED-END B.M. (NEG) AT A (ETC) DUE TO LOAD ON AB (ETC)		L_{AB}, ETC. = LENGTH OF MEMBER AB, ETC. I_{AB}, ETC. = MOMENT OF INERTIA OF MEMBER AB, ETC.	

BEAMS WITH SPLAYS AT THE SUPPORTS.

The formulæ in *Tables* 26 to 32 apply to beams in which the moment of inertia is uniform throughout any span but may differ in each span. If the moment of inertia varies within the span, for example, beams with splays as illustrated in *Table* 33, the fixed-end moments (the span-load-factor), the carry-over factor, and the stiffness of the beam are affected, each of these items being increased if the moment of inertia is greater at the support than at the middle of the span as in the case a beam with haunches at the supports.

The span-load-factors for supports A and B of a beam AB with non-uniform moment of inertia can be expressed as $F'_{AB} = G_{AB}C_{AB}L_{AB}$ and $F'_{BA} = G_{BA}C_{BA}L_{AB}$ respectively. Therefore the coefficient G_{AB} is $\frac{F'_{AB}}{C_{AB}L_{AB}}$ and G_{BA} is $\frac{F'_{BA}}{C_{BA}L_{AB}}$ and, if the magnitudes of these coefficients are known, F'_{AB} and F'_{BA} can be calculated from C_{AB} and C_{BA} for a beam of uniform moment of inertia (see *Table* 18). (For a beam of uniform moment of inertia $G_{AB} = G_{BA} = 1$.) For a symmetrical beam with symmetrical load $G_{AB} = G_{BA}$.

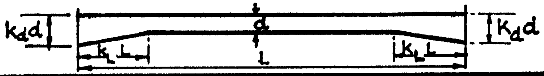
The carry-over factor CO_{AB} for support A is defined as the moment produced at the fixed end B of a beam AB when unit moment is applied at the freely-supported end A. (If the moment of inertia of the beam is uniform throughout the span, $CO_{AB} = CO_{BA} = 0{\cdot}5$.) For a symmetrical beam $CO_{AB} = CO_{BA}$.

The stiffness of a beam is the moment which when applied at the freely-supported end A produces unit slope at A if the beam is fixed at B. (For a beam of uniform moment of inertia the stiffness is $\frac{4EI_{AB}}{L_{AB}}$.)

For a beam with straight splays of the same size at each support (k_d and k_L being factors representing the dimensions of the splays), the values of coefficients $G_{AB} = G_{BA}$ (for uniformly-distributed load and for a central concentrated load), carry-over factors $CO_{AB} = CO_{BA}$, and stiffness coefficients K_{AB} are given in *Table* 33 for beams with splays of common proportions and symmetrical loads. Intermediate values can be interpolated (plotting is recommended).

The alteration of the span-load-factors, the carry-over factors, and the stiffness due to variation in the moment of inertia affect the basic and derived formulæ for the bending moments at the supports of a beam continuous over two or more spans. The general formula for the bending moment at any interior support C (except a penultimate support) are given in *Table* 33, together with the formulæ for the bending moments at other supports such as the supports at a fixed end and penultimate supports.

SYMMETRICAL SPLAYS. SYMMETRICAL LOAD



BENDING MOMENTS AT SUPPORTS

SUPPORT		BENDING MOMENTS	APPLICATION
ANY INTERIOR SUPPORT C (EXCEPT PENULTIMATE)		$-D_{CD}\left[F'_{BC}-CO_{BC}D_{BC}(F'_{AB}-F'_{BC})\right]-D_{CB}\left[F'_{CD}-CO_{CD}D_{DC}(F'_{DE}-F'_{CD})\right]$	FOUR OR MORE SPANS
END SUPPORT A (FIXED)		$-F'_{AB}-CO_{AB}D_{BA}(F'_{AB}-F'_{BC})$	TWO OR MORE SPANS
PENULTIMATE SUPPORT B	FIXED AT A	$-D_{BC}F'_{AB}-D_{BA}\left[F'_{BC}-CO_{BC}D_{CB}(F'_{CD}-F'_{BC})\right]$	THREE OR MORE SPANS
	FREE AT A	$-D_{BC}F'_{AB}(1+CO_{AB})-D_{BA}\left[F'_{BC}-CO_{BC}D_{CB}(F'_{CD}-F'_{BC})\right]$	
MIDDLE SUPPORT B	FIXED AT A AND C	$-D_{BC}F'_{AB}-D_{BA}F'_{BC}$	TWO SPANS ONLY
	FREE AT A AND C	$-D_{BC}F'_{AB}(1+CO_{AB})-D_{BA}F'_{BC}(1+CO_{BC})$	

SYMBOLS

D_{BA}, D_{BC}, ETC. = DISTRIBUTION FACTOR AT SUPPORT B FOR SPANS AB AND BC, ETC.

$$D_{BA} = \frac{1}{1+\dfrac{K_{AB}I_{AB}L_{BC}}{K_{BC}I_{BC}L_{AB}}} = 1 - D_{BC}$$

I_{AB}, I_{BC}, ETC. = MOMENT OF INERTIA AT MIDDLE OF SPANS AB, BC, ETC.

K_{AB}, K_{BC}, ETC. = STIFFNESS COEFFICIENTS OF SPANS AB AND BC, ETC.

L_{AB}, L_{BC}, ETC. = LENGTH OF SPANS AB, BC, ETC.

CO_{AB}, CO_{BC}, = CARRY-OVER FACTORS FOR SPANS AB AND BC.

C_{AB}, C_{BC}, ETC. = LOAD FACTOR AT A AND B (SYMMETRICAL LOAD ON SPAN AB) AND AT B AND C (SYMMETRICAL LOAD ON SPAN BC), ETC.

F'_{AB}, F'_{BC}, ETC. = SPAN-LOAD FACTOR FOR SUPPORTS A AND B FOR LOAD ON SPAN AB, AND B AND C FOR SPAN BC ETC.

= $G_{AB}\cdot L_{AB}\cdot C_{AB}$; $G_{BC}\cdot L_{BC}\cdot C_{BC}$; ETC.

= NUMERICAL VALUE OF B.Ms. AT A AND B ASSUMING BOTH ENDS FIXED.

SUPPORT REFERENCES:— TWO SPANS A B C; THREE OR MORE SPANS A B C D.

FACTORS

LENGTH OF SPLAY k_L	$k_d = 1\cdot4$ CO	K STIFFNESS	G		$k_d = 1\cdot6$ CO	K STIFFNESS	G		$k_d = 2\cdot0$ CO	K STIFFNESS	G	
0·1	0·55	1·21	1·06	1·07	0·57	1·28	1·08	1·09	0·59	1·39	1·11	1·12
0·2	0·59	1·44	1·11	1·13	0·62	1·63	1·15	1·17	0·66	1·95	1·19	1·22
0·3	0·61	1·64	1·13	1·17	0·65	2·01	1·18	1·23	0·71	2·71	1·24	1·31

INFLUENCE LINES FOR CONTINUOUS BEAMS.

For the determination of the bending moments at any of the critical sections in a system of beams due to a train of loads in any given position, the procedure is as follows:

Draw the beam system to a convenient linear scale.

With the ordinates given in *Tables* 34, 35, 36 or 37 construct the influence line (for unit load) for the section to be considered, selecting a convenient scale for the bending moment.

Plot on this diagram the train of loads in what appears to be the most adverse position.

Tabulate the value of (ordinate × load) for each load.

Algebraically add the products (ordinate × load) to give the resultant bending moment at the section considered.

Repeat for other positions of loads to ensure that the most adverse position has been assumed.

The following example shows the direct use of the tabulated influence lines for calculating the bending moments on a continuous beam subjected to concentrated loads in specified positions.

Example.—Determine the bending moment at the penultimate (left-hand end) support of a system of four spans (constant moment of inertia, freely supported on end supports) subject to a central load of 10 tons on the first and third spans (counting from left-hand end); the end spans are 20 ft. and the interior spans 30 ft.; hence the span-ratio is 1 : $1\frac{1}{2}$: $1\frac{1}{2}$: 1.

With load on 1st span (ordinate *c*): ft.-lb.

Bending moment = − (0·082 × 10 × 2240 × 20) = 36,800 (neg.)

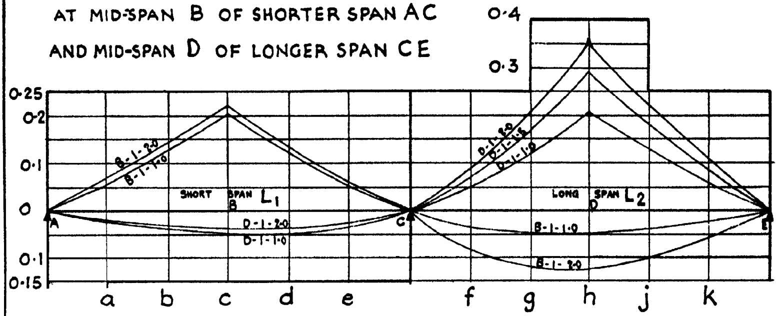
With load on 3rd span (ordinate *m*):

Bending moment = + (0·035 × 10 × 2240 × 20) = 15,700 (pos.)

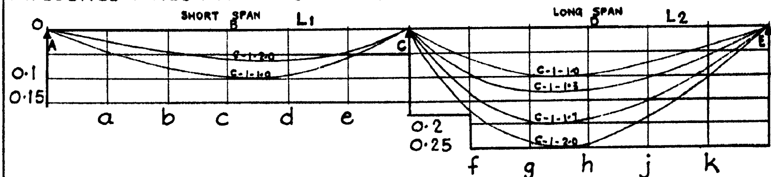
Net bending moment at penultimate support = 21,100 (neg.)

TWO SPANS (EQUAL OR UNEQUAL)

INFLUENCE LINES FOR BENDING MOMENTS
AT MID-SPAN B OF SHORTER SPAN AC
AND MID-SPAN D OF LONGER SPAN CE



INFLUENCE LINES FOR BENDING MOMENTS AT CENTRAL SUPPORT C



SECTION	RATIO OF SPANS $L_1:L_2$	ORDINATES									
		SHORTER SPAN					LONGER SPAN				
		a	b	c	d	e	f	g	h	j	k
SHORTER SPAN MIDSPAN B	1:1	·063	·130	·203	·121	·052	·032	·046	·047	·037	·020
	1:1½	·067	·137	·213	·130	·058	·058	·083	·084	·067	·037
	1:2	·070	·142	·219	·136	·062	·085	·124	·125	·099	·054
CENTRAL SUPPORT C	1:1	·041	·074	·094	·093	·064	·064	·093	·094	·074	·041
	1:1½	·032	·059	·075	·074	·051	·115	·167	·169	·133	·073
	1:2	·027	·049	·063	·062	·042	·170	·247	·250	·198	·108
LONGER SPAN MIDSPAN D	1:1	·020	·037	·047	·046	·032	·052	·121	·203	·130	·063
	1:1½	·016	·030	·038	·037	·025	·067	·167	·291	·183	·088
	1:2	·014	·025	·031	·031	·021	·082	·210	·375	·235	·113

UNEQUAL SPANS.

DATA ENABLES INFLUENCE LINES TO BE DRAWN FOR THE BENDING MOMENTS PRODUCED BY A SINGLE UNIT LOAD MOVING OVER TWO UNEQUAL SPANS.

ORDINATES FOR INTERMEDIATE RATIOS OF SPANS CAN BE INTERPOLATED.

EQUAL SPANS.

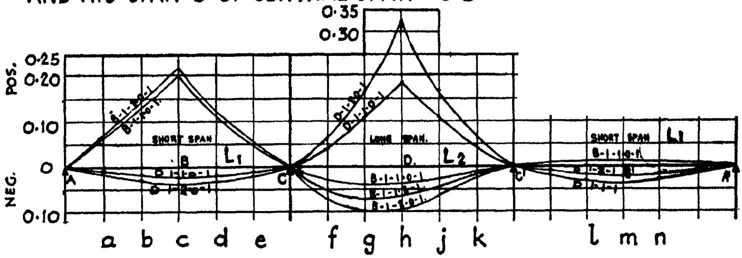
INFLUENCE LINES MARKED 1 – 1·0 CAN BE USED DIRECTLY. (DIAGRAM OF A SUCCESSION OF LOADS MUST BE DRAWN TO THE SAME LINEAR SCALE.)

BENDING MOMENT DUE TO LOAD W CONCENTRATED AT ANY POINT.

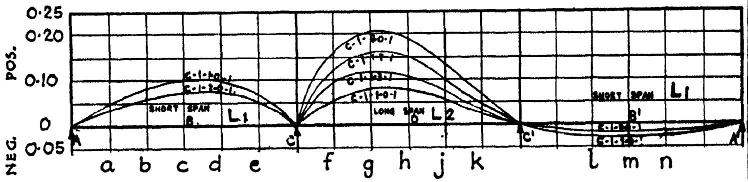
= (ORDINATE OF APPROPRIATE INFLUENCE LINE) × (SHORTER SPAN L_1) × W.

THREE SPANS (EQUAL OR SYMMETRICAL INEQUALITY)

INFLUENCE LINES FOR BENDING MOMENTS
AT MID-SPAN B OF END SPAN AC
AND MID-SPAN D OF CENTRAL SPAN CC'



INFLUENCE LINES FOR BENDING MOMENTS
AT INTERIOR SUPPORT C.



SECTION	RATIO OF SPANS	ORDINATES												
		END SPAN AC					CENTRAL SPAN CC'					END SPAN C'A'		
	$L_1:L_2:L_1$	a	b	c	d	e	f	g	h	j	k	l	m	n
MID-SPAN B OF END SPAN AC	1 : 1 : 1	0·062	·127	.200	·117	·050	.029	·040	·038	·027	·013	·012	·013	·010
	1 : $1\frac{1}{2}$: 1	0·066	·134	·209	·126	·056	·051	·070	·065	·046	·021	·012	·012	·010
	1 : 2 : 1	0·068	·139	·215	·132	·060	·075	·102	·094	·065	·029	·012	·012	·009
INTERIOR SUPPORT C	1 : 1 : 1	0·043	·079	·100	·099	·068	·057	·079	·075	·054	·026	·025	·025	·020
	1 : $1\frac{1}{2}$: 1	0·036	·065	·082	·081	·056	·102	·139	·130	·092	·042	·024	·025	·020
	1 : 2 : 1	0·030	·056	·070	·069	·048	·151	·204	·188	·129	·058	·023	·023	·019
MID-SPAN D OF CENTRAL SPAN CC'	1 : 1 : 1	0·016	·030	·038	·037	·025	·042	·100	·175	·100	·042	·037	·038	·030
	1 : $1\frac{1}{2}$: 1	0·013	·023	·029	·028	·020	·053	·135	·245	·135	·053	·028	·029	·023
	1 : 2 : 1	0·010	·019	·023	·023	·016	·063	·167	·313	·167	·063	·023	·023	·019

UNEQUAL SPANS.

DATA ENABLES INFLUENCE LINES TO BE DRAWN FOR BENDING MOMENTS PRODUCED BY A SINGLE UNIT LOAD MOVING OVER THREE UNEQUAL SPANS.

ORDINATES FOR INTERMEDIATE RATIO OF SPANS CAN BE INTERPOLATED.

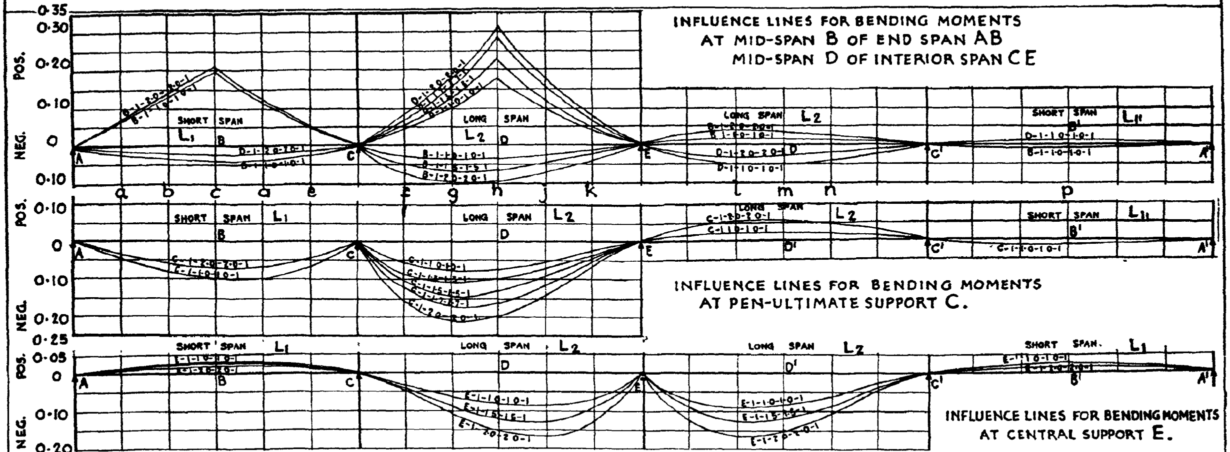
EQUAL SPANS.

INFLUENCE LINES MARKED 1 : 1·0 : 1 CAN BE USED DIRECTLY (DIAGRAM OF A SUCCESSION OF LOADS MUST BE DRAWN TO THE SAME LINEAR SCALE.)

BENDING MOMENT DUE TO LOAD W CONCENTRATED AT ANY POINT
= (ORDINATE OF APPROPRIATE INFLUENCE LINE) × (END SPAN L_1) × W.

TABLE 36.—CONTINUOUS BEAMS: INFLUENCE LINES. FOUR SPANS.

FOUR SPANS (EQUAL OR SYMMETRICAL INEQUALITY)



	SECTION	RATIO OF SPANS	END SPAN AC					INTERIOR SPAN CE					INTERIOR SPAN EC'			END SPAN C'A'
		$L_1 : L_2 : L_2 : L_1$	a	b	c	d	e	f	g	h	j	k	l	m	n	p
ORDINATES	MID-SPAN B OF END SPAN AC	1 : 1 : 1 : 1	0·062	·127	·200	·117	·049	·028	·039	·037	·027	·013	·011	·010	·007	0·003
		1 : 1½ : 1½ : 1	0·066	·135	·209	·126	·056	·052	·071	·067	·048	·023	·019	·017	·012	0·003
		1 : 2 : 2 : 1	0·069	·140	·216	·133	·060	·077	·106	·100	·072	·034	·027	·025	·017	0·003
	PEN-ULTIMATE SUPPORT C	1 : 1 : 1 : 1	0·043	·079	·100	·099	·068	·057	·078	·074	·053	·025	·021	·020	·015	0·007
		1 : 1½ : 1½ : 1	0·035	·065	·082	·081	·055	·103	·142	·134	·096	·046	·037	·035	·025	0·007
		1 : 2 : 2 : 1	0·030	·054	·069	·068	·047	·155	·213	·200	·143	·068	·055	·050	·035	0·006
	MID-SPAN D OF INTERIOR SPAN CE	1 : 1 : 1 : 1	0·016	·029	·037	·036	·025	·041	·099	·175	·098	·040	·032	·030	·022	0·010
		1 : 1½ : 1½ : 1	0·013	·024	·030	·029	·020	·054	·138	·250	·140	·057	·043	·041	·029	0·007
		1 : 2 : 2 : 1	0·011	·020	·025	·025	·017	·066	·175	·325	·180	·073	·054	·050	·035	0·006
	CENTRAL SUPPORT E	1 : 1 : 1 : 1	0·012	·021	·027	·027	·018	·028	·058	·080	·085	·061	·085	·080	·058	0·026
		1 : 1½ : 1½ : 1	0·010	·017	·022	·022	·015	·038	·082	·116	·124	·091	·124	·116	·082	0·022
		1 : 2 : 2 : 1	0·008	·015	·019	·019	·013	·046	·104	·150	·163	·121	·163	·150	·104	0·019

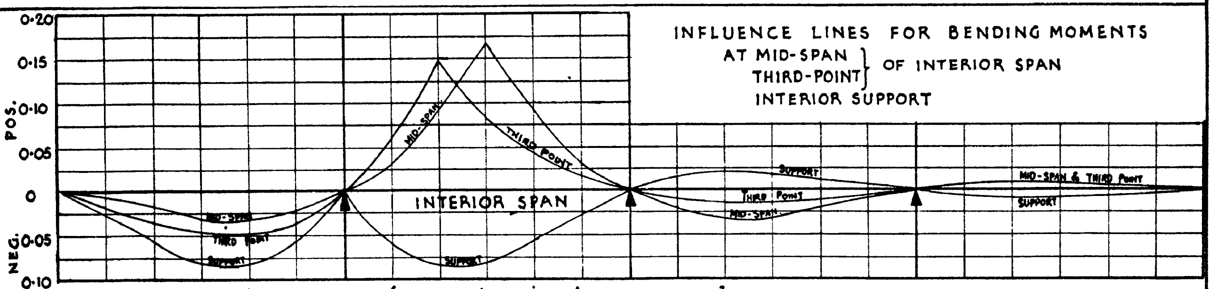
UNEQUAL SPANS.
DATA ENABLES INFLUENCE LINES TO BE DRAWN FOR BENDING MOMENTS PRODUCED BY A SINGLE UNIT LOAD MOVING OVER FOUR UNEQUAL SPANS.

ORDINATES FOR INTERMEDIATE RATIOS OF SPANS CAN BE INTERPOLATED.

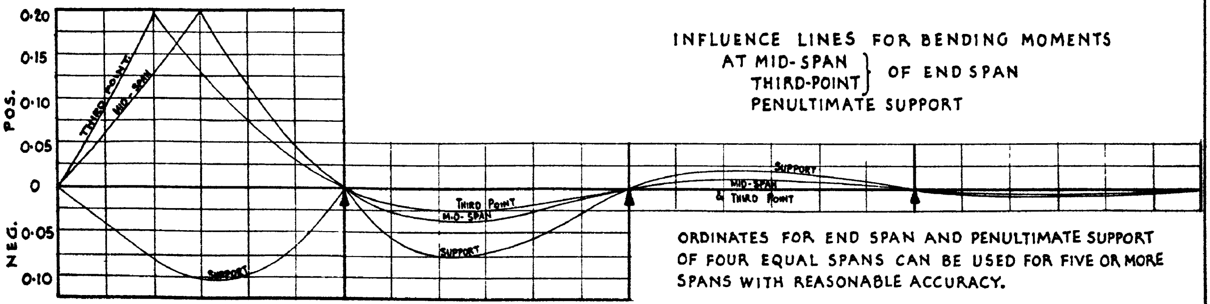
EQUAL SPANS.
INFLUENCE LINES MARKED 1:1·0:1·0:1 CAN BE USED DIRECTLY. (DIAGRAM OF A SUCCESSION OF LOADS MUST BE DRAWN TO SAME LINEAR SCALE)

BENDING MOMENT DUE TO LOAD W CONCENTRATED AT ANY POINT
= (ORDINATE OF APPROPRIATE INFLUENCE LINE) × (END SPAN L_1) × W.

FIVE OR MORE EQUAL SPANS



ORDINATES	a	b	c	d	e	f	g	h	j	k	l	m	n	p	SIGNS (NEG. OR POS.) OF ORDINATES AS IN DIAGRAMS
INTERIOR SPAN:— MIDSPAN	0·010	0·021	0·029	0·031	0·022	0·039	0·096	0·171	0·096	0·039	0·031	0·029	0·021	0·008	
THIRD POINT	0·016	0·033	0·046	0·049	0·035	0·062	0·147	0·087	0·045	0·017	0·013	0·012	0·009	0·003	
INTERIOR SUPPORT	0·027	0·057	0·079	0·084	0·061	0·061	0·084	0·079	0·057	0·027	0·023	0·021	0·015	0·006	



ORDINATES FOR END SPAN AND PENULTIMATE SUPPORT OF FOUR EQUAL SPANS CAN BE USED FOR FIVE OR MORE SPANS WITH REASONABLE ACCURACY.

INFLUENCE LINES ABOVE CAN BE USED DIRECTLY (DIAGRAM OF A SUCCESSION OF LOADS MUST BE DRAWN TO THE SAME LINEAR SCALE.)
BENDING MOMENT DUE TO LOAD W CONCENTRATED AT ANY POINT = (ORDINATE OF APPROPRIATE INFLUENCE LINE) x (SPAN) x W.

SLABS SPANNING IN TWO DIRECTIONS.

Notation.—The symbols used in the following and in *Tables* 38, 39, 41, 42 and 43 are as follows. (The corresponding symbols used in B.S. Code No. 114, where different, are given in brackets.) The symbols used in *Tables* 40 and 44 are given in the respective tables.

W = total load (lb.) on the slab and equal to wL_BL_L for a completely-loaded panel, and equal to wuv for a partially-loaded panel; w = uniformly-distributed load (lb. per sq. ft.).

$L_B(l_x)$ and $L_L(l_y)$ = short and long spans (ft.) respectively, $k = \frac{L_L}{L_B}$.

M_B and M_L = maximum bending moments at the midspan of the short and long spans respectively; M_{BA} and M_{BC} = bending moments at supports A and C respectively of the short span; M_{LD} and M_{LE}, the bending moments at supports D and E respectively of the long span. Bending moments are in ft.-lb. per foot width.

K_B and K_L = bending-moment reduction factors for short and long spans respectively, corners not held down; K_B' and K_L' = corresponding factors with corners held down.

$m_B(=\beta_x)$ and $m_L\left[=\beta_y\left(\frac{l_x}{l_y}\right)^2\right]$ = coefficients for positive bending moments on short and long snaps respectively, with corners held down; m_B' and m_L' = corresponding coefficients for negative bending moments. $m_{BO}(=\alpha_x)$ and $m_{LO}\left[=\alpha_y\left(\frac{l_x}{l_y}\right)^2\right]$ = coefficients for positive bending moments on short and long spans respectively with corners not held down.

Rectangular Panels Freely Supported along All Edges with Uniformly-distributed Load.—For a rectangular panel that is freely supported along all four edges in such a manner that the corners are free to lift, the Grashof and Rankine method is applicable and the bending moment reduction coefficients are $K_B = \frac{k^4}{k^4 + 1}$ and $K_L = 1 - K_B$. The midspan bending moments per foot width M_B and M_L are calculated from the formulæ in *Table* 38. The usual limit of application of this method is when the length of the panel is equal to twice the breadth, that is when $k = 2$. Beyond this limit the slab is considered to span across the short span only, the bending moment per foot width then being $\frac{wL_B^2}{8}$.

For the condition "corners not held down", the bending-moment coefficients in the B.S. Code correspond to m_{BO} and m_{LO} in *Table* 39.

In cases near the limit of $k = 2$, it is necessary to ensure that the amount of reinforcement in the long direction is not less than the minimum amount of distribution bars required.

For panels that are freely supported along all four edges but with the corners prevented from lifting, the corresponding coefficients K'_B and K'_L in *Table* 38 conform to a more exact analysis but with Poisson's ratio equal to zero.

The bending moments at midspan based on Dr. Marcus's method are the midspan bending moments calculated by the Grashof and Rankine method multiplied by a factor C; for a slab freely supported along all four edges $C = 1 - \frac{5}{6}\frac{k^2}{(1 + k^4)}$; the midspan bending moments per foot width are $M_B = CK_B\frac{wL_B^2}{8}$ and $M_L = \frac{M_B}{k^2}$.

The resultant bending moments by the method of Dr. Marcus and the exact theory (with Poisson's ratio equal to zero) are almost identical. If Poisson's ratio is assumed to be 0·15, the midspan bending moments per foot of width are $M_B = \frac{wCK_BL_B^2}{8}\left(1 + \frac{0{\cdot}15}{k^2}\right)$ and $M_L = \frac{wCK_BL_B^2}{8}\left(0{\cdot}15 + \frac{1}{k^2}\right)$. Alternatively the appropriate coefficients can be obtained from the curves in *Table* 42 for $\frac{u}{L_B} = \frac{v}{L_L} = 1$ for a slab completely covered with a load of intensity $w = \frac{W}{L_BL_L}$. The bending-moment coefficients given in the B.S. Code for this case correspond to m_B and m_L in the top left-hand corner in *Table* 39.

Rectangular Panels Fixed along Four Sides with Uniformly-distributed Load.—If a panel is completely fixed along all four sides, the bending moments are as follows.

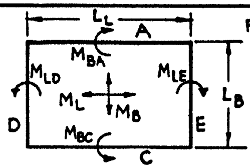
Short span: Midspan $M_B = +\,0{\cdot}8M_{BA}$; Support $M_{BA} = -\,K'_B\frac{wL_B^2}{8}$.

Long span: Midspan $M_L = +\,0{\cdot}8M_{LD}$; Support $M_{LD} = -\,K'_L\frac{wL_L^2}{8}$,

where K'_B and K'_L are as in *Table* 38. (See also B.S.-code method on page 208.)

(*Continued on page* 208.)

RATIO OF SPANS	CONDITION ALONG FOUR EDGES				
	FREE CORNERS NOT HELD DOWN.		FREE OR FIXED CORNERS HELD DOWN		FIXED (DR. MARCUS)
k	K_B	K_L	K'_B	K'_L	C'
1·0	0·50	0·50	0·30	0·30	0·861
1·05	0·55	0·45	0·33	0·27	0·862
1·1	0·59	0·41	0·36	0·24	0·864
1·15	0·64	0·36	0·39	0·22	0·866
1·2	0·68	0·33	0·42	0·19	0·871
1·25	0·71	0·29	0·45	0·17	0·874
1·3	0·74	0·26	0·48	0·15	0·879
1·4	0·79	0·21	0·53	0·13	0·888
1·5	0·84	0·16	0·58	0·11	0·898
1·6	0·87	0·13	0·63	0·09	0·907
1·75	0·90	0·10	0·68	0·07	0·919
2·0	0·94	0·06	0·76	0·05	0·935
2·5	0·97	0·03	0·87	0·03	0·957
3·0	0·98	0·02	0·94	0·02	0·970



RATIO OF SPANS: $k = \frac{\text{LONG SPAN}}{\text{SHORT SPAN}} = \frac{L_L\ (\text{FT.})}{L_B\ (\text{FT.})}$

UNIFORMLY-DISTRIBUTED LOAD = w LB. PER SQ. FT.

FREELY-SUPPORTED ALONG FOUR EDGES.

CORNERS NOT HELD DOWN: $M_B = +K_B \frac{wL_B^2}{8}$; $M_L = +K_L \frac{wL_L^2}{8} = \frac{M_B}{k^2}$

CORNERS HELD DOWN: $M_B = +K'_B \frac{wL_B^2}{8}$; $M_L = +K'_L \frac{wL_L^2}{8}$

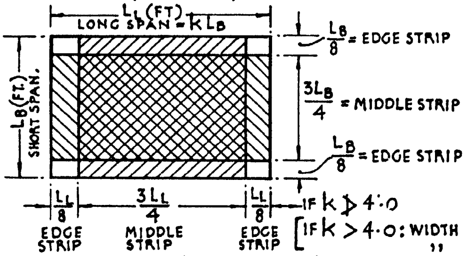
FIXITY ALONG FOUR EDGES. CORNERS HELD DOWN. (DR. MARCUS)

$M_B = +C'K_B \frac{wL_B^2}{24}$; $M_L = +C'K_L \frac{wL_L^2}{24} = \frac{M_B}{k^2}$

$M_{BA} = M_{BC} = -K_B \frac{wL_B^2}{12}$

$M_{LD} = M_{LE} = -K_L \frac{wL_L^2}{12} = \frac{M_{BA}}{k^2}$

CONTINUITY (OR FIXITY) ALONG ONE OR MORE EDGES. (B.S. CODE.)



CONDITIONS:
CORNERS HELD DOWN.
TORSIONAL RESISTANCE PROVIDED
NO REINFORCEMENT REQUIRED IN EDGE STRIPS TO RESIST BENDING MOMENT PARALLEL TO EDGES OF PANEL.

[IF $k > 4·0$: WIDTH OF MIDDLE STRIP = $L_L - L_B$
" " EDGE STRIPS = $0·5 L_B$]

BENDING MOMENTS (FT.-LB. PER FOOT) IN MIDDLE STRIPS:—

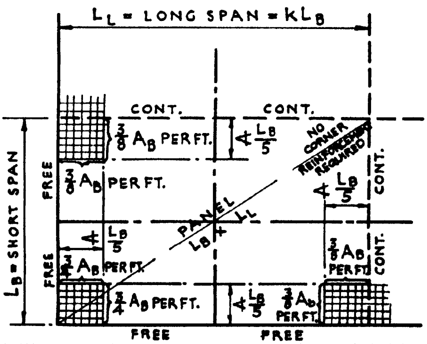
	AT MIDSPAN.	AT CONTINUOUS EDGE.	AT DISCONTINUOUS EDGE. (SLAB MONOLITHIC WITH SUPPORT)
SHORT SPAN:	$+m_B w L_B^2$	$-m'_B w L_B^2$	$-\frac{2}{3} m_B w L_B^2$
LONG SPAN:	$+m_L w L_L^2$	$-m'_L w L_L^2$	$-\frac{2}{3} m_L w L_L^2$

CORNER REINFORCEMENT FOR TORSIONAL RESISTANCE. (B. S. CODE)

A_B & A_L = CROSS SECTIONAL AREA (PER FT.) OF REINFORCEMENT FOR POSITIVE B.M. AT MIDSPAN OF SHORT AND LONG SPANS RESPECTIVELY.

$\frac{3}{4}A_B$ & $\frac{3}{8}A_B$ = CROSS SECTIONAL AREA (PER FT.) OF CORNER REINFORCEMENT IN EACH OF TWO LAYERS (ONE NEAR TOP FACE OF SLAB; ONE NEAR BOTTOM FACE).

IF $A_L > A_B$ SUBSTITUTE $\frac{3}{4}A_L$ & $\frac{3}{8}A_L$ FOR $\frac{3}{4}A_B$ & $\frac{3}{8}A_B$



NOTE.—For values of m_B, m'_B, m_L and m'_L (for calculation of bending moments on middle strips) see *Table* 39.

SLABS SPANNING IN TWO DIRECTIONS (*continued from page* 206).

Formulæ based on Dr. Marcus's method are given in *Table* 38; the coefficient $C' = 1 - \frac{5}{18}\cdot\frac{k^2}{(1 + k^4)}$ and values are given in *Table* 38; K_L and K_B are the Grashof and Rankine reduction coefficients given in *Table* 38.

B.S. Code Method for Rectangular Panels with Uniformly-distributed Load.—For the conditions common in reinforced concrete construction, that is continuity over the supports along some edges and discontinuity at the other edges, the method recommended in the B.S. Code No. 114 is applicable, and the coefficients given by the curves in *Table* 39 should be substituted in the appropriate formulæ for bending moments in the middle strip shown in *Table* 38. Provision is made at the discontinuous edge for the possible restraining moment due to the slab being cast monolithically with reinforced concrete beams supporting the panel or with concrete encasing steel beams, or due to embedment in a brick or masonry wall; if the restraint is of such magnitude that full fixity or conditions of continuity exist, the slab should be considered as continuous, instead of discontinuous, at the edge concerned.

The B.S. Code recommend also other bases of analysis, such as the theory of thin plates (which gives the factors K'_B and K'_L in *Table* 38) and the yield-line method.

Shearing Force on Rectangular Panel with Uniformly-distributed Load.—The maximum shearing forces at the edges of a panel spanning in two directions and carrying a uniformly-distributed load are, according to M. Pigeaud, approximately $0{\cdot}333wL_B$ at the middle of the short edge and $\frac{wL_Bk}{2k + 1}$ at the middle of the long edge. The same values are applicable to a panel fixed or continuous along all four edges, but for other conditions the distribution of the shearing force, the stresses due to which are rarely critical, must be adjusted on the principle that the shearing force is slightly greater at a side where there is continuity or fixity than at an opposite freely-supported edge.

Examples.

(*a*) Determine the bending moments on a rectangular slab AECD freely supported on all four sides (corners not held down) and subject to an inclusive load of 200 lb. per sq. ft., the lengths of the sides D and E being $L_B = 10$ ft. and A and C being $L_L = 12$ ft. 6 in.

Ratio of sides $= k = \frac{12{\cdot}5}{10{\cdot}0} = 1{\cdot}25$.

(i) From *Table* 38 the coefficients are $K_B = 0{\cdot}71$ and $K_L = 0{\cdot}29$.

Bending moments:

Midspan, short span: $0{\cdot}71 \times \frac{1}{8} \times 200 \times 10^2 \times 12 = 21{,}300$ in.-lb.

,, long span: $0{\cdot}29 \times \frac{1}{8} \times 200 \times 12{\cdot}5^2 \times 12 = 13{,}600$ in.-lb.

Bending moments at the supports are zero.

(ii) From *Table* 39 using the coefficients m_{BO} and m_{LO} corresponding to B.S. Code for "corners not held down".

Bending moments.—Midspan, short span $= 0{\cdot}09 \times 200 \times 10^2 \times 12 = 21{,}600$ in.-lb.

long span $= 0{\cdot}036 \times 200 \times 12{\cdot}5^2 \times 12 = 13{,}400$ in.-lb.

(*b*) Find the bending moment in the panel in (*a*) if it is constructed monolithically with the supports, but discontinuous along all edges (that is, nominally freely supported but with the corners held down).

(i) From Table 38, using "exact" coefficients $K'_B = 0{\cdot}45$ and $K'_L = 0{\cdot}17$.

Bending moments:

Midspan of short span: $0{\cdot}45 \times \frac{1}{8} \times 200 \times 10^2 \times 12 = 13{,}500$ in.-lb.

,, ,, long span: $0{\cdot}17 \times \frac{1}{8} \times 200 \times 12{\cdot}5^2 \times 12 = 8000$ in.-lb.

(ii) From *Table* 39, using the B.S. Code coefficients (curves in top left-hand corner).

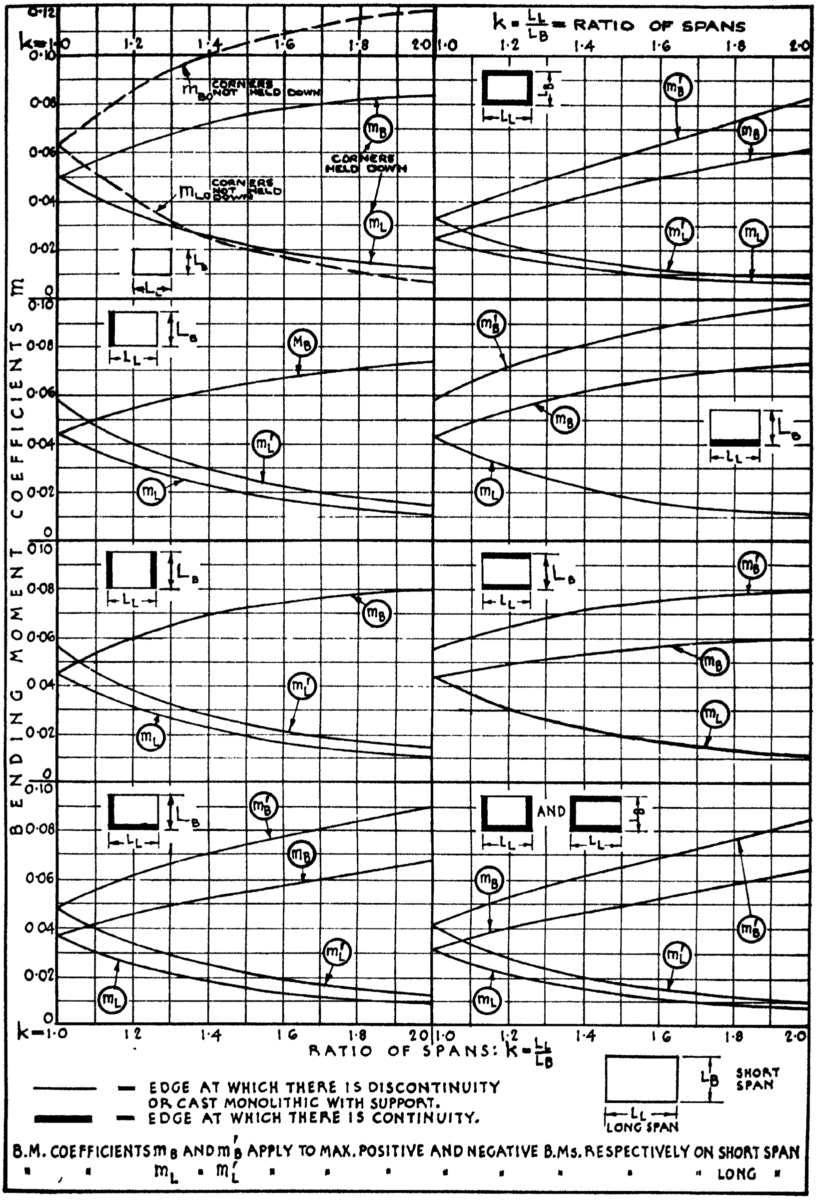
Bending moments:

		in.-lb.
Short span—middle strip, at midspan:	$0{\cdot}065 \times 200 \times 10^2 \times 12$ =	15,600
,, ,, at edge:	$-\frac{2}{3} \times 15{,}600$ =	− 10,400
Long span—middle strip, at midspan:	$0{\cdot}032 \times 200 \times 12{\cdot}5^2 \times 12$ =	12,000
,, ,, at edge:	$-\frac{2}{3} \times 12{,}000$ =	− 8,000

(*c*) Determine the bending moments if the panel in (*a*) is nominally freely supported on side C only and is continuous over the other three sides.

Table 39 applies (curves in bottom right-hand corner): $\frac{L_L}{L_B} = \frac{12{\cdot}5}{10} = 1{\cdot}25$.

			in.-lb.
Bending moments:	Short span, midspan:	$0{\cdot}043 \times 200 \times 10^2 \times 12$ =	10,300
	,, ,, support:	$0{\cdot}055 \times 200 \times 10^2 \times 12$ =	− 13,400
	Long span, midspan:	$0{\cdot}020 \times 200 \times 12{\cdot}5^2 \times 12$ =	7500
	,, ,, support:	$0{\cdot}026 \times 200 \times 12{\cdot}5^2 \times 12$ =	− 9750



NOTE.—Curves give coefficients to be used in formulæ in *Table* 38. See also page facing *Table* 38 for corresponding symbols used in B.S. Code.

SLABS SPANNING IN TWO DIRECTIONS (*continued*).

Rectangular Panels with Triangularly-distributed Loads.—The intensity of pressure on the walls of containers is uniform at any given level, but vertically may vary from zero near the top to a maximum p at the bottom. If there is a support along the top of a rectangular panel spanning in two directions, the curves and expressions in the lower part of *Table* 40 enable the probable maximum bending moments on vertical and horizontal strips of unit width to be calculated, whether the slab is fixed (Case 3) or freely supported (Case 2) along the top edge. Similar curves are given in the upper part of *Table* 40 for the condition when there is no support along the top edge (Case 1). In all cases it is assumed that the slab is continuous over the two vertical edges of the panel and is fixed along the bottom edge.

The upper curves for each case indicate the approximate positions of the maximum bending moments. The values given at the ends of the curves are the values to which the coefficients approach as the ratio of spans k approaches zero, or to which they are asymptotic if k is infinity. Although the conditions $k = 0$ and $k = \infty$ have no practical significance, the coefficients for these cases apply when the slab spans wholly horizontally or vertically respectively. If k is less than 0·3 in Cases 2 and 3 or less than 0·5 in Case 1, the bending moments should be calculated on the assumption that the slab spans entirely horizontally. If k exceeds 2 for Case 1, or $2\frac{1}{2}$ for Cases 2 and 3, the slab should be assumed to span entirely vertically.

At a nominally freely-supported top edge (Case 2), resistance to negative bending moment equal to two-thirds of the positive bending moment on the vertical span should be provided.

If the slab is assumed to span entirely vertically or entirely horizontally, the amount of reinforcement provided horizontally and vertically respectively, and in other cases at sections where the calculated bending moment is small, should be not less than the minimum amount required in a slab (see page 266).

Since it is common in a container to provide 45-deg. splays at the corners, it should be noted that the critical negative bending moments are not necessarily at the edges of the splays.

A pressure which is distributed trapezoidally can be dealt with by adding the bending moment due to a triangularly-distributed load (*Table* 40) to the bending moment due to a uniformly-distributed load (*Table* 38 or 39); this applies to the negative bending moment, but only approximately to the positive bending moment.

Example.—Find the maximum bending moments in a wall panel of a rectangular tank that can be considered as freely supported along the top edge and continuous along the bottom edge and along the two vertical sides. The height of the panel is 8 ft. (L_V) and the horizontal span is 12 ft. (L_H), the intensity of pressure being 500 lb. per square foot along the bottom edge and decreasing uniformly to zero at the top edge.

Ratio of spans: $k = \dfrac{L_H}{L_V} = \dfrac{12}{8} = 1{\cdot}5$. For Case 2, the bending moments are as follows.

At height ($v = 0{\cdot}45$) of $0{\cdot}45 \times 8 = 3$ ft. $7\frac{1}{2}$ in., the maximum positive bending moment on the vertical span is $+ 500 \times 8^2 \times 0{\cdot}027 \times 12 = 10{,}400$ in.-lb.

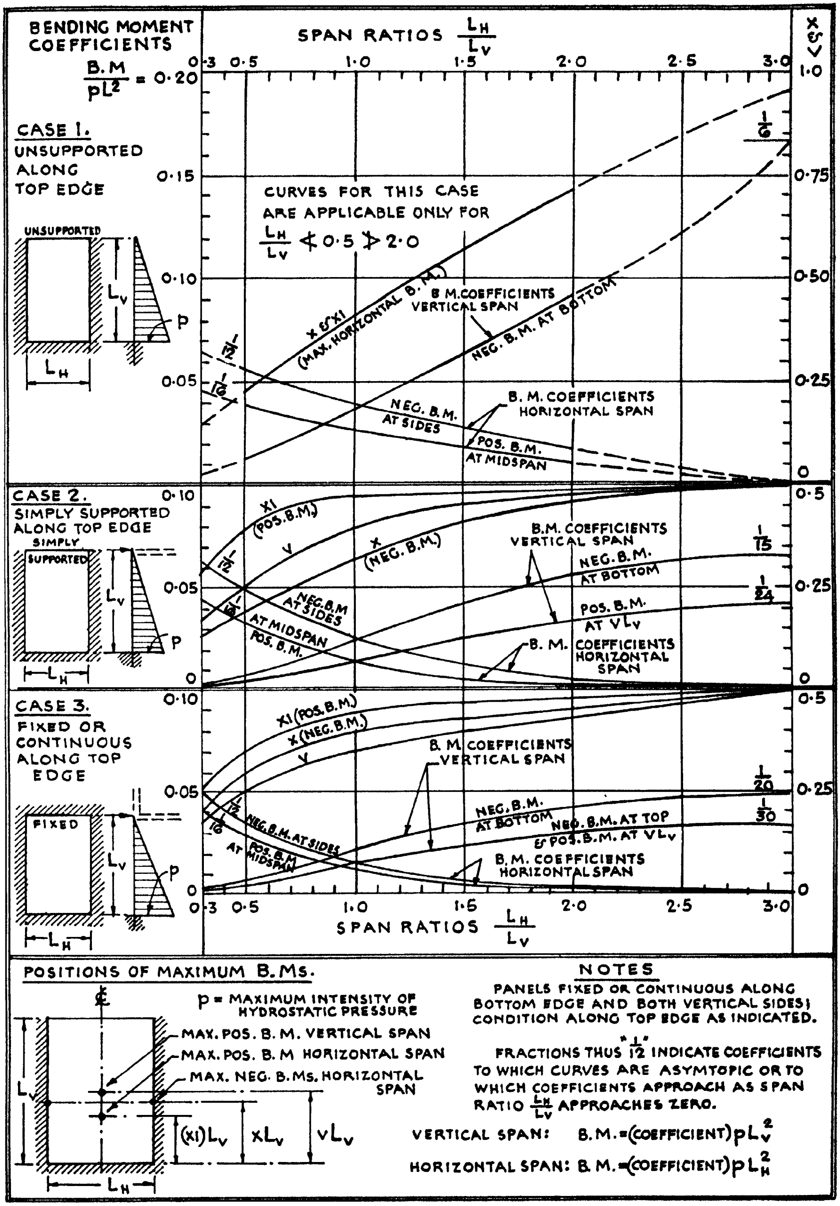
At mid-point of the bottom of the panel, the maximum negative bending moment on the vertical span is $- 500 \times 8^2 \times 0{\cdot}043 \times 12 = 16{,}500$ in.-lb.

At mid-span and at a height ($x_1 =$ about $\frac{1}{2}$) of $\frac{1}{2} \times 8 = 4$ ft., the maximum positive bending moment on the horizontal span is $+ 500 \times 12^2 \times 0{\cdot}004 \times 12 = 3450$ in.-lb., which is negligible; the minimum amount of reinforcement required in a slab forming part of a liquid-container will provide a greater resistance.

At height ($x = 0{\cdot}42$) of $0{\cdot}42 \times 8 = 3$ ft. 4 in., the maximum negative bending moment at the vertical edges of the horizontal span is $- 500 \times 12^2 \times 0{\cdot}013 \times 12 = 11{,}230$ in.-lb.

The bending moments on the horizontal span have generally to be combined with a direct tension.

SLABS SPANNING IN TWO DIRECTIONS: RECTANGULAR PANELS.—TABLE 40.
TRIANGULARLY-DISTRIBUTED LOAD.



NOTES.—Scale on right-hand side is for values of x_1, x, and v. Ratio of spans $= k = \dfrac{L_H}{L_V}$.

SLABS SPANNING IN TWO DIRECTIONS (*continued*).

Rectangular Panel with Free Support along One, Two, or Three Sides with Uniformly-distributed Load.—Bending moments on panels where continuity or fixity occur along one, two, or three edges and free support along the remaining edges, but with corner restraint, can be based on the following approximate method, which gives results reasonably close to those obtained by more accurate methods.

Substitute for $k = \dfrac{\text{long span}}{\text{short span}}$ the expression $k_e = \dfrac{\text{longer equivalent span}}{\text{shorter equivalent span}}$, where the equivalent span is f times the actual span. Values of f for various conditions of supports are approximately as follows: unity for a span freely supported at both ends, 0·8 for a span continuous at one end and freely supported at the other, and 0·67 for a span continuous over both supports. The shorter equivalent span may not be the shorter actual span. If the corners of a slab are prevented from lifting and torsional restraint is provided, the coefficients K'_B and K'_L (*Table* 38) for the ratio k_e, instead of k, are combined with the following bending-moment factors.

Slab freely supported along one edge and continuous over the other three supports.—The bending moment factor is $\frac{1}{8}$ for the negative bending moment over the three supports where continuity exists, $\frac{1}{10}$ for the positive midspan bending moment for the span that is continuous at both ends, and $\frac{1}{8}$ for the positive midspan bending moment for the span that is continuous at one end and free at the other.

Slab freely supported along two opposite edges and continuous over the other two opposite supports.—The bending moment factors are $\frac{1}{8}$ for the negative bending moments over the two supports where continuity exists and for the positive bending moment at midspan of the span that is freely supported at both ends, and $\frac{1}{10}$ for the positive bending moment for the span that is continuous at both ends.

Slab continuous over two adjacent supports and freely supported along the other two.—The bending moment factors are $\frac{1}{8}$ for the negative bending moments and $\frac{1}{8}$ for the positive bending moments.

Slab continuous over one support and freely supported along the remaining three edges.—The bending-moment factors are $\frac{1}{8}$ for the negative bending moment and for the positive bending moment for the span freely supported at both ends, and $\frac{1}{8}$ for the positive bending moment for the span that is continuous at one end and freely supported at the other.

Slab continuous over two adjacent supports and freely supported along the remaining edges, but with the corner where the freely supported edges meet not held down.—Combine with the coefficients K_B and K_L (*Table* 38) for the appropriate ratio of k_e, bending moment factors of $\frac{1}{10}$ for the positive and negative bending moments.

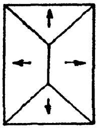
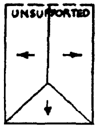
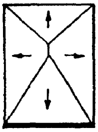
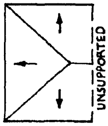
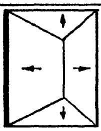
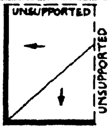
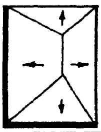
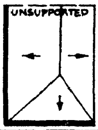
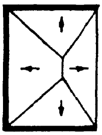
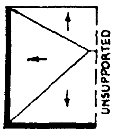
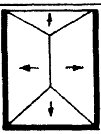
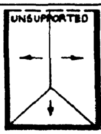
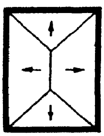
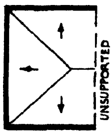
Slab continuous over one support and freely supported along the remaining three edges, but with the corners where the freely-supported edges meet not held down.—The bending moment factors are $\frac{1}{10}$ for the negative bending moment and for the positive midspan moment of the span continuous at one end and free at the other, and $\frac{1}{8}$ for the positive bending moment in the span that is free at both ends.

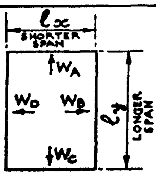
The foregoing method is an alternative method in the B S. Code, the latter method should be used where applicable.

Beams Supporting Rectangular Panels.—Beams supporting slabs spanning in two directions are subjected to loading from the slab that is distributed approximately triangularly on the beam along each shorter edge (span $L_B = l_x$) and trapezoidally on the beam along each longer edge (span $L_L = l_y$) as shown in the diagrams in *Table* 41. For the calculation of the bending moments only on the beams, if the span of a beam is equal to the length (or width) of the panel, and the beam supports one panel only, the equivalent total uniformly-distributed loads on the beams supporting a panel which is freely-supported along all four edges or is subjected to the same degree of restraint along all four edges are as follows.

$$\text{Short-span beam: } \frac{wL_B}{3}; \text{ long-span beam: } \frac{wL_B}{2}\left[1 - \frac{1}{3}\left(\frac{L_B}{L_L}\right)^2\right],$$

where w is the intensity of total uniformly-distributed load on the slab. If a beam supports two identical panels, one on either side, the foregoing equivalent loads are doubled. If a beam supports more than one panel in the direction of its length, the distribution of the load is in the form of two or more triangles (or trapeziums), and the foregoing formulæ do not apply; in such case, however, it is accurate enough to assume that the total load on the beam is uniformly distributed.

PANELS SUPPORTED ALONG FOUR EDGES.		PANELS UNSUPPORTED ALONG ONE (OR TWO) EDGES	
	$k>1$: $W_A = W_C = \frac{1}{4} w l_x^2$; $W_B = W_D = \frac{1}{2}(k-\frac{1}{2}) w l_x^2$ $k=1$: $W_A = W_B = W_C = W_D = \frac{1}{4} w l_x^2$		$W_A = 0$ $W_B = W_D = \frac{1}{2}(k-\frac{1}{4}) w l_x^2$ $W_C = \frac{1}{4} w l_x^2$
	$k \nless 1\frac{1}{3}$: $W_A = \frac{1}{4} w l_x^2$ (MIN.) $W_B = W_D = \frac{1}{2}(k-\frac{2}{3}) w l_x^2$ $W_C = \frac{5}{12} w l_x^2$ (MAX.) $k \ngtr 1\frac{1}{3}$: $W_A = \frac{3}{5} W_C$ APPROX (MIN.) $W_B = W_D = \frac{3}{16} k^2 w l_x^2$ $W_C = \frac{5}{8} k(1-\frac{3}{8}k) w l_x^2$ APPROX (MAX.)		$k \ngtr 2$ $W_A = W_C = \frac{1}{2} k(1-\frac{1}{4}k) w l_x^2$ $W_B = 0$ $W_D = \frac{1}{4} k^2 w l_x^2$
	$W_A = W_C = \frac{3}{16} w l_x^2$ $W_B = \frac{3}{5} W_D$ (MIN.) $W_D = \frac{5}{8}(k-\frac{3}{8}) w l_x^2$ (MAX)		$W_A = W_B = 0$ $W_C = \frac{1}{2} w l_x^2$ $W_D = (k-\frac{1}{2}) w l_x^2$
	$W_A = \frac{3}{16} w l_x^2$ (MIN.) $W_B = \frac{3}{5} W_D$ (MIN) $W_C = \frac{5}{16} w l_x^2$ (MAX) $W_D = \frac{5}{8}(k-\frac{1}{2}) w l_x^2$ (MAX)		$W_A = 0$ $W_B = \frac{3}{5} W_D$ (MIN) $W_C = \frac{5}{16} w l_x^2$ $W_D = \frac{5}{8}(k-\frac{5}{16}) w l_x^2$ (MAX)
	$k \nless 1\frac{1}{4}$: $W_A = W_C = \frac{5}{16} w l_x^2$ $W_B = \frac{3}{5} W_D$ (MIN) $W_D = \frac{5}{8}(k-\frac{5}{8}) w l_x^2$ (MIN) $k \ngtr 1\frac{1}{4}$: $W_A = W_C = \frac{1}{2} k(1-\frac{2}{5}k) w l_x^2$ $W_B = \frac{3}{20} k^2 w l_x^2$ (MIN.) $W_D = \frac{1}{4} k^2 w l_x^2$ (MAX)		$k \ngtr 1\frac{1}{3}$: $W_A = \frac{3}{5} W_D$ (MIN.) $W_B = 0$ $W_C = \frac{5}{8} k(1-\frac{5}{16}k) w l_x^2$ (MAX) $W_D = \frac{5}{16} k^2 w l_x^2$ $k \nless 1\frac{1}{3}$: $W_A = \frac{3}{16} w l_x^2$ (MIN.) $W_B = 0$ $W_C = \frac{1}{2} w l_x^2$ $W_D = (k-\frac{4}{5}) w l_x^2$ (MAX.)
	$W_A = \frac{3}{20} w l_x^2$ (MIN) $W_B = W_D = \frac{1}{2}(k-\frac{2}{5}) w l_x^2$ $W_C = \frac{1}{4} w l_x^2$ (MAX.)		$W_A = 0$ $W_B = W_D = \frac{1}{2}(k-\frac{1}{4}) w l_x^2$ $W_C = \frac{1}{4} w l_x^2$
	$k>1$: $W_A = W_C = \frac{1}{4} w l_x^2$ $W_B = W_D = \frac{1}{2}(k-\frac{1}{2}) w l_x^2$ $k=1$: $W_A = W_B = W_C = W_D = \frac{1}{4} w l_x^2$		$k \ngtr 2$ $W_A = W_C = \frac{1}{2} k(1-\frac{1}{4}k) w l_x^2$ $W_B = 0$ $W_D = \frac{1}{4} k^2 w l_x^2$



$k = \frac{l_x}{l_y} = \frac{\text{LONGER SPAN}}{\text{SHORTER SPAN}}$

w = INTENSITY OF UNIFORMLY-DISTRIBUTED LOAD

W_A, W_B, W_C, W_D = TOTAL LOAD CARRIED BY EACH SUPPORT OF PANEL

CONDITION OF SUPPORTS.

- - - - - - = NO SUPPORT

———— = FREELY SUPPORTED

▬▬▬▬ = CONTINUITY OR FIXITY

LOADS MARKED (MIN.) APPLY IF PANEL IS ENTIRELY FREELY SUPPORTED ALONG EDGE INDICATED: IF PARTIALLY RESTRAINED LOAD WILL BE SLIGHTLY GREATER THAN GIVEN AND LOAD MARKED (MAX.) ON OPPOSITE EDGE WILL BE CORRESPONDINGLY REDUCED.

SLABS SPANNING IN TWO DIRECTIONS (*continued*).

Square Panels ($k = 1\cdot0$).—The expression in *Table* 42 that the bending moment in both directions is $W(m_1 + 0\cdot15m_2)$ applies only if load is over entire panel, or if $u = v$.

Other conditions.—u and v can be in either direction; m_1 is the bending moment coefficient in the direction of u; m_2 is the coefficient in the direction of v. Coefficient m_1 is based on $\frac{u}{L}$ and $\frac{v}{L}$ as selected; for coefficient m_2 reverse u and v.

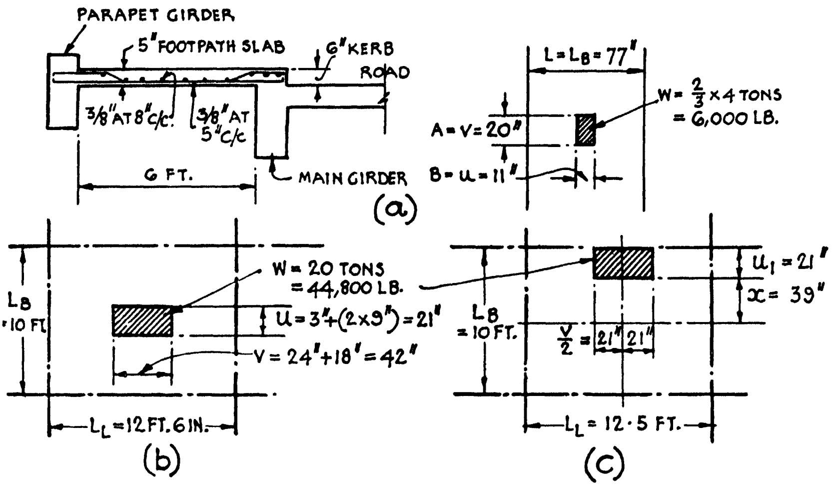
Example.—If $\frac{u}{L} = 0\cdot8$ and $\frac{v}{L} = 0\cdot2$, $m_1 = 0\cdot072$; for $\frac{u}{L} = 0\cdot2$ and $\frac{v}{L} = 0\cdot8$, $m_2 = 0\cdot103$.

Bending moments.

On span in direction of u: $W[0\cdot072 + (0\cdot15 \times 0\cdot103)] = 0\cdot087W$ ft.-lb. per ft.

On span in direction of v: $W[0\cdot103 + (0\cdot15 \times 0\cdot072)] = 0\cdot114W$ ft.-lb. per ft.

Examples of Panels Supporting Concentrated Loads.—The following examples illustrate the use of *Tables* 42 and 43 for slabs supporting a load which is concentrated uniformly over an area less than the entire area of the panel. Notes on these tables are given on page 32.



(*a*) The footpath of a bridge spans 6 ft. between a parapet girder and a main longitudinal girder, and is monolithic with both girders [diagram (*a*)] The live load is either 100 lb per sq. ft. uniformly distributed, or a load of 4 tons from a wheel the contact area of which is 12 in. by 3 in. (With the latter load the stresses may be increased by 50 per cent.; that is at ordinary working stresses the wheel load can be assumed to be about 6000 lb.) These loads comply with the recommendations of the Ministry of Transport.

(i) Assume a 5-in. slab; total uniformly-distributed load = 63 + 100 = 163 lb. per sq. ft. With continuity at both supports, bending moment at midspan and at each support is $\frac{1}{12} \times 163 \times 6\cdot3^2 \times 12 = 6400$ in.-lb. per ft. width.

(ii) Contact area of 12 in. by 3 in. at the wheel can be increased to 20 in. by 11 in. (*Table* 6); depth to the reinforcement is about 4 in.

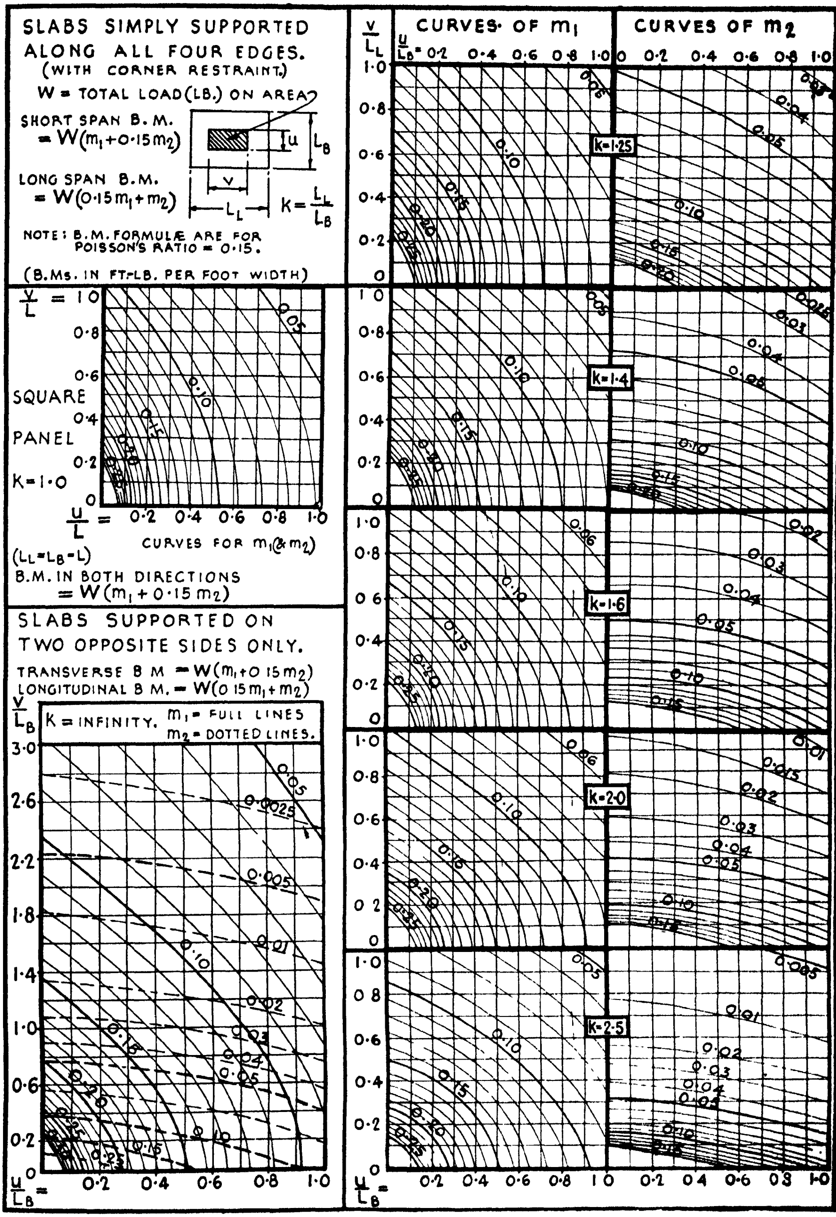
The slab spans mainly in one direction; the curves in the lower left-hand corner of *Table* 42 apply.—$\frac{u}{L_B} = \frac{11}{77} = 0\cdot143$; $\frac{v}{L_B} = \frac{20}{77} = 0\cdot26$; $m_1 = 0\cdot22$ and $m_2 = 0\cdot12$.

Free transverse bending moment $= 6000[0\cdot22 + (0\cdot15 \times 0\cdot12)]12 + (\frac{1}{8} \times 63 \times 6\cdot3^2 \times 12)$ $= 17{,}150 + 3750 = 20{,}900$ in.-lb. per ft. width.

Allow for continuity (partial fixity) by reducing the free bending moment due to the dead load by one-third, and that due to the live load by 20 per cent.; the transverse bending

(*Continued on page* 216.)

CONCENTRIC CONCENTRATED LOADS.



Note.—See note regarding square panels on page 214.

SLABS SPANNING IN TWO DIRECTIONS (*continued*).

Examples of Panels Supporting Concentrated Loads (*continued from page* 214).
moment = 16,100 in.-lb. [This is greater than the bending moment in case (i); therefore case (ii) controls the design.] With stresses of 1000 lb. and 18,000 lb. per sq. in., the effective depth required is about 3 in., but it is practical to make the footpath slab 5 in. thick, which gives $d_1 = 4$ in. ($\frac{3}{4}$-in. cover) and l_a = about $3\frac{1}{2}$ in.

Transverse reinforcement: $A_{st} = \dfrac{16{,}100}{18{,}000 \times 3{\cdot}5} = 0{\cdot}26$ sq. in., say, $\frac{3}{8}$-in. bars at 5-in. centres.

Longitudinal bending moment $= 6000[(0{\cdot}15 \times 0{\cdot}22) + 0{\cdot}12]12 = 11{,}000$ in.-lb.

Allowing 20 per cent. for continuity: net bending moment = about 8800 in.-lb.

Longitudinal reinforcement: $A_{st} = \dfrac{8800}{18{,}000 \times 3{\cdot}125} = 0{\cdot}156$ sq. in., say, $\frac{3}{8}$-in. bars at 8-in. centres.

(iii) Another method is to calculate the transverse bending moment due to the concentrated load by estimating the width of slab assumed to carry the load from the formula

$$\tfrac{2}{3}(L + B) + A = 0{\cdot}67(77 + 11) + 20 = 79 \text{ in.} = 6{\cdot}55 \text{ ft.}$$

Load carried by 1 ft. of slab is $\dfrac{6000}{6{\cdot}55} = 920$ lb.

Free transverse bending moment $= \dfrac{920}{4}\left(77 - \dfrac{11}{2}\right) = 16{,}400$ in.-lb.; reducing by 20 per per cent. for continuity, the net bending moment (after adding 2400 in.-lb. approximately due to the dead load) = 15,500 in.-lb.; this is about the same as calculated from *Table* 43.

Longitudinal reinforcement in accordance with the requirement of the Ministry of Transport for distribution bars (see page 268): for a span of about 6 ft., 50 per cent. of the transverse reinforcement, that is, $\frac{3}{8}$-in. bars at 10-in. centres are sufficient; therefore $\frac{3}{8}$-in. bars at 8-in. centres are satisfactory.

(iv) Another method is an adaptation of B.S. Code method. From bottom left-hand corner of *Table* 43.

$$e = 20 + [2{\cdot}4 \times \tfrac{1}{2} \times 77(1 - \tfrac{1}{2})] \text{ or } = 20 + (0{\cdot}6 \times 77) = 66 \text{ in.} = 5{\cdot}5 \text{ ft.}$$

Free bending moment (symmetrical load) $= \dfrac{6000}{4 \times 5{\cdot}5}\left(77 - \dfrac{11}{2}\right) = 19{,}200$ in.-lb. per ft.; reducing by 20 per cent. for continuity and adding 2400 in.-lb. for the dead load, maximum positive bending moment = 17,760 in.-lb., which exceeds slightly the bending moments as calculated by the preceding methods.

(*b*) A panel of the deck slab of a bridge is 12 ft. 6 in. long by 10 ft. wide and is supported on all four sides and is fully continuous over all supports. Calculate the maximum bending moments due to a load of 20 tons symmetrically placed at the centre of the panel, the contact area being 3 in. by 24 in.

Assume a slab 8 in. thick ($d_1 = 7$ in.) with 2 in. of tarmacadam. Loaded area, allowing for dispersion down to reinforcement, = 21 in. by 42. in [diagram (*b*) on page facing *Table* 42].

$k = \dfrac{L_L}{L_B} = \dfrac{12{\cdot}5}{10} = 1{\cdot}25$. $\dfrac{u}{L_B} = \dfrac{21}{10 \times 12} = 0{\cdot}175$; $\dfrac{v}{L_L} = \dfrac{42}{12{\cdot}5 \times 12} = 0{\cdot}28$. From *Table* 42, $m_1 = 0{\cdot}18$, $m_2 = 0{\cdot}13$. Allow a reduction of 20 per cent. for continuity. For the uniformly-distributed dead load of about 120 lb. per sq. ft. (from *Table* 38) for $k = 1{\cdot}25$, $K'_B = 0{\cdot}45$ and $K'_L = 0{\cdot}17$.

Bending moments (in.-lb. per ft. width).

Midspan of shorter span.—

Live load: $44{,}800[0{\cdot}18 + (0{\cdot}15 \times 0{\cdot}13)] \times 12 \times 0{\cdot}8 = 86{,}000$
Dead load: $0{\cdot}45 \times 0{\cdot}100 \times 120 \times 10^2 \times 12 = 6{,}480$

Total = 92,480

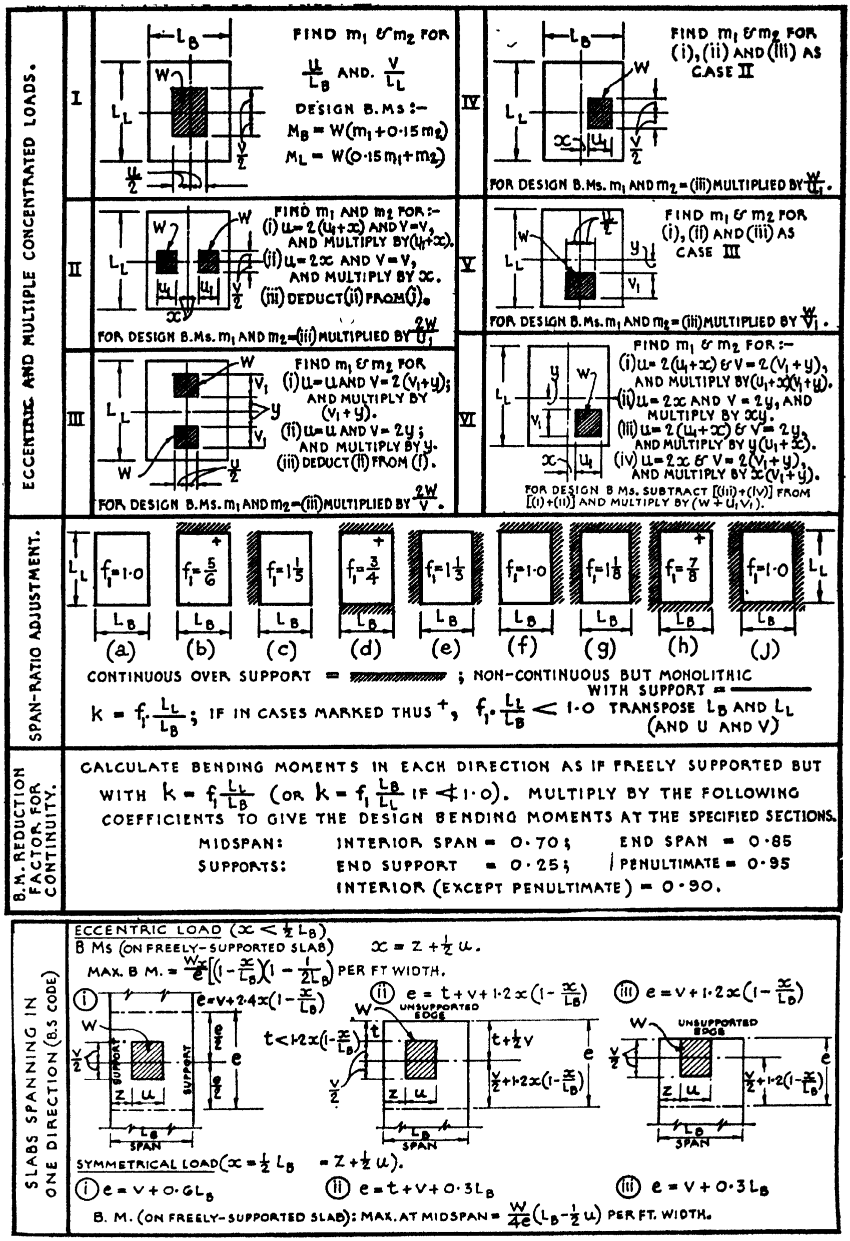
Supports of shorter span.—Live load: As at midspan = 86,000

Dead load: $6480 \times \dfrac{0{\cdot}125}{0{\cdot}100} = 8{,}100$

Total = 94,100

(*Continued on page* 218.)

ECCENTRIC CONCENTRATED LOADS.



SLABS SPANNING IN TWO DIRECTIONS (*continued*).

Examples of Panels Supporting Concentrated Loads (*continued from page* 216).

Midspan of longer span.—

$$\text{Live load: } 44{,}800[(0{\cdot}15 \times 0{\cdot}18) + 0{\cdot}13] \times 12 \times 0{\cdot}8 = 67{,}600$$

$$\text{Dead load: } 0{\cdot}17 \times 0{\cdot}100 \times 120 \times 12{\cdot}5^2 \times 12 = 3{,}830$$

$$\text{Total} = 71{,}430$$

Supports of the longer span.—Live load: As at midspan $= 67{,}600$

$$\text{Dead load: } 3830 \times \frac{0{\cdot}125}{0{\cdot}100} = 4{,}790$$

$$\text{Total} = 72{,}390$$

(*c*) Find the bending moments due to the live load in the position shown in diagram (*c*) (page 214) if the panel in (*b*) is not continuous over two of its adjacent sides, but is monolithic with the supporting beams on those two sides; continuity exists over the supports on the remaining two sides [Case (*f*) in *Table* 43; $f_1 = 1{\cdot}0$]. Adopt the procedure for Case IV, *Table* 43.

(i) $u = 2(21 + 39) = 120$ in.; $v = 42$ in.; $\frac{u}{L_B} = \frac{120}{120} = 1$; $\frac{v}{L_L} = \frac{42}{150} = 0{\cdot}28$; from *Table* 42, with $k = 1{\cdot}25$, $m_1 = 0{\cdot}073$ and $m_2 = 0{\cdot}063$; these are the factors for a load of $\frac{2W}{u_1}(u_1 + x)$ on an area of $2(u_1 + x)$ by v. $u_1 + x = 21 + 39 = 60$ in.;

$$m_1(u_1 + x) = 0{\cdot}073 \times 60 = 4{\cdot}38;\ m_2(u_1 + x) = 0{\cdot}063 \times 60 = 3{\cdot}78;$$

these are the corresponding basic bending moments $\div \frac{2W}{u_1}$.

(ii) $u = 2 \times 39 = 78$ in.; $v = 42$ in.; $\frac{u}{L_B} = \frac{78}{120} = 0{\cdot}65$; $\frac{v}{L_L} = \frac{42}{150} = 0{\cdot}28$; from *Table* 42, $m_1 = 0{\cdot}10$ and $m_2 = 0{\cdot}088$, these are the factors for a load of $\frac{2Wx}{u_1}$ on an area $2x$ by v symmetrically disposed at the centre of the panel.

$$m_1x = 0{\cdot}10 \times 39 = 3{\cdot}9;\ m_2x = 0{\cdot}088 \times 39 = 3{\cdot}43;$$

these are the corresponding basic bending moments $\div \frac{2W}{u_1}$.

(iii) $m_1 = 4{\cdot}38 - 3{\cdot}9 = 0{\cdot}48$; $m_2 = 3{\cdot}78 - 3{\cdot}43 = 0{\cdot}35$, these are the basic bending moments for a total load of $\frac{2W}{u_1}(u_1 + x) - \frac{2Wx}{u_1} = 2W$ on two symmetrically disposed areas of u_1 by v (as in Case II); the case under consideration, Case IV, is one half of Case II and thus the immediately preceding bending moments m_1 and m_2 must be multiplied by $\frac{1}{2} \times \frac{2W}{u_1} = \frac{W}{u_1}$ to give the basic bending moments for the specified load $\frac{W}{u_1} = \frac{44{,}800}{21} = 2140$ lb. per in.

Free bending moment across short span: $M_B = \frac{W}{u_1}(m_1 + 0{\cdot}15m_2)$

$$= 2140[0{\cdot}48 + (0{\cdot}15 \times 0{\cdot}35)]12 = 13{,}650 \text{ in.-lb. per ft.}$$

Free bending moment across long span: $M_L = \frac{W}{u_1}(0{\cdot}15m_1 + m_2)$

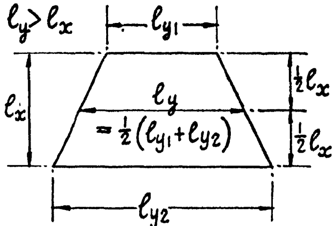
$$= 2140[(0{\cdot}15 \times 0{\cdot}48) + 0{\cdot}35]12 = 10{,}800 \text{ in.-lb. per ft.}$$

The maximum bending moments for the live load, allowing for continuity by using the factors given in *Table* 43, are:

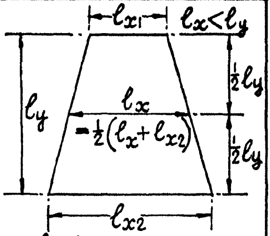
Shorter span.—Midspan:	$+ 0{\cdot}85 \times 13{,}650 =$	$+ 11{,}600$ in.-lb.
Inner support:	$- 0{\cdot}95 \times 13{,}650 =$	$- 12{,}950$ in.-lb.
Outer support:	$- 0{\cdot}25 \times 13{,}650 =$	$- 3410$ in.-lb.
Longer span.—Midspan:	$+ 0{\cdot}85 \times 10{,}800 =$	$+ 9180$ in.-lb.
Inner support:	$- 0{\cdot}95 \times 10{,}800 =$	$- 10{,}250$ in.-lb.
Outer support:	$- 0{\cdot}25 \times 10{,}800 =$	$- 2700$ in.-lb.

The bending moments due to the uniformly distributed dead load must be added.

TRAPEZIUM

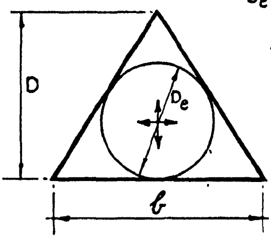


CALCULATE B.Ms. AS FOR RECTANGULAR PANEL WITH $K = \dfrac{\ell_y}{\ell_x}$



IF ℓ_{y1} IS SMALL COMPARED WITH ℓ_{y2} } APPLY RULES FOR TRIANGULAR PANEL
OR ℓ_{x1} " " " " ℓ_{x2}

ISOSCELES TRIANGLE



D_e = DIAMETER OF INSCRIBED CIRCLE = $\dfrac{2bD}{b+\sqrt{4b^2+D^2}}$

FREELY-SUPPORTED ALONG ALL EDGES (CORNERS RESTRAINED).

B.M. (IN TWO DIRECTIONS AT CENTRE OF CIRCLE) $= +\dfrac{wD_e^2}{16}$

CONTINUOUS ALONG ALL SIDES.

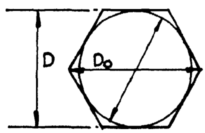
B.M. (IN TWO DIRECTIONS AT CENTRE OF CIRCLE) $= +\dfrac{wD_e^2}{30}$

B.M. (AT SIDES) $= -\dfrac{wD_e^2}{30}$

w = INTENSITY OF UNIFORMLY-DISTRIBUTED LOAD (OR INTENSITY OF PRESSURE AT CENTRE OF CIRCLE IF PRESSURE VARIES UNIFORMLY.)

REGULAR POLYGON

FIVE OR MORE SIDES.



D = DIAMETER OF INSCRIBED CIRCLE = DISTANCE ACROSS FLATS.

D_o = DIAMETER OF CIRCUMSCRIBED CIRCLE = DISTANCE ACROSS CORNERS.

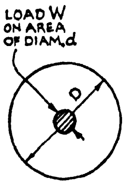
$D_e = \frac{1}{2}(D + D_o)$ = 1·08D FOR HEXAGON
1·04D FOR OCTAGON

CALCULATE B.Ms. AS FOR CIRCLE OF DIAMETER D_e.

CIRCLE (DIAMETER = D)

CONCENTRIC CONCENTRATED LOAD

LOAD W ON AREA OF DIAM. d



FREELY SUPPORTED AROUND EDGE.

M_T = TOTAL POSITIVE B.M. ACROSS DIAMETER $= \dfrac{WD}{2\pi}\left(1-\dfrac{2d}{3D}\right)$

M_A = AVERAGE POS. B.M. " " $= \dfrac{M_T}{D}$ PER FT.

OR M_m = MAX. POS B.M. AT CENTRE = 1·5 M_T (APPROX.) PER FT.

RESTRAINED AROUND EDGE.— NEG. B.M. AT EDGE $\nless \frac{1}{3} M_A$ PER FT.

POS. B.M. — AVERAGE ACROSS DIAMETER $= KM_A$ PER FT.

OR MAX. AT CENTRE $= KM_M$ PER FT.

$K = \frac{2}{15}\left(\frac{d}{D}\right) + \frac{2}{3}$

UNIFORMLY-DISTRIBUTED LOAD OVER ENTIRE PANEL w LB. PER SQ. FT.

CONDITION AT CIRCUMFERENCE	POSITIVE BENDING MOMENT: AVERAGE ACROSS DIAMETER	POSITIVE BENDING MOMENT: ALTERNATIVE MAX. AT CENTRE	NEGATIVE BENDING MOMENT AROUND EDGE
FREELY SUPPORTED	$\frac{wD^2}{24}$ PER FOOT OF DIAMETER	$\frac{wD^2}{16}$ PER FOOT AT CENTRE	—
PARTIALLY RESTRAINED (CONTINUOUS)	$\frac{wD^2}{30}$ " "	$\frac{wD^2}{20}$ " "	$\frac{wD^2}{48}$ (MIN) PER FOOT OF CIRCUMFERENCE
FIXED	$\frac{wD^2}{48}$ (MIN) " "	$\frac{wD^2}{32}$ " "	$\frac{wD^2}{36}$ " "

NOTES.

REINFORCEMENT TO RESIST POSITIVE B.M. TO BE PROVIDED IN TWO DIRECTIONS MUTUALLY AT RIGHT-ANGLES.

REINFORCEMENT TO RESIST NEGATIVE B.M. AROUND EDGE TO BE RADIAL (OR EQUIVALENT).

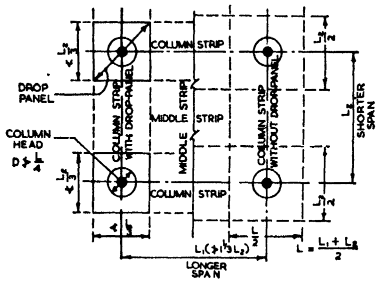
IF MAX. POS

IF MAX. POS. B.M. IS DESIGNED FOR REINF'T (OR THICKNESS OF SLAB) CAN BE REDUCED TOWARDS EDGE.

For notes on this table, see page 34.

FLAT SLABS.

The notes on flat slabs in the following and the data in *Table* 45 are in accordance with the recommendations for the empirical method of analysis described in B.S. Code No. 114 (1957).



Thickness of Slab.—The slab should be not less than 5 in. thick and the ratio of mean span L to the thickness should be not greater than the following.

Internal panels.	Without drops . .	36.	With drops . .	40.
Edge panels.	,, ,, . .	32.	,, ,, . .	36.

Adjustment of Bending Moments.—If the width of the drop-panel is less than half the panel width, the total bending moment on the middle strip should be increased in proportion to the increased width of the middle strip, and the additional bending moment can be deducted from the total for the corresponding column strips. This adjustment ensures that the total bending moments along the principal axes are unaltered. In edge panels the bending moments should be increased to the values given in *Table* 45.

Columns.—The diameter of the column head D (*Table* 45) should not exceed $0{\cdot}25L$. The angle between the slope of the head and the horizontal should be not less than 45 deg. The diameter D of the head is the diameter $1\frac{1}{2}$ in. below the underside of the slab; if the head is not circular, D is the diameter of the largest circle that can be drawn within the head. Exterior columns should be provided with as much as possible of the head prescribed for interior columns.

Columns should be designed for bending moments equal to 50 per cent. in the case of interior columns, and 90 per cent. in the case of exterior columns, of the negative bending moment required in the column strip. The bending moment should be divided between the upper and lower columns in proportion to their stiffnesses. If the floor panels are cantilevered beyond the outer row of columns, the bending moment on the exterior column can be reduced by the cantilever bending moment due to the dead load.

BENDING MOMENT COEFFICIENTS AND GENERAL DATA.

TOTAL B.M. ON STRIP OF WIDTH $\frac{L_2}{2}$ $= fwL_2(L_1-\frac{2}{3}D)^2$	COEFFICIENT f									
	INTERIOR PANELS				EDGE PANELS					
	COLUMN STRIP		MIDDLE STRIP		COLUMN STRIP			MIDDLE STRIP		
	NEG. B.M.	POS. B.M.	NEG. B.M.	POS. B.M.	NEG. B.M. INNER	NEG. B.M. OUTER EDGE	POS. B.M.	NEG. B.M. INNER	NEG. B.M. OUTER EDGE	POS. B.M.
WITHOUT DROPS	0·046	0·022	0·016	0·016	0·046	0·041	0·028	0·016	0·010	0·020
WITH DROPS	0·050	0·020	0·015	0·015	0·050	0·045	0·025	0·015	0·010	0·019

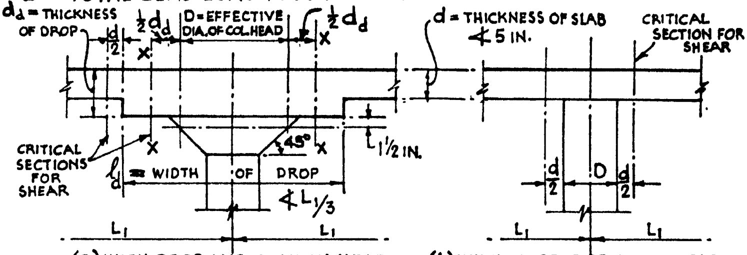
L_1 = DISTANCE BETWEEN COLUMN CENTRES IN DIRECTION OF SPAN.

L_2 = DISTANCE BETWEEN COLUMN CENTRES AT RIGHT-ANGLES TO L_1.

MAXIMUM RATIO OF $L_1 : L_2$ (OR $L_2 : L_1$) = $1\frac{1}{3}$.

SQUARE PANELS: $L = L_1 = L_2$.

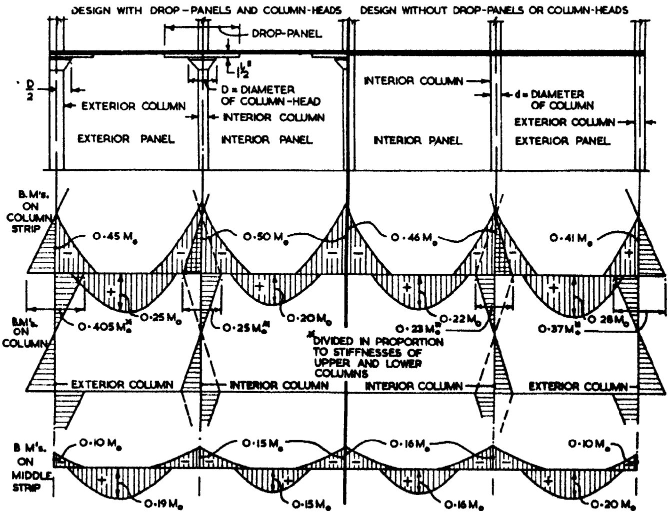
w = TOTAL DEAD LOAD PLUS LIVE LOAD PER UNIT AREA.



(a) WITH DROP AND COLUMN HEAD. (b) WITHOUT DROP OR COLUMN HEAD.

NOTE.– OTHER VARIANTS ARE BASED ON THESE TWO LIMITING CASES.

BENDING MOMENT DIAGRAMS



FRAME ANALYSIS BY SLOPE-DEFLECTION.

Bending Moments.—The principles of the slope-deflection method of analysing a restrained member are that the difference in slope between any two points in the length of the member is equal to the area of the $\frac{M}{EI}$ diagram between these two points; and that the distance of any point on the member from a line drawn tangentially to the elastic curve at any other point, the distance being measured normal to the initial position of the member, is equal to the moment (taken about the first point) of the $\frac{M}{EI}$ diagram between these two points. In the foregoing M represents the bending moment, E the modulus of elasticity of the material, and I the moment of inertia of the member. The bending moments at the ends of a member subject to the deformation and restraints shown in the diagram at the top of *Table* 46 are given by the corresponding formulæ which are derived from a combination of the basic principles and which are given in the general form and in the special form for members on non-elastic supports.

The symbol K is the stiffness factor, $\frac{I}{L}$; the stiffness is proportional to EK, a term in the formulæ, but as E is assumed to be constant the term which varies in each member is K. The terms P and Q relate to the load on the member. When there is no external load the factors P and Q are zero, and when the load is symmetrically disposed on the member $P = Q = \frac{A}{L}$. Values of P, Q and $\frac{A}{L}$ are given in *Table* 18.

The conventional signs for slope-deflection analyses are: an external restraint moment acting clockwise is positive; a slope is positive if the rotation of the tangent to the elastic line is clockwise; a deflection in the same direction as a positive slope is positive.

Examples of Use of Tables 46 and 49.

(*a*) Derive the formulæ for the bending moments in a column CAD into which is framed a beam AB. The beam is hinged at B and the column is fixed at C and D (see diagram in *Table* 46). The beam only is loaded. Assume there is no displacement of the joint A.

Selecting the general formulæ from *Table* 46,

$$M_{AB} = 3EK_{AB}\theta_A - \left(P_{AB} + \frac{Q_{BA}}{2}\right). \qquad M_{AC} = 4EK_{AC}\theta_A. \qquad M_{AD} = 4EK_{AD}\theta_A.$$

Therefore $M_{AB} + M_{AC} + M_{AD} = E\theta_A(3K_{AB} + 4K_{AC} + 4K_{AD}) - \left(P_{AB} + \frac{Q_{BA}}{2}\right) = 0.$

Thus $E\theta_A = \frac{P_{AB} + 0{\cdot}5Q_{BA}}{3K_{AB} + 4K_{AC} + 4K_{AD}}$, and $M_{AC} = \frac{4K_{AC}(P_{AB} + 0{\cdot}5Q_{BA})}{3K_{AB} + 4K_{AC} + 4K_{AD}}$.

$M_{AD} = \frac{K_{AD}}{K_{AC}}.M_{AC}$; $M_{AB} = -(M_{AC} + M_{AD})$; $M_{CA} = 2EK_{AC}\theta_A = \frac{M_{AC}}{2}$; $M_{DA} = 0{\cdot}5M_{AD}$.

For symmetrical load, $P_{AB} + \frac{Q_{BA}}{2} = 1{\cdot}5\frac{A_{AB}}{L_{AB}}$; $M_{AC} = \frac{6K_{AC}}{3K_{AB} + 4K_{AC} + 4K_{AD}}.\frac{A_{AB}}{L_{AB}}$.

(See also numerical example in *Table* 47.)

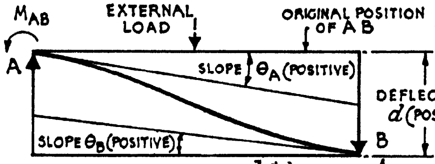
(*b*) Find the bending moments in a rectangular portal frame ABCD subjected to a horizontal load $W = 5000$ lb. at B (that is, $a = 1$, $z_1 = 0$, and $N = 0$), if $H = 12$ ft., $L = 10$ ft., and $R = 1$, if the frame is fixed at A and D.

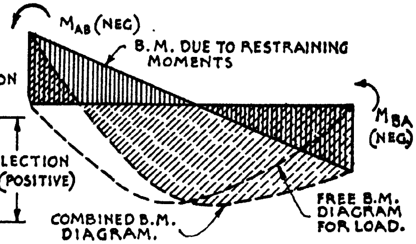
From *Table* 49,

$$H_D = 0{\cdot}5 \times 5000 = 2500 \text{ lb.; therefore } H_A = 5000 - 2500 = 2500 \text{ lb.}$$

Also $K_2 = 7$, and $K_4 = 4$; $C_1 = 0$; $M_A = M_D = C_2 = \frac{1}{2 \times 7}(5000 \times 12 \times 4) = 17{,}150$ ft.-lb. (anti-clockwise restraint moment); $M_B = M_C = (12 \times 2500) - 17{,}150 = 12{,}850$ in.-lb. acting at the head of the column of the frame in opposite sense to M_B.

BASIC FORMULÆ
(SLOPE DEFLECTION)





$$M_{AB} = 2EK_{AB}\left(2\theta_A + \theta_B - \frac{3d}{L_{AB}}\right) - P$$

$$M_{BA} = 2EK_{AB}\left(2\theta_B + \theta_A - \frac{3d}{L_{AB}}\right) + Q$$

NOTATION

THE SUFFIXES AB RELATE TO JOINT OR SUPPORT A OF ANY MEMBER AB } SIMILARLY FOR OTHER
" BA " " " B " AB } MEMBERS BC, AC, ETC.

L_{AB} = LENGTH OF MEMBER AB

I_{AB} = MOMENT OF INERTIA (CONCRETE UNITS) OF AB

K_{AB} = STIFFNESS FACTOR OF AB $= \dfrac{I_{AB}}{L_{AB}}$

E = YOUNG'S MODULUS FOR CONCRETE

θ_A = SLOPE OF DEFORMED MEMBER AB AT A.

WITH NO LOAD ON AB, $P_{AB} = Q_{BA} = 0$

$$P_{AB} = \frac{2A_{AB}}{L^2_{AB}}(2L_{AB} - 3Z_{AB})$$

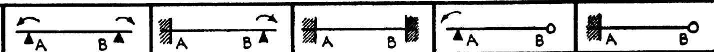
$$Q_{BA} = \frac{2A_{AB}}{L^2_{AB}}(3Z_{AB} - L_{AB})$$

FOR SYMMETRICAL LOAD ON AB $P_{AB} = Q_{BA} = \dfrac{A_{AB}}{L_{AB}}$

A_{AB} = AREA OF FREE B.M. DIAGRAM FOR LOAD ON AB

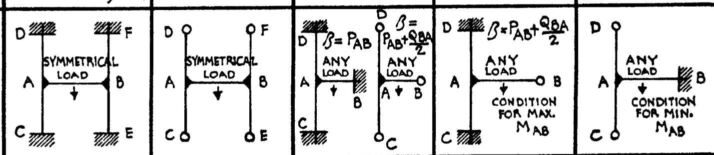
Z_{AB} = DISTANCE FROM A TO CENTROID OF FREE B.M. DIAGRAM FOR LOAD ON AB.

SLOPE-DEFLECTION GENERAL FORMULÆ NON-ELASTIC SUPPORTS



-	(1)	(2)	(3)	(4)	(5)
M_{AB}	$2EK_{AB}(2\theta_A+\theta_B)-P_{AB}$	$2EK_{AB}\theta_B - P_{AB}$	$-P_{AB}$	$3EK_{AB}\theta_A - \left(P_{AB}+\frac{Q_{BA}}{2}\right)$	$-\left(P_{AB}+\frac{Q_{BA}}{2}\right)$
M_{BA}	$2EK_{AB}(2\theta_B+\theta_A)+Q_{BA}$	$4EK_{AB}\theta_B+Q_{BA}$	$+ - Q_{BA}$	ZERO	ZERO

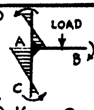
EXTERIOR COLUMNS



-	(1)	(2)	(3)	(4)	(5)
M_{AC}	$\frac{2K_{AC}}{2K_{AC}+2K_{AD}+K_{AB}}\cdot\frac{A_{AB}}{L_{AB}}$	$\frac{3K_{AC}}{3K_{AC}+3K_{AD}+2K_{AB}}\cdot\frac{A_{AB}}{L_{AB}}$	$\frac{K_{AC}}{K_{AC}+K_{AD}+K_{AB}}\beta$	$\frac{4K_{AC}}{4K_{AC}+4K_{AD}+3K_{AB}}\beta$	$\frac{3K_{AC}}{3K_{AC}+3K_{AD}+4K_{AB}}P_{AB}$

$M_{AD} = \dfrac{K_{AD}}{K_{AC}}\cdot M_{AC}$ $\quad M_{AB} = -(M_{AC}+M_{AD})$ $\quad$ IF FIXED AT C: $M_{CA} = 0\cdot5\,M_{AC}$, ETC. AT HINGES: B.M. = 0

B.M. IN EXTERNAL COLUMNS OF BUILDING FRAMES (B.S. CODE)



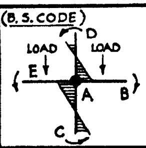
$$M_{AC} = \frac{K_{AC}}{K_{AC}+K_{AD}+nK_{AB}}P_{AB}$$

FOR FRAMES OF ONE BAY WIDTH ONLY : $n = \frac{1}{2}$
" " MORE THAN ONE BAY WIDTH : $n = 1$

$M_{AD} = \dfrac{K_{AD}}{K_{AC}}\cdot M_{AC}$ $\quad M_{AB} = -(M_{AC}+M_{AD})$ $\quad$ FOR TOP-STORY (E.G. ROOF BEAM AND TOP-STORY COLUMN): $K_{AD} = 0$

INTERIOR COLUMNS

(B.S. CODE)



$$M_{AC} = \frac{K_{AC}}{K_{AC}+K_{AD}+K_{AB}+K_{AE}}M_{es}$$

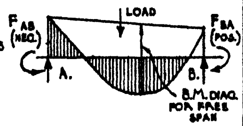
$$M_{AD} = \frac{K_{AD}}{K_{AC}}\cdot M_{AC}$$

$M_{es} = P_{AB}\,\binom{\text{DEAD}}{+\text{LIVE}} - Q_{AE}\,\binom{\text{DEAD}}{\text{ONLY}}$ OR $Q_{AE}\,\binom{\text{DEAD}}{+\text{LIVE}} - P_{AB}\,\binom{\text{DEAD}}{\text{ONLY}}$ } WHICHEVER IS THE GREATER

(M_{AB} AND M_{AE} ARE IN ALL CASES LESS THAN B.M. ASSUMING KNIFE-EDGE SUPPORT FOR BEAM EAB AT A.)

NOTE.—For values of P_{AB} and Q_{AB} see *Table* 18.

MOMENT - DISTRIBUTION METHOD.

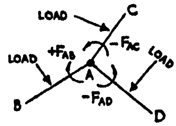


① CONSIDER EACH MEMBER AS FIXED ENDED; END MOMENTS $(F = CL)$ DUE TO EXTERNAL LOADS ON GIVEN MEMBER CAN BE DERIVED FROM * OR CALCULATED.

CONVENTION OF SIGNS FOR B.M.:- RESTRAINING MOMENT CLOCKWISE = POSITIVE.

IF SYMMETRICAL LOAD, $F_{AB} = -F_{BA} = \frac{\text{AREA OF FREE B.M. DIAG}}{\text{SPAN}} = \frac{A}{L_{AB}}$

IF NO EXTERNAL LOAD, $F_{AB} = F_{BA} = $ ZERO.



② WHERE TWO OR MORE MEMBERS MEET, SUM OF B Ms AT JOINT = ZERO (FOR EQUILIBRIUM)

i.e. $M_{AB} + M_{AC} + M_{AD} + \cdots = 0$.

SINCE $\Sigma F \; (= +F_{AB} - F_{AC} - F_{AD} \text{ etc})$ IS UNLIKELY TO EQUAL ZERO, A BALANCING-MOMENT $= -\Sigma F$ MUST BE INTRODUCED AT A.

③ THE BALANCING MOMENT IS DISTRIBUTED BETWEEN THE MEMBERS MEETING AT A IN PROPORTION TO THEIR RELATIVE STIFFNESSES $(K = \frac{I}{L})$ BY MULTIPLYING $-\Sigma F$ BY THE DISTRIBUTION-FACTOR (D) FOR EACH MEMBER. $D_{AB} = \frac{K_{AB}}{\Sigma K}$; $D_{AC} = \frac{K_{AC}}{\Sigma K}$; ETC. $\Sigma D = 1 \cdot 0$

AT FREE END $D = 1 \cdot 0$
AT FIXED END $D = 0$

DISTRIBUTED MOMENT FOR AB $= D_{AB}(-\Sigma F)$, ETC.

④ EFFECT OF APPLYING DISTRIBUTED MOMENT AT ONE END OF MEMBER, IS TO PRODUCE A MOMENT OF HALF THE MAGNITUDE BUT THE SAME SIGN AT THE REMOTE END (TERMED CARRY-OVER). THUS DISTRIBUTED MOMENT $-\Sigma F \cdot D_{AB}$ APPLIED AT A ON AB, PRODUCES MOMENT OF $-0{\cdot}5\,\Sigma F \cdot D_{AB}$ AT B.

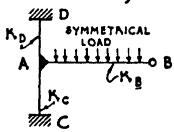
⑤ THE CARRY-OVER MOMENTS AT A FROM THE DISTRIBUTED MOMENTS AT B, C, D, ETC, WILL PRODUCE FURTHER UNBALANCED MOMENTS AT A, WHICH MUST BE DISTRIBUTED AND THE CARRY-OVER PROCESS REPEATED. THESE SUCCESSIVE OPERATIONS ARE REPEATED UNTIL THE UNBALANCED MOMENT IS NEGLIGIBLE. (SEE EXAMPLE BELOW).

EXAMPLE

(THIS EXAMPLE IS WORKED BY SLOPE-DEFLECTION AND BY MOMENT-DISTRIBUTION.)

ASSUME:- FOR SYMMETRICAL LOAD ON AB, VALUE OF $\frac{A}{L} = 24{,}000$ FT-LB.

RATIO OF STIFFNESS FACTORS:- $K_B = K$; $K_C = \frac{2}{3}K$; $K_D = \frac{1}{3}K$



FORMULA DERIVED FROM SLOPE - DEFLECTION

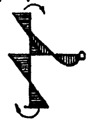
$$M_{AC} = \frac{4K_C}{4K_C + 4K_D + 3K_B} \cdot \frac{1{\cdot}5A}{L} = \frac{4}{7} \times 24{,}000 = +13{,}714 \text{ FT-LB.}$$

$$M_{AD} = \frac{K_D}{K_C} \cdot M_{AC} = \tfrac{1}{2}(13{,}714) = +6{,}857 \text{ FT-LB.} \quad M_{DA} = \tfrac{1}{2} M_{AD} = +3{,}428 \text{ FT-LB.}$$

$$M_{AB} = -M_{AC} - M_{AD} = -(13{,}714 + 6{,}857) = -20{,}571 \text{ FT-LB.} \quad M_{CA} = \tfrac{1}{2} M_{AC} = +6{,}857 \text{ FT-LB.}$$

BY MOMENT-DISTRIBUTION.

$$D_{AB} = \frac{K}{K + \frac{2}{3}K + \frac{1}{3}K} = \frac{1}{2}; \quad D_{AC} = \frac{1}{3}; \quad D_{AD} = \frac{1}{6}\left[= 1 - (\tfrac{1}{2} + \tfrac{1}{3})\right]$$



	M_{CA}	M_{AC}	M_{AB}	M_{AD}	M_{DA}	M_{BA}
DISTRIBUTION FACTORS (D)	0	⅓	½	⅙	0	1·0
*FIXED END B.Ms. (F)	0	0	−24,000	0	0	+24,000
1ST. DISTRIBUTION	0	+8,000	+12,000	+4,000	0	−24,000
1ST. CARRY-OVER	+4,000	0	−12,000	0	+2,000	+6,000
2ND. DISTRIBUTION	0	+4,000	+6,000	+2,000	0	−6,000
2ND. CARRY-OVER	+2,000	0	−3,000	0	+1,000	+3,000
3RD. DISTRIBUTION	0	+1,000	+1,500	+500	0	−3,000
3RD. CARRY-OVER	+500	0	−1,500	0	+250	+750
4TH. DISTRIBUTION	0	+500	+750	+250	0	−750
SUMMATIONS	+6,500	+13,500	−20,250	+6,750	+3,250	0 FT-LB.

NOTE: M_{AB} AFTER 1ST DISTRIBUTION = −12,000 FT.LB
" 2ND. " = −18,000 " "
" 3RD. " = −19,500 " "
" 4TH. " = −20,250 " "
" 5TH " (NOT SHOWN) = −20,437 " "

ACCURACY INCREASES WITH INCREASE IN NUMBER OF DISTRIBUTION OPERATIONS, BUT RESULTS ARE NORMALLY SUFFICIENTLY ACCURATE AFTER TWO DISTRIBUTIONS FOR BEAMS (SEE TABLE No. 11A) AND FOUR FOR OTHER CASES.

ETC. COMPARED WITH $M_{AB} = -20{,}571$ FT-LB. BY SLOPE DEFLECTION.

NOTES.—*$F(= CL)$ can be derived from factors in *Table* 18.
See *Table* 46 for formulæ for slope-deflection method.

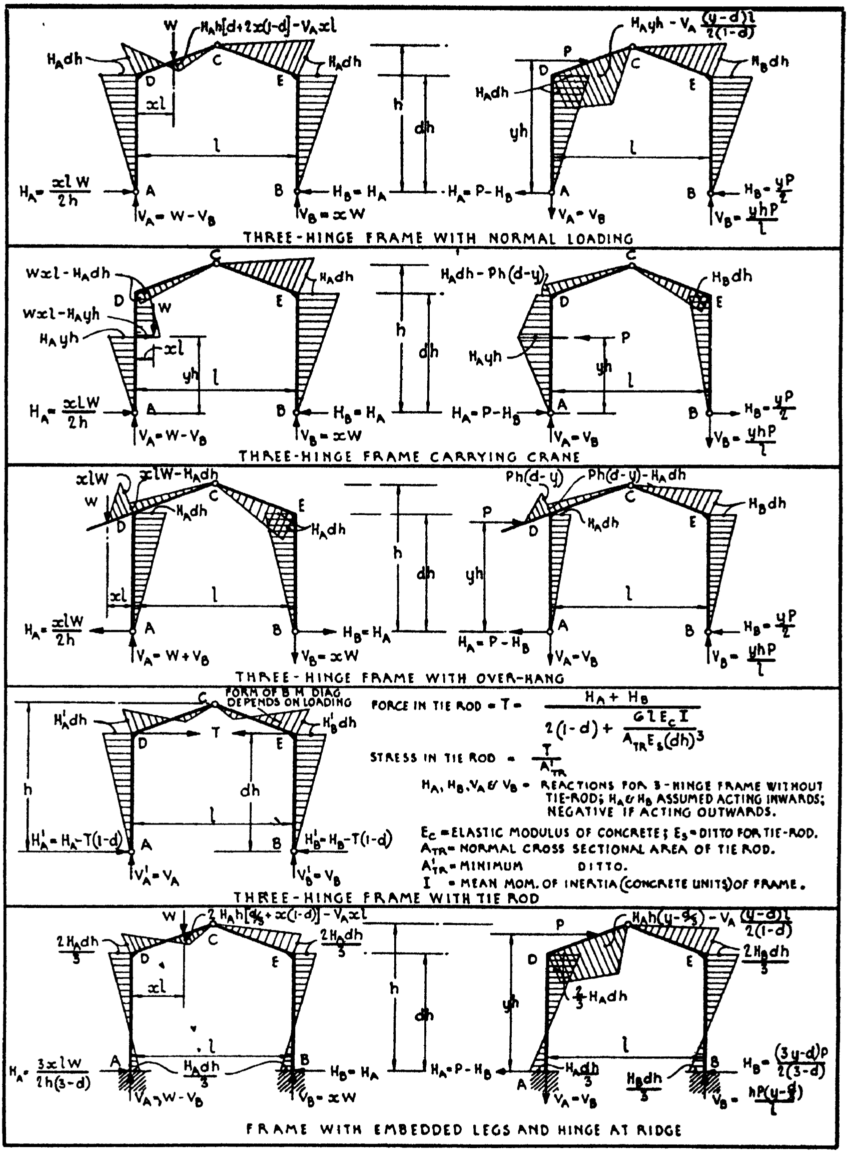
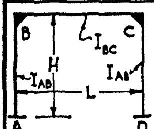
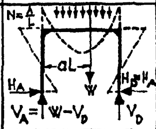
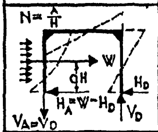
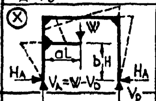
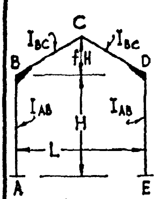
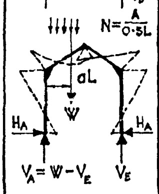
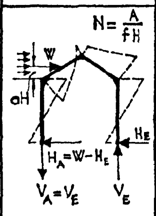
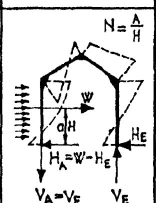
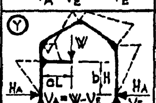

THREE-HINGE FRAME WITH NORMAL LOADING
THREE-HINGE FRAME CARRYING CRANE
THREE-HINGE FRAME WITH OVER-HANG
FORM OF B M DIAG DEPENDS ON LOADING
FORCE IN TIE ROD = T = $\dfrac{H_A + H_B}{2(1-d) + \dfrac{6 l E_c I}{A_{TR} E_s (dh)^3}}$
STRESS IN TIE ROD = $\dfrac{T}{A'_{TR}}$
H_A, H_B, V_A & V_B = REACTIONS FOR 3-HINGE FRAME WITHOUT TIE-ROD; H_A & H_B ASSUMED ACTING INWARDS; NEGATIVE IF ACTING OUTWARDS.
E_c = ELASTIC MODULUS OF CONCRETE; E_s = DITTO FOR TIE-ROD.
A_{TR} = NORMAL CROSS SECTIONAL AREA OF TIE ROD.
A'_{TR} = MINIMUM DITTO.
I = MEAN MOM. OF INERTIA (CONCRETE UNITS) OF FRAME.
THREE-HINGE FRAME WITH TIE ROD
FRAME WITH EMBEDDED LEGS AND HINGE AT RIDGE

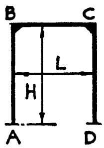
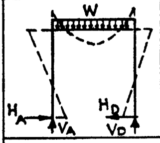
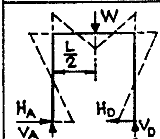
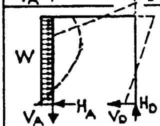
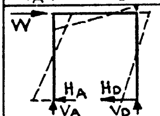
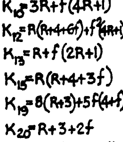
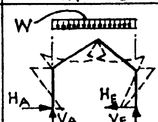
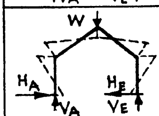
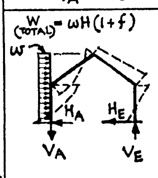
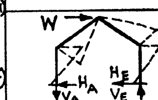
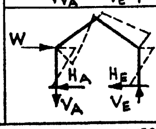
TABLE 49.—FRAMED STRUCTURES: HINGED AND FIXED PORTALS. GENERAL CASES.

FRAME FORM	LOADING	ENDS	REACTIONS AND BENDING MOMENTS.
 $R = \frac{I_{BC} H}{I_{AB} L}$, $K_1 = R + 2$, $K_2 = 6R + 1$, $K_3 = 2R + 3$, $K_4 = 3R + 1$, $K_5 = 2R + 1$, $K_6 = R + 1$		HINGED	$H_A = \frac{3N}{K_3 H}$ $V_D = aW$ $M_A = M_D = 0$ $M_B = M_C = H_A H$
		FIXED	$H_A = \frac{3N}{K_1 H}$ $V_D = aW - 2K_2 N$ $M_B = H_A H - M_A$ $M_C = H_A H - M_D$; $K'_2 = \frac{3(1-2z_1)}{K_2}$ $\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = \left(\frac{1}{K_1} \mp K'_2\right) N$
		HINGED	$H_D = 0.5aW + \frac{3Rz_1}{K_3 H} N$ $V_D = \frac{WaH}{L}$ $M_C = H_D H$ $M_A = M_D = 0$ $M_B = H[H_A - W(1-a)]$
		FIXED	$H_D = 0.5aW + \frac{3N}{K_1 H}(K_5 z_1 - K_6)$ $C_1 = \frac{N}{K_1}[K_3(1-z_1) - Rz_1]$ $V_D = \frac{1}{L}(M_B + M_C)$ $M_C = H_D H - M_D$ $C_2 = \frac{1}{2K_2}[WaHK_4 + 6RN]$ $\left.\begin{matrix} M_A \\ M_D \end{matrix}\right\rangle = C_1 \pm C_2$ $M_B = H(H_D - aW) + M_A$
	⊗ 	HINGED	$H_D = 1{\cdot}5\frac{WaL}{HK_3}\left[(1-b^2)R + 1\right]$ $M_B = WaL - H_A H$ $M_C = H_A H$ $V_D = aW$ $M_A = M_D = 0$
 $R = \frac{I_{BC} H}{I_{AB}\sqrt{\left(\frac{L}{2}\right)^2 + (fH)^2}}$, $K_7 = R + 3 + f(3+f)$, $K_8 = 2R + 6 + 3f + af(3+2f)$, $K_9 = 2R + 6 + 3f$, $K_{10} = 3R + f(4R+1)$, $K_{11} = R - f + 2z_1 f(1+R)$, $K_{12} = R(R + 4 + 6f) + f^2(4R+1)$, $K_{13} = R + f(2R+1)$, $K_{14} = R - f(3+2f) + 3z_1 f(2+R+f)$, $K_{15} = (R + 4 + 3f)R$, $K_{16} = \frac{3(1-z_1)N}{3R+1}$, $K_{17} = R + 2 + f - 2z_1(R+f)$, $K_{18} = 2(R+3) + 2f(3+f) - 3z_1(R+2) - 3z_1 f$		HINGED	$H_A = \frac{WaL(3+2f) + 6(1+z_1 f)N}{4HK_7}$ $V_E = aW$ $M_A = M_E = 0$ $M_C = 0{\cdot}5WaL - H_A H(1+f)$ $M_B = M_D = H_A H$
		FIXED	$H_A = \frac{0{\cdot}5WaL\,K_{10} + 3K_{11}N}{HK_{12}}$ $V_E = Wa - \frac{K_{16}}{L}$ $M_B = H_A H - M_A$ $M_C = 0{\cdot}5(WaL + M_A + M_E) - H_A H(1+f)$ $\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = \left(\frac{0{\cdot}5WaL\,K_{13} + K_{14}N}{K_{12}}\right) \mp 0{\cdot}5K_{16}$ $M_D = H_A H - M_E$
		HINGED	$H_E = \frac{WHK_8 + 6(1+z_1 f)N}{4HK_7}$ $V_E = \frac{WH}{L}(1+a)$ $M_A = M_E = 0$ $M_C = 0{\cdot}5WH(1+a) - H_E H(1+f)$ $M_B = H_A H$ $M_D = H_E H$
		FIXED	$H_E = \frac{WH(K_{15} + aK_{10}) + 6K_{11}N}{2HK_{12}}$ $V_E = \frac{1}{L}[WH(1+a) - M_E - M_A]$ $M_D = H_E H - M_E$ $M_C = 0{\cdot}5[WH(1+a) - M_A + M_E] - H_E H(1+f)$ $M_B = H_A H - M_A$ $\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = \frac{1}{K_{12}}\left[\frac{WH}{2}(f-a)K_{13} - NK_{14}\right] \mp \frac{WH(3f+2) + 6(1-z_1)N}{4K_4}$
		HINGED	$H_E = \frac{WaHK_9 + 6Rz_1 N}{4HK_7}$ $V_E = \frac{WaH}{L}$ $M_D = H_E H$ $M_C = 0{\cdot}5WaH - H_E H(1+f)$ $M_B = H(Wa - H_E)$ $M_A = M_E = 0$
		FIXED	$H_E = \frac{1}{2HK_{12}}(WaHK_{15} - 6RK_{17}N)$ $V_E = \frac{1}{L}(WaH - M_E - M_A)$ $M_B = H(Wa - H_E) - M_A$ $M_C = 0{\cdot}5[WaH - M_A + M_E] - H_E H(1+f)$ $M_D = H_E H - M_E$ $\left.\begin{matrix} M_A \\ M_E \end{matrix}\right\rangle = \frac{1}{K_{12}}(0{\cdot}5WaHfK_{13} + RK_{18}N) \mp \frac{1}{4K_4}[WaH(3R+2) + 6RN]$
	Ⓨ 	HINGED	WHEN ALSO HINGED AT C $H_A = \frac{aLW}{2H(1+f)}$ $M_C = 0$ $V_E = aW$
		HINGED	$H_A = \frac{3WaL}{4H}\cdot\frac{R(1-b^2) + 2 + f}{f(3+f) + 3 + R}$ $M_B = WaL - H_A H$ $M_D = H_A H$ $M_A = M_E = 0$ $M_C = 0{\cdot}5WaL - H_A H(1+f)$

FORMULÆ APPLY TO ANY TYPE OF LOAD (EXCEPT CASES ⊗ AND Ⓨ) AND GIVE NUMERICAL VALUES OF REACTIONS AND MOMENTS; SEE LOADING DIAGRAMS FOR DIRECTION OF ACTION.
CENTRE OF GRAVITY OF TOTAL LOAD W IS AT aL OR aH.
CENTROID OF FREE B.M. DIAGRAM IS AT $z_1 L$, OR $z_1 H$, OR $z_1 f H$, OR $z_1 \frac{L}{2}$ FROM L.H.S. (VERTICAL LOADS) OR LOWER SUPPORT (HORIZ. LOADS)
$\frac{A}{L}, \frac{A}{0{\cdot}5L}, \frac{A}{H}$ OR $\frac{A}{fH} = \frac{\text{AREA OF FREE B.M. DIAGRAM}}{\text{LOADED LENGTH}} = N$

NOTE.—For values of $N\left(= \frac{A}{L}, \text{ etc.}\right)$ see *Table* 18.

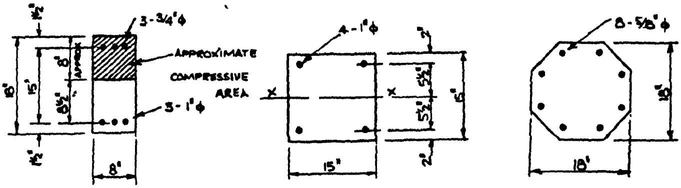
For example of use of this table see page facing *Table* 46.

FORM	LOADING	HINGED AT A AND D (OR E) $M_A = M_D = 0$ OR $M_A = M_E = 0$	FIXED AT A AND D (OR E)
$W =$ TOTAL LOAD $I_{AB} = I_{DC}$ $R = \frac{I_{BC}H}{I_{AB}L}$  $K_1 = R+2$ $K_2 = 6R+1$ $K_3 = 2R+3$ $K_4 = 3R+1$		$H_A = H_D = \frac{WL}{4HK_3}$ $V_A = V_D = \frac{1}{2}W$ $M_B = M_C = H_AH = \frac{WL}{4K_3}$	$H_A = H_D = \frac{WL}{4HK_1}$ $V_A = V_D = \frac{1}{2}W$ $M_B = M_C = \frac{WL}{6K_1}$ $M_A = M_D = \frac{WL}{12K_1}$
		$H_A = H_D = \frac{3WL}{8HK_3}$ $V_A = V_D = \frac{1}{2}W$ $M_B = M_C = H_AH = \frac{3WL}{8K_3}$	$H_A = H_D = \frac{3WL}{8HK_1}$ $V_A = V_D = \frac{1}{2}W$ $M_B = M_C = \frac{WL}{4K_1}$ $M_A = M_D = \frac{WL}{8K_1}$
		$V_D (= -V_A) = \frac{WH}{2L}$ $H_A = \frac{W}{8}\left(\frac{6K_3 - R}{K_3}\right)$; $H_D = W - H_A$ $M_B = H(\frac{1}{2}W - H_D) = \frac{3WHK_1}{8K_3}$ $M_C = H_DH = \frac{WH}{8}\left(\frac{2K_3+R}{K_3}\right)$	$H_D = \frac{WK_3}{8K_1}$ $H_A = W - H_D$ $V_D (= -V_A) = \frac{WHR}{LK_2}$ $\left.\begin{matrix}M_A\\M_D\end{matrix}\right\rangle \frac{WH}{4}\left[\frac{R+3}{6K_1} \pm \frac{4R+1}{K_2}\right]$ $M_B = (H_A - \frac{1}{2}W)H - M_A$ $M_C = H_DH - M_D$
		$H_A = H_D = \frac{1}{2}W$ $V_D (= -V_A) = \frac{WH}{L}$ $M_B = M_C = \frac{1}{2}WH$	$H_A = H_D = \frac{1}{2}W$ $V_D (= -V_A) = \frac{3WHR}{LK_2}$, $M_A = M_D = \frac{WHK_4}{2K_2}$ $M_B = M_C = \frac{3WHR}{2K_2}$
$I_{AB} = I_{ED}$ $I_{BC} = I_{CD}$ $BC = CD = \ell$ $= \sqrt{\left(\frac{L}{2}\right)^2 + (fH)^2}$ $R = \frac{I_{BC}H}{I_{AB}\ell}$  $K_7 = R+3+f(3+f)$ $K_9 = 2R+3(2+f)$ $K_{10} = 3R+f(4R+1)$ $K_{12} = R(R+4+6f)+f^2(4R+1)$ $K_{13} = R+f(2R+1)$ $K_{15} = R(R+4+3f)$ $K_{19} = 8(R+3)+5f(4+f)$ $K_{20} = R+3+2f$ $K_{21} = 2R(R+4+5f)$ $+f^2(5R+1)$ $K_{22} = 2(10R+3)+$ $f(9R+1)$		$H_A = H_E = \frac{WL}{32H}\left[\frac{8+5f}{K_7}\right]$ $V_A = V_E = \frac{1}{2}W$ $M_B = M_D = H_AH$ $M_C = H_AH(1+f) - \frac{WL}{8}$	$H_A = H_E = \frac{WL}{8H}\left[\frac{4R+f(5R+1)}{K_{12}}\right]$ $V_A = V_E = \frac{1}{2}W$ $M_A = M_E = \frac{WL}{48}\left[\frac{R(8+15f)+f(6-f)}{K_{12}}\right]$ $M_B = M_D = H_AH - M_A$ $M_C = H_AH(1+f) - \frac{WL}{8} - M_A$
		$H_A = H_E = \frac{WL}{8H}\left[\frac{3+2f}{K_7}\right]$ $V_A = V_E = \frac{1}{2}W$ $M_B = M_D = H_AH$ $M_C = H_AH(1+f) - \frac{WL}{4}$	$H_A = H_E = \frac{WL}{4H}\left[\frac{K_{10}}{K_{12}}\right]$ $V_A = V_E = \frac{1}{2}W$ $M_A = M_E = \frac{WL}{4}\left[\frac{K_{13}}{K_{12}}\right]$ $M_B = M_D = H_AH - M_A$ $M_C = H_AH(1+f) - \frac{1}{4}WL - M_A$
		$V_E (= -V_A) = \frac{\omega H^2}{2L}\left[1+f(2+f)\right]$ $H_E = \frac{\omega H}{16K_7}\left[2K_9+R+fK_{19}\right]$ $H_A = \omega H(1+f) - H_E$ $M_B = H_AH - \frac{1}{2}\omega H^2$; $M_D = H_EH$ $M_C = H_EH(1+f) - \frac{V_EL}{2}$	$H_E = \frac{\omega H}{4K_{12}}\left[RK_{20}+fK_{21}\right]$ $H_A = W - H_E$ $V_E (= -V_A) = \frac{\omega H^2}{8LK_4}\left[4R(1+3f)+f^2(12R+5)\right]$ $\left.\begin{matrix}M_A\\M_E\end{matrix}\right\rangle \frac{\omega H^2}{24}\left[\frac{R(R+6+15f)+f^2K_{22}}{K_{12}}\right.$ $\left.\pm \frac{12K_1+f\left[12(3R+2)+3f\right]}{2K_4}\right]$ $M_B = H_AH - M_A - \frac{1}{2}\omega H^2$ $M_D = H_EH - M_E$ $M_C = H_EH(1+f) - \frac{1}{2}LV_E - M_E$
		$H_A = H_E = \frac{1}{2}W$ $V_E (= -V_A) = \frac{WH(1+f)}{L}$ $M_B = M_D = \frac{1}{2}WH$ $M_C = 0$	$H_A = H_E = \frac{W}{2}$ $V_E (= -V_A) = \frac{1}{L}\left[WH(1+f) - 2M_E\right]$ $M_A = M_E = \frac{WH}{4}\left[\frac{3R+2}{2R+1}\right]$ $M_B = M_E = H_AH - M_A$; $M_C = 0$
		$H_E = \frac{WK_9}{4K_7}$; $H_A = W - H_E$ $V_E (= -V_A) = \frac{WH}{L}$ $M_B = H_AH$; $M_D = H_EH$ $M_C = H_EH(1+f) - \frac{WH}{2}$	$H_E = \frac{WR}{2}\left[\frac{K_{15}}{K_{12}}\right]$; $H_A = W - H_E$ $V_E (= -V_A) = \frac{3WH}{2L}\left[\frac{R}{3R+1}\right]$ $\left.\begin{matrix}M_A\\M_E\end{matrix}\right\rangle \frac{WH}{2}\left[\frac{fK_{13}}{K_{12}} \pm \frac{3R+2}{6R+2}\right]$ $M_B = H_AH - M_A$; $M_D = H_EH - M_E$ $M_C = H_EH(1+f) - \frac{1}{2}LV_E - M_E$

NOTE.—FORMULÆ GIVE NUMERICAL VALUES OF REACTIONS AND MOMENTS; SEE DIAGRAMS FOR DIRECTION OF ACTION.

NOTES ON FRAMED STRUCTURES.

Ratio of Moments of Inertia of Members of Frames.—In the formulæ for the bending moments on the members of framed structures, only the relative stiffness factors of adjacent members are required, that is the relative moments of inertia are required. The following examples show the accuracy of various methods of comparison. ($m = 15$.)



(a) RECTANGULAR BEAM (b) SQUARE COLUMN BENDING ABOUT X.X (COMPRESSION ONLY) (c) OCTAGONAL COLUMN (COMPRESSION ONLY)

(1) Find the ratio of the moments of inertia of the column in (*b*) and the beam in (*a*).

(i) Complete concrete areas only.—$\frac{I_C}{I_B} = \frac{15^4}{8 \times 18^3} = 1{\cdot}08.$

(ii) Stressed areas only.—Column: Concrete, $0{\cdot}083 \times 15^4 = 4210$ in.^{4}
Steel, $14 \times 3{\cdot}14 \times 5{\cdot}5^2 = 1330$,, $I_C = 5540$ in.^{4}
Beam: Concrete $0{\cdot}33 \times 8 \times 8^3 = 1360$,,
Compression steel, $14 \times 1{\cdot}33 \times 6{\cdot}5^2 = 790$,,
Tension steel, $15 \times 2{\cdot}36 \times 8{\cdot}5^2 = 2550$,, $I_B = 4700$ in.^{4}

$\frac{I_C}{I_B} = \frac{5540}{4700} = 1{\cdot}18.$ (Compare with 1·08.)

(2) Find the ratio of the moments of inertia of the beam in (*a*) and the column in (*c*).

(i) Complete concrete areas only.—From *Table* 64 the moment of inertia of an octagonal section is $0{\cdot}055D^4$. Therefore $\frac{I_C}{I_B} = \frac{0{\cdot}055 \times 18^4}{0{\cdot}083 \times 8 \times 18^3} = 1{\cdot}49.$

(ii) Stressed areas only: I_C [by calculation similar to (1)] $= 6780$ in.^{4}
I_B as calculated in (1) $= 4700$ in.^{4} $\frac{I_C}{I_B} = \frac{6780}{4700} = 1{\cdot}44.$ (Compare with 1·49)

Columns in Buildings: Approximate Method.—For preliminary calculations for the design of columns in a multiple-story building frame, it is often convenient to design for resistance to direct load only and to allow for the effects of bending either (i) by decreasing the maximum stresses, or (ii) by increasing the load to a value which is equivalent to that producing the same stresses as would occur if the actual load and bending moment were combined. The factors in the accompanying table apply to method (ii) and are applicable to most normal building frames. The diagrams indicate arrangements of beams supported by the column.

EQIVALENT DIRECT LOAD ON COLUMN

TO ALLOW FOR BENDING ON COLUMS, EQUIVALENT DIRECT LOAD = F X (LOAD BASED ON STATIC REACTIONS)

	STORY	TOP	TOP BUT ONE	LOWER
VALUES OF F		1·0	1·0	1·0
		4·5	2·0	1·4
		6·0	2·3	1·8

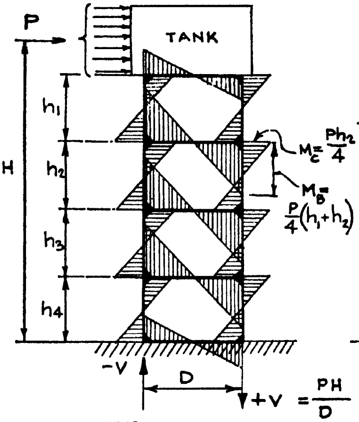
Example of Use of Table 51.—Find the approximate bending moments and forces in the columns of a gantry; the vertical load on each pair of columns is 100 tons, and the horizontal wind load is 5 tons. The height of the columns is 48 ft. braced at top and bottom and with intermediate braces at 12-ft. centres; the distance apart of the columns at the base is 20 ft.; thus $P = 5$ tons; $W = \frac{1}{2} \times 100 = 50$ tons. $H = 48{\cdot}0$; $h = 12{\cdot}0$; $D = 20{\cdot}0$.

Bending moment on columns at junction with brace, $\frac{5 \times 12{\cdot}0}{4} = 15$ ft.-tons.

Bending moment in brace at junction with columns, $2 \times 15 = 30$ ft.-tons.

The shearing force on the brace (constant throughout length) $= \frac{2 \times 30}{20} = 3$ tons.

The vertical load on the leeward column is $50 + \frac{5 \times 48}{20} = 62$ tons plus the dead load of the column and half of the braces. The vertical load on the windward column is $50 - \frac{5 \times 48}{20} = 38$ tons plus the dead load of the column and half of the braces.



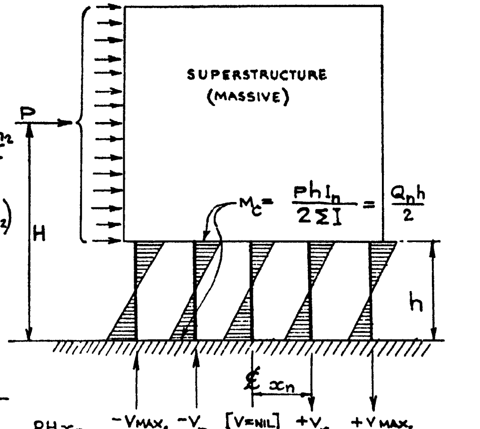
SHEARING FORCES.

ON COLUMN $Q_C = \frac{1}{2}P$

ON BRACE $Q_B = \frac{2M_B}{D}$

ASSUMPTION – BOTH COLUMNS OF SAME SIZE AND VERTICAL OR NEARLY VERTICAL.

(a).– OPEN BRACED TOWER



$V_n = \frac{PHx_n}{\Sigma x^2}$

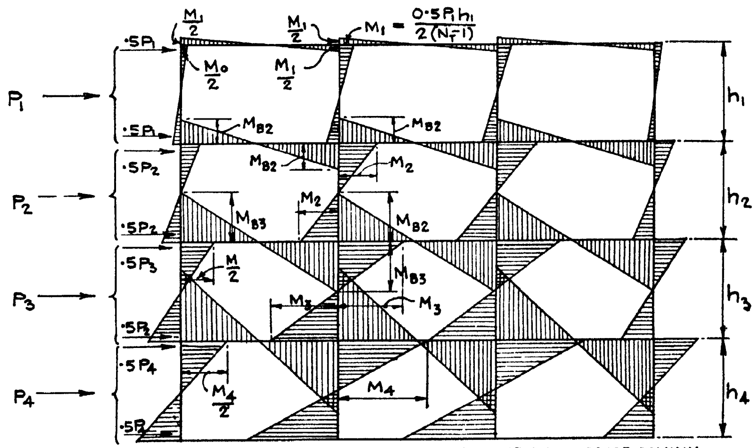
I_n = MOMENT OF INERTIA OF ANY COLUMN n. (COLUMNS OF DIFFERENT SIZES.)

N = TOTAL NUMBER OF COLUMNS SUPPORTING SUPERSTRUCTURE

SHEARING FORCE ON COLUMN n: $Q_n = \frac{PI_n}{\Sigma I}$

IF ALL COLUMNS ARE SAME SIZE: $M_C = \frac{Ph}{2N}$.

(b).– SUBSTRUCTURE OF SILO (OR SIMILAR STRUCTURE)



ASSUMPTION. – EXTERIOR COLUMN SUBJECTED TO HALF SHEARING FORCE ON INTERIOR COLUMN.

N_T = TOTAL NUMBER OF COLUMNS IN FRAME.

$$M_n = \frac{\Sigma_1^{n-1}P + \frac{1}{2}P_n}{N_T - 1}\left(\frac{h_n}{2}\right); \quad M_{Bn} = \frac{1}{2}(M_n + M_{n+1})$$

EXAMPLE WITH $N_T = 4$.– $M_1 = \frac{P_1h_1}{12}$; $M_2 = (P_1 + \frac{1}{2}P_2)\frac{h_2}{6}$; $M_3 = (P_1 + P_2 + \frac{1}{2}M_3)\frac{h_3}{6}$; ETC.

$M_{B1} = \frac{1}{2}M_1$; $M_{B2} = \frac{1}{2}(M_1 + M_2)$; $M_{B3} = \frac{1}{2}(M_2 + M_3)$; ETC.

(c).– FRAME OF MULTIPLE-STORY BUILDING.

NOTE.—Explanations of the effects of wind forces (or other lateral loads) on the types of structures illustrated in *Table* 51 are given on page 40.

ARCHES.

Two-hinge Arch.—Referring to the diagram in *Table* 52, there is no bending moment at the springings of a two-hinge arch. The vertical component of the thrust at the springings is the same as for a freely-supported beam; the horizontal component H is as given by the formula in *Table* 52, in which M_x is the bending moment on a section at a distance x from the crown considering the arch as a freely-supported beam, that is M_x is as given by the corresponding expression in *Table* 52. The summations $\Sigma M_x y s_I$ and $\Sigma y^2 s_I$ are taken over the whole length of the arch. The formula for H allows for the elastic contraction of the arch. A is the average equivalent area of the arch rib or slab; s is the length of a short segment of the axis of the arch or slab; the ordinates of s are x and y as shown in *Table* 52, and if I is the moment of inertia of the arch at x, $s_I = \frac{s}{I}$. The bending moment at any section is $M_x = M - Hy$.

The procedure is to divide the axis of the arch into an even number of segments and to calculate the value of H. The calculation can be facilitated by tabulation of the successive steps. The total bending moment need only be determined generally for the crown ($x = 0$, $y = R$) and the first quarter-point ($x = 0{\cdot}25L$). The bending moment M_c at the crown is the bending moment for a freely-supported beam minus HR. For the maximum positive bending moment at the crown the sum of the values of M_c for all elements of dead load is added to the values of M_c for only those elements of live load that give positive values of M_c. For the maximum negative bending moment at the crown the sum of the values of M_c for all elements of dead load is added to the values of M_c for those elements of live load only that give negative values of M_c. The bending moment at the first quarter-point is the bending moment for a freely-supported beam minus Hy_Q, where y_Q is the vertical ordinate of the first quarter-point. The bending moment is combined with the normal component of H.

For an arch of large span it is worth while drawing the influence lines ($W = 1$) for the bending moments at the crown and at the first quarter-point.

Approximate Determination of Thickness of a Fixed Arch.—Referring to the diagram at the top of *Table* 53, draw a horizontal line through the crown C, and find G, the point of intersection with the vertical through the centre of gravity of the total load on half the span of the arch. Set off GT equal to the dead load on the half-span, drawn to a convenient scale; draw a horizontal through T to intersect GS produced at R. Draw RK perpendicular to GR, and GK parallel to the tangent to the arch axis at S. With the same unit of weight used in drawing GT, scale off TR, which equals H_C, and GK which equals H_S. If c is the maximum allowable compressive stress in the concrete, d is the thickness of the arch at the crown, d_s the thickness of the arch at the springing, and b the assumed breadth of the arch (12 in. for a slab), then approximately $d = \frac{1{\cdot}7H_C}{cb}$, and $d_s = \frac{2H_S}{cb}$.

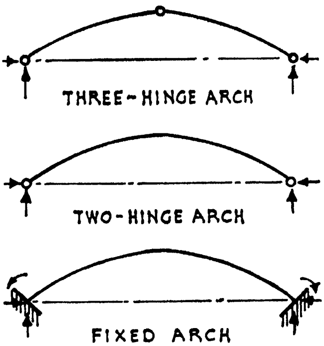
The method applies only to spans from 40 ft. to 200 ft., and span-rise ratios between 4 and 8. The method does not depend on knowing the profile of the arch (except for solid-spandrel earth-filled arches where the dead load is largely dependent on the shape of the arch), but the span and rise must be known. With d and d_s thus approximately determined, the thrusts and bending moments at the crown, springing, and quarter-points can be determined and the stresses on the assumed sections calculated. If this calculation shows the sections to be unsuitable, other dimensions must be assumed and the calculations reworked.

Stresses in a Fixed Arch of Any Profile.—The following method is suitable for determining the stresses in any symmetrical fixed arch if the dimensions and shape of the arch are known, or assumed, and if the shape of the arch must conform to a specified profile. Reference should be made to page 44 for general comments on this method.

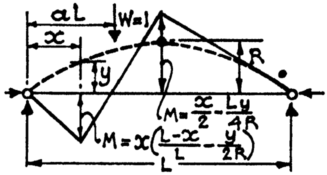
On half the arch drawn to scale, as in *Table* 53, plot the arch axis. Divide the half-arch into a number n of segments such that each segment has the same ratio s_I of length s to mean moment of inertia I based on the thickness of the arch measured normal to the axis, allowance being made for the reinforcement; $s_I = \frac{s}{I}$.

The ordinates x and y about the axis of the arch at the crown are determined by measurement to the centre of the length of each segment. Calculate the dead load and the live load separately on each segment. Assume the loads act at the centre of the length of each segment. In an open spandrel arch, the dead and live loads are concentrated on the arch at the positions of the columns; these positions should be taken as the centre of the segments; but it may not then be possible to maintain a constant value of s_I, and the value of s_I for each segment must be calculated; the general formulæ in *Table* 53 are then applicable.

(*Continued on page* 232.)

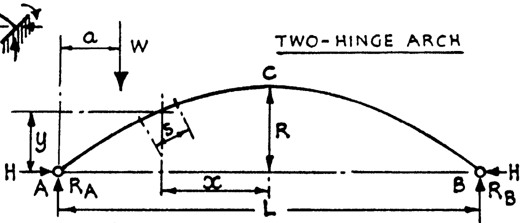


TYPES OF ARCHES



THREE-HINGE ARCH
INFLUENCE LINE FOR SECTION AT x

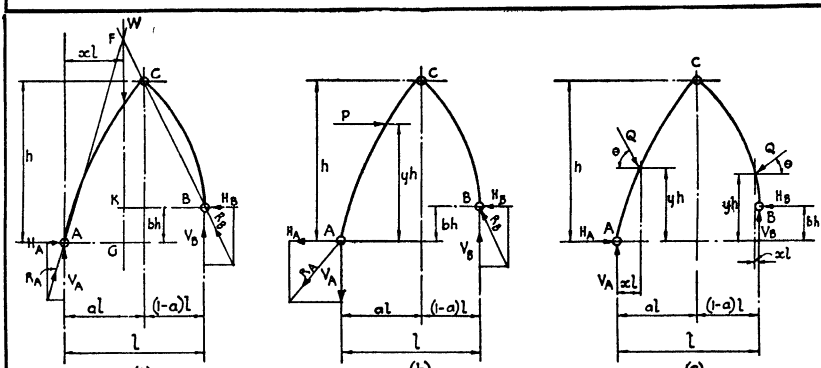
TWO-HINGE ARCH



B.M. AT x: $M_x = Wa\left(\frac{x}{L}+\frac{1}{2}\right)$

HORIZONTAL THRUST: $H = \frac{\Sigma M_x y s_1}{(\Sigma y^2 s_1) + L/A}$

UNSYMMETRICAL THREE-HINGE ARCHES.



(a) (b) (c)

REACTION FORMULÆ FOR GENERAL CASE (c):—

$$V_A = \sum_A^C\left[1-\frac{(1-b)x}{1-ab}\right]Q\sin\theta + \frac{1}{1-ab}\sum_B^C xQ\sin\theta + \frac{h}{(1-ab)l}\left[\sum_B^C(y-b)Q\cos\theta - (1-b)\sum_A^C yQ\cos\theta\right]$$

$$V_B = \frac{1-b}{1-ab}\sum_A^C xQ\sin\theta + \sum_B^C\left[1-\frac{x}{1-ab}\right]Q\sin\theta + \frac{h}{(1-ab)l}\left[(1-b)\sum_A^C yQ\cos\theta - \sum_B^C(y-b)Q\cos\theta\right]$$

$$H_A = \frac{l}{(1-ab)h}\left[(1-a)\sum_A^C xQ\sin\theta + a\sum_B^C xQ\sin\theta\right] + \frac{a}{1-ab}\sum_B^C(y-b)Q\cos\theta - \sum_A^C\left[1-\frac{(1-a)y}{1-ab}\right]Q\cos\theta$$

$$H_B = \frac{l}{(1-ab)h}\left[(1-a)\sum_A^C xQ\sin\theta + a\sum_B^C xQ\sin\theta\right] + \frac{1-a}{1-ab}\sum_A^C yQ\cos\theta - \sum_B^C\left[1-\frac{(y-b)a}{1-ab}\right]Q\cos\theta$$

NOTE.—See facing page for explanation of symbols in, and notes on, the expressions for two-hinge arches.

ARCHES (*continued*).

Stresses in a Fixed Arch of Any Profile (*continued from page* 230).

For constant values of s_I, the effects at the crown are as follows.

$$H_C = \frac{n\Sigma M_L y - \Sigma y.\Sigma M_L}{2[n\Sigma y^2 - (\Sigma y)^2]}; \quad V_C = \frac{\Sigma M_L x}{2\Sigma x^2}; \quad M_C = \frac{\Sigma M_L - 2H_C\Sigma y}{2n}.$$

The summations are taken over one-half of the arch. The term M_L is the moment at the centre of the segment of all loads between the centre of the segment and the crown. Summations are also made for the loads on the other half, for which V_C is negative.

Due to elastic shortening of the arch due to H_C:

$$H_{CS} = -\frac{H_C n\Sigma\frac{s}{A}}{s_I[n\Sigma y^2 - (\Sigma y)^2]}; \quad M_{CS} = -\frac{H_{CS}\Sigma y}{n},$$

in which A is the cross-sectional area of the segment calculated on the same basis as I.

Due to a rise $(+t)$ or a fall $(-t)$ in temperature:

$$H_{CT} = \frac{\pm\, telnE_c}{2s_I[n\Sigma y^2 - (\Sigma y)^2]}; \; M_{CT} = -\frac{H_{CT}\Sigma y}{n},$$

in which l is the length of the arch axis. Arch shortening due to H_{CT} is neglected. Shrinking of the concrete is equal to a fall of 15 deg. F.

The procedure for applying the foregoing formulæ is to calculate $(H_C - H_{CS})$, V_C, and $(M_C - M_{CS})$ for the dead load. Calculate separately H_{CT} and M_{CT} for a rise and fall of temperature of 30 deg. F. (or any other suitable amount) and separately for shrinking. Calculate $(H_C - H_{CS})$ and $(M_C - M_{CS})$ for the live load (reduced to an equivalent uniformly-distributed load) which, for the purpose of finding the maximum bending moment at the crown (and the corresponding horizontal thrust), should be considered as operating on the segments in the middle-third of the arch. (By considering the effect of the live load on one segment more and one less than those in the middle third, the number of segments that should be loaded to give the maximum positive bending moment due to live load at the crown can be determined.) With the live load only on those segments which are unloaded when calculating the maximum positive bending moment, the maximum negative bending moment at the crown due to the live load is obtained. These maximum bending moments are then each combined with the bending moments due to dead load and arch shortening, and with the bending moments due to change of temperature and to shrinking, in such a way that absolute maximum and minimum values are obtained. The corresponding thrusts are also calculated and are combined with the appropriate bending moments to determine the stresses at the crown.

The bending moment at the springing due to a load at a point between the springing and the crown of the arch and at a distance aL from the springing (L being the span of the arch) is

$$M_S = (M_C - M_{CS}) + (H_C - H_{CS})R + 0{\cdot}5LV_C - WaL,$$

where R is the rise of the arch. For the dead load the values determined for the crown are substituted in this expression, with the term WaL replaced by $\Sigma W(0{\cdot}5L - x)$. To give the maximum negative bending moment at the springing, the live load is considered as acting only on those segments in the part of the arch extending 0·4 of the span from the support. (As before, the effect of the live load on one more and one less segment should be determined to ensure that the most adverse disposition of the loading has been considered.) By loading only those segments that are unloaded when finding the maximum negative bending moment, the maximum positive bending moment is obtained. These maximum moments are each combined with the bending moments due to dead load, temperature, and shrinking to give the most adverse combination, and the stresses at the springing are obtained by combining the bending moments and normal thrusts.

Thrust normal to the section at the springing: $T_S = (H_C - H_{CS})\cos\phi + V_S\sin\phi$.

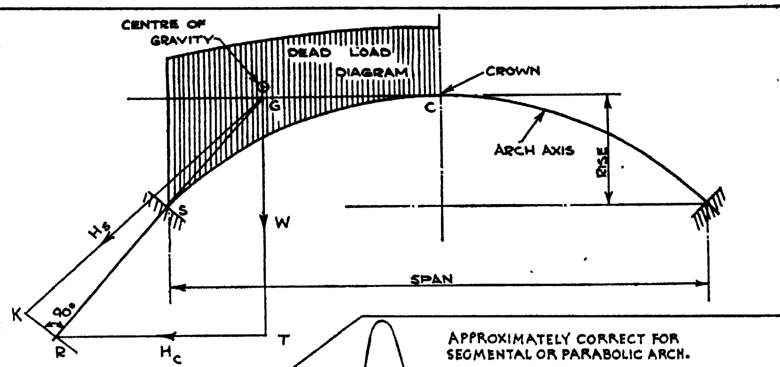
Vertical component of thrust at springing: V_S = [total load on half arch giving the appropriate value of $(H_C - H_{CS})$] minus [V_C corresponding to $(H_C - H_{CS})$].

Shearing force at crown = maximum value of V_C due to any combination of dead and live load.

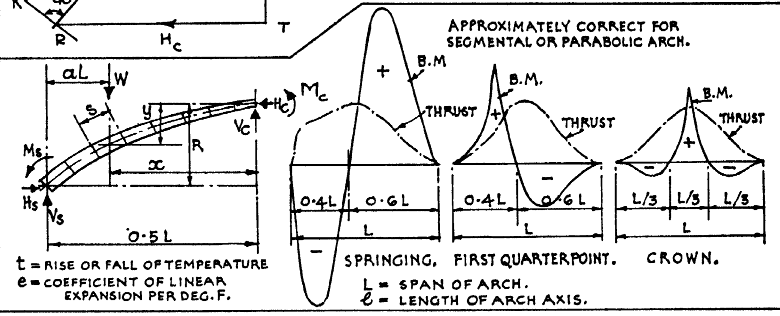
Shearing force at springing $= (H_C - H_{CS})\sin\phi + V_S\cos\phi$; the values of $(H_C - H_{CS})$ and V_S are those due to the incidence of dead plus live load that gives the maximum value for the shearing force, which is generally when the live load extends over the whole arch.

Quarter-point ($x_Q = 0{\cdot}25L$ from the crown).—Bending moment due to a single load W placed between the section and the crown at a distance $(x_Q - x')$ from the section: $M_Q = M_C + H_C y_Q + V_C x_Q - W(x_Q - x')$. The procedure is then similar to the analysis of the section at the springing.

DETERMINATION OF TRIAL SIZES — APPROXIMATE METHOD



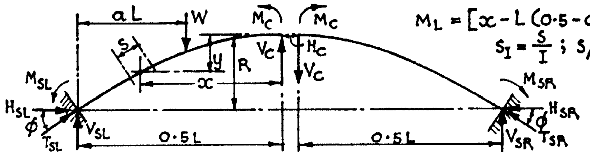
NOTATION AND TYPICAL INFLUENCE LINES



t = RISE OR FALL OF TEMPERATURE
e = COEFFICIENT OF LINEAR EXPANSION PER DEG. F.

L = SPAN OF ARCH.
ℓ = LENGTH OF ARCH AXIS.

GENERAL FORMULÆ



$$M_L = [x - L(0{\cdot}5 - a)]W. \qquad s_I = \frac{s}{I}\,;\quad s_A = \frac{s}{A}.$$

LOAD W ON LEFT-HAND SIDE:

$$H_c = \frac{\Sigma s_I \Sigma M_L y s_I - \Sigma y s_I \Sigma M_L s_I}{2[\Sigma s_I \Sigma y^2 s_I - (\Sigma y s_I)^2]}, \qquad M_c = \frac{\Sigma M_L s_I - 2H_c \Sigma y s_I}{2\Sigma s_I};$$

$$V_c = \frac{\Sigma M_L x s_I}{2\Sigma x^2 s_I}.$$

TEMPERATURE AND SHRINKING:

$$H_{CT} = \frac{(\pm t)e\ell E_c \Sigma s_I}{2[\Sigma s_I \Sigma y^2 s_I - (\Sigma y s_I)^2]}; \qquad M_{CT} = -\frac{H_{CT}\Sigma y s_I}{\Sigma s_I}.$$

E_c = ELASTIC MODULUS OF CONCRETE

ARCH SHORTENING:

$$H_{CS} = -\frac{\Sigma s_A \Sigma s_I (H_c + H_{CT})}{\Sigma s_I \Sigma y^2 s_I - (\Sigma y s_I)^2}; \qquad M_{CS} = -\frac{H_{CS}\Sigma y s_I}{\Sigma s_I}$$

SUMMATIONS ARE TAKEN OVER LEFT-HAND HALF OF SPAN; NOTE THAT SUMMATIONS INVOLVING M_L ONLY INCLUDE SEGMENTS WITHIN aL.

AT CROWN. NET THRUST $= H'_c = H_c + H_{CT} + H_{CS}$ (NOTE SIGNS OF H AND M.)
NET B. M. $= M'_c = M_c + M_{CT} + M_{CS}$

AT SPRINGINGS. $M_{SL} = M'_c + H'_c R + 0{\cdot}5LV_c - WaL$; $V_{SL} = W - V_c$
$M_{SR} = M'_c + H'_c R - 0{\cdot}5LV_c$; $V_{SR} = V_c$; $H_{SL} = H_{SR} = H'_c$.
$T_{SL} = H_{SL}\cos\phi + V_{SL}\sin\phi$; $T_{SR} = H_{SR}\cos\phi + V_{SR}\sin\phi$.

AT ANY SECTION x_Q FROM CROWN.
IN LEFT-HAND HALF: $M_Q = M'_c + H'_c y_Q + V_c x_Q - W[x_Q - L(0{\cdot}5 - a)]$ (INCLUDE W-TERM ONLY IF WITHIN x_Q)
$H_Q = H'_c$; $V_Q = V_c - W$.
IN RIGHT-HAND HALF: $M_{QR} = M'_c + H'_c y_Q - V_c x_Q$; $H_{QR} = H'_c$; $V_{QR} = V_c$

NOTE.—For explanation of the approximate method of determining the size of a fixed arch, see page facing *Table* 52.

FIXED PARABOLIC ARCHES.

Reference should be made to page 45 for comments on the method of analysing a fixed parabolic arch as given by the data in *Table* 54. The formulæ, basis of the coefficients, and an example are given in the following.

Dead Load and Elastic Contraction.—Horizontal thrust due to the dead load alone, $H = \frac{k_1 w_d L^2}{R}$, in which w_d = dead load per unit length at the crown; L = span, R = rise of the arch axis.—The coefficient k_1 depends on the dead load at the springing, which varies with the ratio of rise to span and the type of structure, that is whether the arch is open-spandrel or solid-spandrel, or whether the dead load is uniform throughout the span.

Elastic contraction produced by the thrust along the arch axis (assuming rigid abutments).—The counter-thrust H_D, while slightly reducing the thrust due to the dead load, renders this thrust eccentric and produces a positive bending moment at the crown and a negative bending moment at the springings. If d is the thickness of the arch at the crown,

$$H_D = -k_2\left(\frac{d}{R}\right)^2 H,$$

in which the coefficient k_2 depends on the relative thicknesses at the crown and springing.

Due to dead load and arch shortening, the resultant thrusts H_C and H_S at the crown and springing respectively, and acting parallel to the arch axis at these points, are

$$H_C = H - H_D; \qquad H_S = \frac{H}{\cos\phi} - H_D \cos\phi,$$

in which ϕ is the angle between the horizontal and the tangent to the arch axis at the springing. Values of $\cos\phi$ are given in *Table* 54.

Bending moments due to the eccentricity of H_C and H_S: $M_C = k_3 R H_D$; $M_S = (k_3 - 1) R H_D$.

Effect of Change of Temperature.—Additional horizontal thrust due to a rise in temperature or the corresponding counter-thrust due to a fall in temperature:

$$H_T = \pm k_4\left(\frac{d}{R}\right)^2 dt,$$

in which t is the rise or fall in temperature in deg. F. If d and R are in feet, H_T is in lb. per foot width of arch. The values of k_4 in *Table* 54 are based on an elastic modulus for concrete E_C of 2,000,000 lb. per square inch, and a coefficient of linear expansion e of 0·0000066 per deg. F. If other values, E_1 and e_1, are adopted, k_4 should be multiplied by 0·076 $E_1 e_1$. At the crown the increment or decrease in normal thrust due to change of temperature is H_T and the bending moment is $-k_3 R H_T$, account being taken of the sign of H_T. The normal thrust at the springing due to change of temperature is $H_T \cos\phi$, and whether the thrusts due to dead load, etc., are increased or decreased thereby depends upon the sign of H_T. At the springing the bending moment is $(1 - k_3) R H_T$, the sign being the same as that of H_T.

Shrinking of Concrete.—Shrinking can be considered to be equivalent to a fall of temperature of 15 deg. F.

Live Load.—The intensity of uniformly-distributed load equivalent to the specified live load = w.

Maximum positive bending moment at the crown = $k_5 w L^2$; horizontal thrust $= k_6 w \frac{L^2}{R}$.

Maximum negative bending moment at springing = $k w_7 L^2$; horizontal thrust = $k_8 w \frac{L^2}{R}$; vertical reaction = $k_9 w L$.

Maximum positive bending moment at springing = $k_{10} w L^2$; horizontal thrust = $k_{11} w \frac{L^2}{R}$; vertical reaction = $k_{12} w L$.

If H and V are the corresponding horizontal thrust and vertical reaction, normal thrust at the springing = $H\cos\phi + V\sqrt{1 - \cos^2\phi}$.

Dimensions of Arch.—The line of pressure, and therefore the arch axis, can now be plotted as described on page 46. The thicknesses of the arch at the crown and springing having been determined, the lines of the extrados and intrados can be plotted to give a parabolic variation of thickness between the two extremes. Thus, the thickness normal to the axis of the arch at any point is given by $[(d_s - d)r + d]$ where r has the following values: if the ratio of the distance of the point from the springing, measured along the axis of the arch, to half the length of the axis of the arch is $\frac{1}{4}$, the value of r is 0·563; if the ratio is $\frac{1}{2}$, r is 0·250; and if the ratio is $\frac{3}{4}$, r is 0·063.

An example of the application of *Table* 54 and the foregoing method is given on the page facing *Table* 55.

TYPE		UNIFORM DEAD LOAD				OPEN SPANDREL				SOLID SPANDREL			
RISE : SPAN		0·10	0·15	0·20	0·25	0·10	0·15	0·20	0·25	0·10	0·15	0·20	0·25
INCLINATION OF AXIS OF ARCH AT SPRINGING $\cos \phi$		0·930	0·848	0·781	0·709	0·918	0·820	0·740	0·650	0·893	0·764	0·665	0·565
HORIZONTAL THRUST DUE TO DEAD LOAD Values of k_1	–	0·125	0·125	0·125	0·125	0·135	0·140	0·144	0·148	0·160	0·176	0·190	0·204
HORIZONTAL THRUST DUE TO ARCH SHORTENING Values of k_2	$\frac{d_s}{d}$ 1·25	1·10	1·07	1·03	0·99	1·13	1·08	1·03	1·00	1·19	1·13	1·08	1·00
	1·50	1·42	1·37	1·32	1·25	1·44	1·39	1·33	1·27	1·53	1·48	1·42	1·33
	1·75	1·68	1·63	1·58	1·53	1·73	1·68	1·63	1·58	1·86	1·82	1·76	1·69
MOMENTS DUE TO ARCH SHORTENING, TEMPERATURE CHANGE AND ECCENTRICITY OF THRUST Values of k_3	1·25	0·284	0·293	0·300	0·307	0·279	0·280	0·281	0·282	0·255	0·261	0·265	0·270
	1·50	0·248	0·253	0·258	0·263	0·240	0·243	0·247	0·251	0·224	0·226	0·228	0·230
	1·75	0·223	0·227	0·231	0·235	0·218	0·220	0·222	0·224	0·200	0·200	0·200	0·200
HORIZONTAL FORCE DUE TO TEMPERATURE CHANGE Values of $k_4 \div 10^3$	1·25	2·54	2·42	2·32	2·22	2·58	2·46	2·33	2·19	2·72	2·53	2·38	2·17
	1·50	3·42	3·27	3·14	3·01	3·49	3·34	3·18	3·02	3·74	3·55	3·34	3·12
	1·75	4·26	4·10	3·94	3·78	4·37	4·21	4·04	3·83	4·69	4·48	4·29	4·12
HORIZONTAL THRUSTS DUE TO LIVE LOAD	k_6	0·059	0·059	0·059	0·059	0·062	0·064	0·065	0·066	0·070	0·074	0·077	0·080
	k_8	0·039	0·039	0·039	0·039	0·038	0·038	0·037	0·037	0·037	0·035	0·033	0·032
	k_{11}	0·086	0·086	0·086	0·086	0·088	0·089	0·090	0·092				
	$\frac{d_s}{d}$ 1·25									0·093	0·097	0·098	0·100
	1·50									0·095	0·098	0·101	0·103
	1·75									0·097	0·100	0·104	0·106
VERTICAL REACTIONS DUE TO LIVE LOAD	k_9	0·358	0·358	0·358	0·358	0·354	0·352	0·350	0·349	0·342	0·337	0·330	0·321
	k_{12}	0·149	0·149	0·149	0·149	0·150	0·151	0·153	0·155	0·160	0·164	0·170	0·177
BENDING MOMENTS DUE TO LIVE LOAD — Values of $k_5 \times 10^4$	$\frac{d_s}{d}$ 1·25	48	49	51	52	52	54	57	60	60	69	77	84
	1·50	45	46	46	47	48	50	52	54	56	62	68	75
	1·75	42	43	43	44	44	46	48	50	52	58	63	68
Values of k_7	1·25	0·019	0·019	0·018	0·018	0·018	0·018	0·017	0·017	0·017	0·015	0·014	0·014
	1·50	0·021	0·021	0·020	0·020	0·020	0·020	0·019	0·018	0·018	0·017	0·016	0·015
	1·75	0·022	0·022	0·022	0·022	0·022	0·021	0·020	0·020	0·020	0·018	0·017	0·016
Values of k_{10}	1·25	0·019	0·019	0·018	0·018	0·020	0·021	0·021	0·021	0·024	0·025	0·025	0·026
	1·50	0·021	0·020	0·020	0·020	0·022	0·023	0·023	0·023	0·026	0·027	0·028	0·028
	1·75	0·022	0·022	0·022	0·022	0·024	0·025	0·025	0·025	0·029	0·030	0·031	0·031

NOTE.—See facing page for formulæ in which coefficients k_1, k_2, etc., should be substituted, as shown in the example on page 236.

FIXED PARABOLIC ARCHES (*continued*).

Example of Use of Table 54.—Design of a fixed arch slab for an open-spandrel bridge.

Span: 150 ft. measured horizontally between the intersection of the axis of the arch and the abutment.

Rise: 22 ft. 6 in., being the rise of the axis of the arch within the 150-ft. span.

Thickness of arch slab: 3 ft. $4\frac{1}{2}$ in. at springings, 2 ft. 3 in. at crown.

Dead load: 250 lb. per square foot excluding the weight of the arch slab.

Live load: 300 lb. per square foot. This is approximately equal to the Ministry of Transport loading; since the live load extends over one-third or four-tenths of the span to give the maximum bending moments, the loaded lengths are 50 ft. and 60 ft.; from *Table* 6 the equivalent distributed load is 220 lb. per sq. ft. The difference between this and 300 lb. per sq. ft. allows for the single knife-edge load of 2700 lb. per ft. width of slab.

Temperature range: $\pm$ 15 deg. F. $E_C = 2{,}000{,}000$ lb. per sq. in. $e = 0{\cdot}0000066$ per deg. F.

Shrinking: Equivalent to a fall in temperature of 15 deg. F.

Geometrical properties: $\dfrac{\text{Thickness at springing}}{\text{Thickness at crown}} = \dfrac{d_s}{d} = \dfrac{3{\cdot}375}{2{\cdot}25} = 1{\cdot}5.$

$$\frac{\text{Rise}}{\text{Span}} = \frac{22{\cdot}5}{150} = 0{\cdot}15.$$

Angle of inclination of arch axis (from *Table* 54): $\cos\phi = 0{\cdot}82$.

Dead load at crown = 250 lb. per sq. ft. plus weight of 27-in. slab (= 338 lb. per sq. ft. at 150 lb. per cu. ft.; $w_d = 250 + 338 = 558$ lb. per sq. ft.

A strip of slab 12 in. wide is considered. The coefficients are taken from *Table* 54 and substituted in the formulæ preceding on page 234.

Horizontal thrusts due to dead load, etc.

Dead load ($k_1 = 0{\cdot}140$): $H = 0{\cdot}140 \times 588 \times \dfrac{150^2}{22{\cdot}5}$ $= +\ 82{,}200$ lb.

Arch shortening ($k_2 = 1{\cdot}39$): $H_D = -\ 1{\cdot}39\left(\dfrac{2{\cdot}25}{22{\cdot}5}\right)^2 82{,}200$ $= -\ 1145$ lb.

Change of temperature ($k_4 = 3{\cdot}34 \times 10^3$):

$$H_T = \pm\ 3{\cdot}34 \times 10^3\left(\frac{2{\cdot}25}{22{\cdot}5}\right)^2 \times 2{\cdot}25 \times 15 = \pm\ 1130 \text{ lb.}$$

Shrinking: As for fall in temperature $= -\ 1130$ lb.

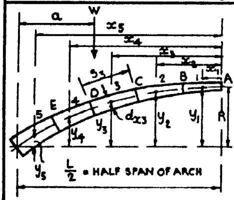
	Bending moment in.-lb.	Thrust lb.
Crown.—Maximum positive bending moment.		
Dead load and arch shortening: $H_C = 82{,}200 - 1145$		+ 80,855
($k_3 = 0{\cdot}243$): $M_C = 0{\cdot}243 \times 22{\cdot}5 \times 1145 \times 12$	+ 75,100	
Fall in temperature : Thrust H_T as above		− 1130
$M_C = 0{\cdot}243 \times 22{\cdot}5 \times 1130 \times 12$	+ 74,300	
Shrinking: As for fall in temperature	+ 74,300	− 1130
Live load: Thrust ($k_6 = 0{\cdot}064$); $H = 0{\cdot}064 \times 300 \times \dfrac{150^2}{22{\cdot}5}$		+ 18,600
($k_5 = 50 \div 10^4$); $M_C = 0{\cdot}0050 \times 300 \times 150^2 \times 12$	+ 405,000	
Totals:	+ 628,700	+ 97,395
Springing.—Maximum negative bending moment.		
Dead load and arch shortening: $H_S = \dfrac{82{,}200}{0{\cdot}820} - 1145 \times 0{\cdot}820$		+ 99,260
$M_S = (1 - 0{\cdot}243) \times 22{\cdot}5 \times 1145 \times 12$	− 234,200	
Fall in temperature : Thrust $= -\ 1130 \times 0{\cdot}820$		− 925
$M_S = -\ 0{\cdot}757 \times 22{\cdot}5 \times 1130 \times 12$	− 231,000	
Shrinking: As for fall in temperature	− 231,000	− 925
Live load: ($k_8 = 0{\cdot}038$); $H = 0{\cdot}038 \times 300 \times \dfrac{150^2}{22{\cdot}5} = 11{,}400$ lb.		
($k_9 = 0{\cdot}352$); $V = 0{\cdot}352 \times 300 \times 150 = 15{,}800$ lb.		
Normal thrust $= (11{,}400 \times 0{\cdot}820) + (15{,}800\sqrt{1 - 0{\cdot}820^2})$		+ 14,500
($k_7 = 0{\cdot}020$); $M_S = 0{\cdot}020 \times 300 \times 150^2 \times 12$	− 1,580,000	
Totals:	− 2,276,200	111,910

(*Continued on page* 237.)

CALCULATION CHART FOR FIXED-END SYMMETRICAL ARCH.

DIMENSIONAL PROPERTIES													UNIT LOAD AT A (CROWN)				UNIT LOAD AT B				UNIT LOAD AT C, ETC.		
SEGMENT No.	x	y	d_x	s	A	I_x	$S_1 = \frac{s}{I}$	S_1x	S_1x^2	S_1y	S_1y^2	$\frac{s}{A} = S_A$	M_L (= x)	M_LS_1 = S_1x	M_LS_1x = S_1x^2	M_LS_1y = S_1xy	M_L	M_LS_1	M_LS_1x	M_LS_1y	C	D	E
1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	-	✓	-	-	-	-	-	-	-
2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	-	✓	✓	✓	✓	✓	-	-	-
3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	-	✓	✓	✓	✓	✓	✓	-	-
4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	-	✓	✓	✓	✓	✓	✓	✓	-
5 ETC.	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	-	✓	✓	✓	✓	✓	✓	✓	✓
SUMMATIONS ON LEFT-HAND HALF OF ARCH						$\Sigma_0^{L/2}$	T_S	T_{1A}	T_{2A}	T_{Sy}	T_{Sy^2}	T_{SA}	-	T_{1A}	T_{2A}	T_{3A}	$\Sigma_a^{L/2}$	T_{1B}	T_{2B}	T_{3B}	TOTALS SIMILAR TO B		

(C, D, E: SUB-HEADINGS AS B)



$$\text{Den} = (T_S \times T_{Sy^2}) - (T_{Sy})^2$$

HORIZONTAL THRUST AT CROWN

$$H_C = \frac{(T_S \times T_3) - (T_{Sy} \times T_1)}{2 \times \text{Den.}}$$

BENDING MOMENT AT CROWN

$$M_C = \frac{T_1 - (2H_C \times T_{Sy})}{2T_S}$$

SHEARING FORCE AT CROWN AND SPRINGING: $V_C = \frac{T_2}{2T_{2A}}$; $V_S = 1 - V_C$

ORDINATES OF INFLUENCE LINES					
UNIT LOAD AT		A (CROWN) (a = 0)	B (a = ✓)	C (a = ✓)	D ETC. (a = ✓)
1	$T_S \times T_3$	✓	✓	✓	✓
2	$T_{Sy} \times T_1$	✓	✓	✓	✓
3	(1) + (2)	✓	✓	✓	✓
4	H_C = (3) ÷ 2 Den.	✓	✓	✓	✓
5	2(4) × T_{Sy}	✓	✓	✓	✓
6	T_1 − (5)	✓	✓	✓	✓
7	M_C = (6) ÷ 2 T_S	✓	✓	✓	✓
8	V_C = T_2 ÷ (2 T_{2A})	✓	✓	✓	✓
9	V_S = 1 − (8)	✓	✓	✓	✓
10	(4) OR H_{CT}	✓	✓	✓	✓
11	(10) × T_{SA} × T_S	✓	✓	✓	✓
12	H_{CS} = −(11) ÷ Den	✓	✓	✓	✓
13	M_{CS} = −(12) × T_{Sy} ÷ T_S	✓	✓	✓	✓
14	[(4) − (12)] × R	✓	✓	✓	✓
15	(7) − (13)	✓	✓	✓	✓
16	(14) + (15)	✓	✓	✓	✓
17	0 5L × (8)	✓	✓	✓	✓
18	M_{SL} = (16) + (17) − a	✓	✓	✓	✓
19	M_{SR} = (16) − (17)	✓	✓	✓	✓

TEMPERATURE AND SHRINKING

$$H_{CT} = \pm\frac{(\pm t)\varepsilon L E_C T_S}{2 \times \text{Den.}} \qquad M_{CT} = -H_{CT}\frac{T_{Sy}}{T_S}$$

ARCH SHORTENING: HORIZONTAL THRUST AT CROWN

$$H_{CS} = -\frac{T_{SA} \times T_S \times (H_C \text{ OR } H_{CT})}{\text{Den.}}$$

BENDING MOMENT AT CROWN $\quad M_{CS} = -H_{CS}\frac{T_{Sy}}{T_S}$

BENDING MOMENTS AT SPRINGINGS.

LEFT-HAND SUPPORT: $M_{SL} = M_C' + H_C'R + V_C\frac{L}{2} - a$

RIGHT-HAND SUPPORT: $M_{SR} = M_C' + H_C'R - V_C\frac{L}{2} - M_L$

$H_C' = H_C - (H_{CS} \text{ DUE TO } H_C)$

$M_C' = M_C - (M_{CS} \text{ FOR } H_{CS} \text{ DUE TO } H_C)$

(M_L = 0 WHEN LOAD IS ON LEFT-HAND HALF OF ARCH)

BENDING MOMENT AT QUARTER-POINT: REPEAT SIMILAR TO FOREGOING WITH CORRESPONDING FORMULÆ

DEAD LOADS (EFFECTS CALCULATED FROM INFLUENCE LINES "ORD." = ORDINATE)

SEGMENT No.	LOADS			AT CROWN								M_{SL} AT SPRINGING				QUARTER POINT	LIVE LOADS
	ARCH: 150 × sd_x	FILL AND OTHER D.L.	TOTAL D.L. (HALF ARCH)	H_C		H_{CS}		M_C		M_{CS}		L-H SEGMENTS		R-H SEGMENTS			
				ORD.	PROD.	ORD.	PROD.	ORD.	PROD.	ORD.	PROD.	ORD.	PROD.	ORD.	PROD.		
1	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	REPEAT FOR M_Q	MAXIMUM THRUSTS, SHEARING FORCES AND B.Ms DUE TO LIVE LOADS ARE CALCULATED FROM THE INFLUENCE LINES FOR THE MOST ADVERSE POSITION OF LOADS.
2	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
3	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
4	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
5	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
		Σ =	W_D	ΣH_C =	✓	ΣH_{CS} =	✓	ΣM_C =	✓	ΣM_{CS} =	✓	ΣM_{SL} =	✓	ΣM_{SR} =	✓		
$V_S = W_D$		$V_C = 0$		TOTAL $H_C' = 2(\Sigma H_C - \Sigma H_{CS})$ = ✓				TOTAL $M_C' = 2(\Sigma M_C - \Sigma M_{CS})$ = ✓				NET $M_S = \Sigma M_{SL} \pm \Sigma M_{SR}$ = ✓					

RESULTANT THRUSTS, B Ms AND SHEARING FORCES DUE TO DEAD AND LIVE LOADS ARE CALCULATED BY SUBSTITUTION IN FORMULÆ

NOTE.—Reference should be made to page 44 for comments on this chart.

Example of Use of Table 54 (*continued from page 236*).

	Bending moment in.-lb.	Thrust lb.
Springing: Maximum positive bending moment.		
Dead load and arch shortening as before	− 234,200	+ 99,260
(Rise of temperature and shrinking neutralise each other.)		
Live load: $(k_{11} = 0{\cdot}089)$; $H = 0{\cdot}089 \times 300 \times \frac{150^2}{22{\cdot}5} = 26{,}700$ lb.		
$(k_{12} = 0{\cdot}151)$; $V = 0{\cdot}151 \times 300 \times 150 = 6800$ lb.		
Normal thrust $(26{,}700 \times 0{\cdot}820) + 6800\sqrt{1 - 0{\cdot}820^2}$		+ 24,200
$(k_{10} = 0{\cdot}023)$ $M_S = 0{\cdot}023 \times 300 \times 150^2 \times 12$	+ 1,830,000	
Totals	+ 1,595,800	+ 123,460

The corresponding bending moments and thrusts should be combined to determine the maximum stresses and reinforcement at the crown and springing.

PROPERTIES OF CONCRETE.

Quality Control.—The standards of control A, B, etc., and the strength categories 1, 2, etc., given under the heading " Grade—I.C.E." in *Table* 56 are from " Quality of Concrete in the Field " (Journal of Inst. Civil Engs., May 1955).

Strength categories 1, 2, and 3 relate respectively to strengths at twenty-eight days exceeding 4000 lb., 2500 lb., and 1000 lb. per sq. in.; strength of category 4 is below 1000 lb. per sq. in.

Standard A is the most rigid control, requiring tests on materials and trial mixtures, and strict control of all site operations; B requires simple trial mixtures only, batching by weight and reasonable site supervision; C permits proportioning by experience and batching by volume; D applies to concrete with no special properties.

Compressive Strength.—The compressive strengths u_c of concretes made with the same cement and cured and tested under the same conditions are shown by Feret to comply approximately with the expression

$$u_c = a\left(\frac{V_c}{V_c + V_w + V_v}\right)^2$$

in which V_c, V_w and V_v are respectively the net volumes of cement, water, and voids in unit volume of mixed concrete; that is if V_a is the net volume of aggregate in unit volume of mixed concrete, $V_c + V_a + V_w + V_v = 1$. The term a is a numerical factor determined by tests and depends on the nature of the materials.

Tensile Strength.—The ratio of the direct tensile strength u_t and the compressive strength u_c may vary from 0·05 to over 0·10, the relation being very approximately of the form $u_t = b(u_c)^n$, where n is between 0·5 and unity. A formula, derived by Feret, is $u_t = c\sqrt{u_c} - d$.

The parameters c and d are obtained by test and depend on the nature of the cement, the type, grading, and largest size of the aggregate, the amount of water, the conditions of curing, and the methods of testing.

Modulus of Rupture.—For the moduli of rupture of various concretes to be comparable, the test piece must have standard dimensions, say, 4 in. deep by 4 in. wide and 16 in. long. If such a test piece is supported on a span of 12 in. and loaded with a centrally applied load, the modulus of rupture u_r is 0·28125Wlb. per sq. in., where W is the load that causes the test piece to break. Factors that contribute to high compressive strength u_c also cause an increase in the modulus of rupture. The relation of u_r to u_c is given by Feret to be approximately

$$u_r = s\sqrt{u_c} - r,$$

where s and r are parameters affected by the same conditions as affect c and d for the direct tensile strength.

Modulus of Elasticity.—The modulus of elasticity of concrete E_c increases with increase of cement content, age, repetition of stress, and some other factors. Actual values lie between 750 and 1500 times the compressive strength. Thus E_c for a 1 : 2 : 4 concrete may average 3,000,000 lb. per sq. in. and, when combined with a modulus of elasticity for steel E_s of 30,000,000 lb. per sq. in., a modular ratio $\left(m = \frac{E_s}{E_c}\right)$ of 10 is obtained. The generally-accepted arbitrary value for the modular ratio is 15 (corresponding to E_c equal to 2,000,000 lb. per sq. in.) for concretes of all proportions, and is recommended in the B.S. Codes. This ratio compensates in part for the errors involved in the consideration of reinforced concrete as a theoretically elastic substance and for the neglect of the tensile resistance of concrete in bending. It only applies within the range of the working stresses. The modular ratio is not taken into account in the load-factor method of design.

The D.S.I.R. Code of 1934 recommended a variable modular ratio of $\frac{40{,}000}{u_p}$, where u_p is the compressive strength at twenty-eight days. The Ministry of Transport adopt modular ratios recommended by the D.S.I.R. for the design of bridges.

A modular ratio of 10 is often used when calculating deflections. For calculations involving actual deformations the ratios in the D.S.I.R. Code might also be used.

Poisson's Ratio.—Poisson's ratio, by means of which can be calculated the secondary stress, produced by a primary stress, in a direction normal to that of the primary stress, is about 0·15 for concrete, although in many cases this effect is ignored. The omission of Poisson's ratio makes a difference to the calculated stresses in ordinary slabs spanning in two directions (if in panels that are almost square) and to the analyses of the stresses in shells and in road slabs.

NOMINAL VOLUMETRIC PROPORTIONS		1:8	1:6	1 : 2 : 4				1 : 1⅔ : 3⅓		1 : 1⅔ : 3⅓	1 : 1½ : 3		1 : 1 : 2	
QUANTITIES PER CWT. OF CEMENT	FINE AGGREGATE	10 (all-in)	7½ (all-in)	2½				2 1/12		2	1⅞		1¼ CU.FT.	
	COARSE AGGREGATE			5				4 1/6		4	3¾		2½ CU.FT.	
APPROX QUANTS PER CU. YD. MIXED CONCRETE	CEMENT	DEPENDS ON GRADING OF ALL-IN AGGREGATE		485				550		560	600		815 LB.	
	FINE AGGREGATE			11 (14⅜)				10½ (13⅝)		10⅜ (13½)	10¼ (13⅜)		9⅓ (9⅝) CU.FT.	
	COARSE AGGREGATE			22				21		20¾	20½		18¾ CU.FT.	
GRADE	LONDON BY-LAWS	V	IV	III	IIIA	HIGH ALUMINA	–	ORD.	A	–	II	IIA	I	IA
	I.C.E.	4D	4D	2B or 2C	2A	–	LIQUID CONTAINERS	2B	2A	LIQUID CONTAINERS	2B	2A	2B	1A
CRUSHING STRENGTH LB PER SQ IN. PRELIM.	2 DAYS	–	–	–	–	6000	–	–	–	–	–	–	–	–
	7	–	–	–	2300	–	3000	–	2665	3600	–	2850	–	3400
	28	–	–	–	3500	–	4500	–	4100	5400	–	4400	–	5250
WORKS	2	–	–	–	–	5000	–	–	–	–	–	–	–	–
	7	800	1000	1500	2000	–	2000	1635	2335	2400	1700	2500	1950	3000
	28	1100	1500	2250	3000	–	3000	2450	3600	3600	2550	3750	2900	4500
WORKING STRESSES LB PER SQ IN. COMPRESSIVE	BENDING	–	–	750	1000	1500	1000	827	1167	1200	850	1250	975	1500
	DIRECT	–	–	600	760	1140	760	654	887	910	680	950	780	1140
	WALLS & PIERS (≯1)	235	310	605	–	–	–	660	–	–	685	–	780	–
	ON FOUNDS.	235	310	605	995	1490	–	660	1168	–	685	1240	780	1490
SHEARING	WITHOUT REINFORCEMENT	–	–	75	100	130	100	82	110	110	85	115	100	130
	WITH REINFORCEMENT	–	–	300	400	520	250*	313	440	280*	340	460	400	520
TENSILE	BENDING	–	–	–	–	–	245	–	–	270	–	–	–	–
	DIRECT	–	–	–	–	–	175	–	–	190	–	–	–	–
BOND	AVERAGE	–	–	100	120	150	120	107	130	130	110	135	125	150
	LOCAL	–	–	150	180	220	180	160	193	195	165	200	190	220
COMMON USES		FOUNDATIONS RETAINING WALLS WALLS; PIERS SITE CONCRETE BLINDING FILLING		FOUNDATIONS RETAINING WALLS BUILDINGS INDUSTRIAL STRUCTURES ROADS (BOTTOM COURSE)		ACID-RESISTANT RAPID-HARDENING OR REFRACTORY CONSTRUCTION	TANKS RESERVOIRS ETC. ≮18" THICK	PRECAST PILES EXPOSED ROOFS ROADS (SINGLE COURSE)		TANKS RESERVOIRS ETC.	RIBBED FLOORS ARCHES (MEDIUM SPAN) COLUMNS (MEDIUM LOAD) PRECAST PRODUCTS ROADS WEARING COURSE		ARCHES (LONG SPAN) COLUMNS (HIGH LOAD)	
		PLAIN CONCRETE		REINFORCED CONCRETE										

NOTES.—For rules for properties of mixtures not tabulated see page 246.

QUANTITIES.—Tabulated quantities per cu. yd. of mixed concrete are approximate, and vary with grading of aggregate, compaction, etc. Quantities of fine aggregates (in brackets) assume material to be damp and bulked by about 30 per cent.

GRADES.—Those marked " London By-laws " relate to London Building (Constructional) By-laws 1952. Those marked " I.C.E." relate to recommendations in " Quality of Concrete in the Field " (Journal Inst. Civil Engs., May 1955); see note on facing page. Those marked " Liquid Containers " relate to B.S. Code No. 2007 (1960), " Design and Construction of Reinforced and Prestressed Concrete Structures for the Storage of Water and other Aqueous Liquids ".

WORKING STRESS.—Increase values (London By-laws) by 10 per cent. if vibrated. Working stresses due to effects of wind increased by one-third (London By-laws). Shearing stresses indicated thus * denote maximum (with reinforcement) in design for no cracking. Bond stresses marked * may be increased by 25 per cent. if deformed bars are used.

B.S. CODE No. 114—See *Table* 57.

THERMAL PROPERTIES OF CONCRETE AND REINFORCED CONCRETE

Temperature Coefficients.—The coefficient of linear expansion and contraction due to temperature changes increases with an increase in the cement content, depends on the type of aggregate, and varies from 0·000004 to 0·000007 per deg. F. A coefficient of 0·0000055 per deg. F., which is about the same as that for mild steel, is commonly used.

Thermal Conductivity.—Average values of the thermal conductivity of concrete (k), expressed in the number of British thermal units that pass in one hour through 1 in. of concrete 1 sq. ft. in area for each deg. F. difference in temperature between the two faces are: 1 : 2 : 4 gravel concrete, 7·0; 1 : 1 : 2 gravel concrete, 6·7; 1 : 2½ : 7½ clinker concrete, 2·3; 1 : 2 : 4 foamed-slag concrete, 2·2; 1 : 2 : 4 pumice concrete, 1·4; 1 : 2 : 4 expanded-shale concrete, 2·1; 1 : 8 vermiculite concrete, 0·6.

Recommendations are given in B.S. Code No. 114 relating to the resistance to fire of reinforced concrete construction. This resistance, which depends primarily on the type of aggregate, the thickness of the member, and the cover of concrete over the reinforcement, is expressed by the number of hours of effective resistance as established by tests made in accordance with B.S. No. 476, " Fire Tests on Building Materials and Structures ", and values are given in the table below.

The degree of resistance required depends on the size of a building and the use to which the building is to be put, and is specified in the local by-laws. In general, the resistances given in the table conform to those in the London Building By-laws (1952); exceptions and additions are noted.

The Code gives no recommendation regarding the fire-resistance of reinforced concrete stairs, but the London By-laws require at least one hour's resistance which is provided by 2½ in. of solid concrete.

LEAST DIMENSIONS TO PROVIDE STATED PERIOD OF FIRE RESISTANCE.

<table>
<tr><th colspan="3" rowspan="2">Member</th><th colspan="5">Period of Resistance (hours)</th><th colspan="2" rowspan="2">Notes</th></tr>
<tr><th>½</th><th>1</th><th>2</th><th>4</th><th>6</th></tr>
<tr><td colspan="3">Walls: Thickness</td><td>in.
3*</td><td>in.
3</td><td>in.
4</td><td>in.
7</td><td>in.
9</td><td colspan="2">With 1 in. (minimum) cover</td></tr>
<tr><td rowspan="5">Floor Slabs: Thickness (minimum)</td><td colspan="2">Solid</td><td>3½</td><td>4</td><td>5</td><td>6</td><td>-</td><td>—</td><td rowspan="5">Cover to be not less than ½ in for 2 hrs (or less); not less than 1 in for 4 hrs.</td></tr>
<tr><td rowspan="3">Pre-cast</td><td>Channel or T-section</td><td>3½</td><td>4</td><td>5</td><td>6</td><td>-</td><td>—</td></tr>
<tr><td>Inverted Channel</td><td>2½
(3½)</td><td>3
(4)</td><td>4
(5)</td><td>6</td><td>-</td><td>Applies if minimum thickness is at crown only. Also applies to similar slabs cast in-situ.</td></tr>
<tr><td>Box or I-section</td><td>2½</td><td>3</td><td>3½</td><td>5</td><td>-</td><td rowspan="2">Thicknesses are total thickness of concrete, tile, and the like.</td></tr>
<tr><td colspan="2">Hollow blocks</td><td>2½</td><td>3</td><td>3½</td><td>5</td><td>-</td></tr>
<tr><td rowspan="2">Columns: Minimum size</td><td colspan="2">Mesh within cover</td><td>-</td><td>-</td><td>9</td><td>12</td><td>-</td><td colspan="2">6-in square mesh (Minimum)</td></tr>
<tr><td colspan="2">Ordinary Construction</td><td>6*</td><td>8
(9)</td><td>12</td><td>18*</td><td>-</td><td colspan="2" rowspan="2">Renderings of cement-lime mortar, or gypsum plaster up to ½ in. thick around columns or beams accepted as equivalent to same thickness of reinforced concrete.</td></tr>
<tr><td colspan="3">Beams: Minimum cover</td><td>½
(1)</td><td>1</td><td>2</td><td>2½</td><td>-</td></tr>
<tr><td colspan="3">Stairs: Minimum thickness</td><td>(2½)</td><td>(2½)</td><td>-</td><td>-</td><td>-</td><td colspan="2">London Building By-Laws only.</td></tr>
</table>

NOTES.—(*a*) Dimensions are for concrete made with aggregates conforming to B.S. No. 882, except that if limestone is used a slightly less thickness is acceptable.

(*b*) Tabulated data are as given in B.S. Code No. 114 and conform to London By-laws (1952) except that (i) Dimensions marked * are excluded from By-laws; (ii) Dimensions in brackets conform to By-laws but differ from Code.

(*c*) Dimensions of reinforced concrete members may be reduced if protected by plaster, sprayed asbestos, or the like.

CLASS OF CONCRETE			STRENGTH (LB. PER SQ. IN.)					PERMISSIBLE STRESSES (LB. PER SQ. IN.)									
MATERIALS	NOMINAL PROPORTIONS QUANTITIES *	QUALITY	AGE DAYS	CRUSHING PRELIM. CUBES	CRUSHING WORKS CUBES	MODULUS OF RUPTURE WITH CUBE TESTS	MODULUS OF RUPTURE WITHOUT CUBE TESTS	ORDINARY CONDITIONS COMPRESSIVE DIRECT	ORDINARY CONDITIONS COMPRESSIVE BEND.	ORDINARY CONDITIONS SHEAR	ORDINARY CONDITIONS BOND PLAIN BARS AV.	ORDINARY CONDITIONS BOND PLAIN BARS LOCAL	INCLUDING EFFECTS OF WIND COMPRESSIVE DIRECT	INCLUDING EFFECTS OF WIND COMPRESSIVE BEND.	INCLUDING EFFECTS OF WIND SHEAR	INCLUDING EFFECTS OF WIND BOND PLAIN BARS AV.	INCLUDING EFFECTS OF WIND BOND PLAIN BARS LOCAL
PORTLAND CEMENT OR PORTLAND BLAST-FURNACE CEMENT AGGREGATE CONFORMING TO B.S. No 882 FINE AGGREGATE IN ZONES 1 TO 3	1:2:4 (BY VOL.)	LOWER	7 28	2025 3000	1500 2250	–	–	570	750	75	90	135	712	937	94	112	168
		ORD.	3 7 28	– 2700 4000	– 2000 3000	250 350 –	275 385 –	760	1000	100	120	180	950	1250	125	150	225
		HIGHER	7 28	3375 5000	2500 3750	–	–	950	1250	115	135	200	1188	1560	143	168	250
	1:1½:3 (BY VOL.)	LOWER	7 28	2512 3750	1875 2812	–	–	712	937	94	112	169	890	1170	117	140	211
		ORD.	3 7 28	– 3350 5000	– 2500 3750	275 400 –	303 440 –	950	1250	115	135	200	1188	1560	143	168	250
		HIGHER	7 28	4187 6250	3125 4687	–	–	1187	1562	134	154	225	1483	1952	168	192	281
	1:1:2 (BY VOL.)	LOWER	7 28	3000 4500	2250 3370	–	–	855	1125	107	127	190	1068	1406	133	158	237
		ORD.	3 7 28	– 4000 6000	– 3000 4500	300 450 –	330 495 –	1140	1500	130	150	220	1425	1875	162	187	275
		HIGHER	7 28	5000 7500	3750 5625	–	–	1425	1875	152	172	250	1781	2343	190	215	312
DITTO BUT FINE AGGREGATE ALSO IN ZONE 4	CEMENT: TOTAL AGG. (BY WT.) 1:8 (MAX.) WITH ¾" AGG. 1:9 (MAX.) WITH 1½" AGG.	SPECIAL	7 28	TWO-THIRDS OF REQUIRED 28-DAY STRENGTH. 1⅓ TIMES WORKS CUBE STRENGTH $\nless 4p_{cb}$	2250 MIN. TO 6000 MAX. $\nless 3p_{cb}$	–	–	570 TO 1520	750 TO 2000	75 TO 160	90 TO 180	135 TO 260	712 TO 1900	937 TO 2500	94 TO 200	112 TO 225	168 TO 325
								INTERMEDIATE STRESSES AS GENERAL RELATION BELOW.					INTERMEDIATE STRESSES AS FOR ORDINARY CONDITIONS PLUS 25%.				
HIGH-ALUMINA CEMENT	1:2:4 (BY VOL.)	ORD.	2 3 7	6000 – –	5000 – –	– 450 500	– 495 550	1140	1500	130	150	220	1425	1875	162	187	275
AGGREGATES CONFORMING TO B.S. No 882 FINE AGG. IN ZONES 1 TO 3	CEMENT: TOTAL AGG. (BY WT.) 1:8 MAX. TO 1:5 MIN. FINE AGG. IN ZONE 4 ALSO	SPECIAL	2	1⅓ TIMES WORKS CUBE STRENGTH $\nless 4p_{cb}$	5000 MIN. TO 6000 MAX. $\nless 3p_{cb}$	–	–	1266 TO 1520	1666 TO 2000	140 TO 160	160 TO 180	233 TO 260	1582 TO 1900	2082 TO 2500	175 TO 200	200 TO 225	291 TO 325
								INTERMEDIATE STRESSES AS GENERAL RELATION BELOW					INTERMEDIATE STRESSES AS FOR ORDINARY CONDITIONS PLUS 25%				
GENERAL RELATION OF STRESSES	NOTATION		–	u_p	u_w	–	–	p_{cc}	p_{cb}	q	s_b	s_{bl}	STRESSES INCLUDING EFFECTS OF WIND 25% IN EXCESS OF STRESSES FOR ORDINARY CONDITIONS				
	AGGREGATES CONFORMING TO B.S. No 882		28	$\nless 4p_{cb}$	$\nless 3p_{cb}$	–	–	$\cdot 76p_{cb}$	$\ngtr 1000$	$\cdot 1p_{cb}$	$\cdot 12p_{cb}$	$\cdot 18p_{cb}$					
	AGGREGATES NOT CONFORMING TO B.S. No 882		28	$\nless 6p_{cb}$	$\nless 4p_{cb}$	–	–		> 1000	$\cdot 06p_{cb} + 40$	$\cdot 08p_{cb} + 60$	$\cdot 08p_{cb} + 100$					

MODIFICATIONS TO TABULATED STRESSES						
	MINIMUM AGE AT LOADING (P.C. CONCRETE) — MONTHS	2	3	6	12	NARROW BEAMS $\frac{\text{SPAN}}{\text{BREADTH}} = \frac{L}{b} > 30 \ngtr 60$ REDUCTION FACTOR APPLIED TO p_{cb} $R_B = 1\cdot75 - 0\cdot025 \frac{L}{b}$
	INCREASE BASIC STRESS p_{cb} BY — PER CENT	10	16	20	24	
	OTHER STRESSES MODIFIED IN ACCORDANCE WITH GENERAL RELATION TO p_{cb}					
	DEFORMED BARS. BOND STRESSES 25% IN EXCESS OF STRESSES FOR PLAIN ROUND BARS					

NOTE.—* For quantities of materials see *Table* 56.
For notes on *Table* 57 see page 246.

TABLE 58.—REINFORCEMENT: PROPERTIES AND STRESSES.

Type of Reinforcement			Stresses in lb. per square inch.							
			Tensile Properties		Tensile Stress (p_{st})				Compressive Stress (p_{sc})	
					Ordinary Conditions		Corrosive Conditions			
	Type of Bar	Size of Bar (in.)	Minimum Tensile Strength t_{ult}	Minimum Yield Stress t_y	Normal	Including Effects of Wind	Normal	Including Effects of Wind	Normal Conditions	Including Effects of Wind
Hot-rolled bars (B.S. No 785)	Mild Steel	≯ 1½	62,720	Not specified	20,000 (18,000)	25,000 (24,000)	20,000	25,000	18,000	22,500 (24,000)
		> 1½			18,000	22,500 (24,000)	18,000	22,500	16,000 (18,000)	20,000 (22,500)
	Medium Tensile Steel	≯ 1"	73,920	44,000	22,000	27,500 (27,000)	20,000	25,000	22,000 (20,000)	27,500 (26,670)
		> 1" ≯ 1½"		41,500	20,750	25,935	20,000	25,000	20,750 (20,000)	25,937
		> 1½" ≯ 2"		39,000	19,500	24,375	19,500	24,375	19,500	24,375
	High Tensile Steel	≯ 1"	82,880	51,500	25,750	30,000 (27,000)	20,000	25,000	23,000 (20,000)	28,750 (26,670)
		> 1" ≯ 1½"		49,500	24,750	30,000 (27,000)				
		> 1" ≯ 2"		47,000	23,500	29,375 (27,004)				
Cold-worked bars	Twisted Square Bars B.S. No 1144	< ⅜"	80,000	70,000	30,000 (27,000)	30,000 (27,000)	20,000	25,000	23,000 (20,000)	28,750 (26,670)
		≮ ⅜"	(70,000)	60,000						
	Twisted Ribbed Bars (No B.S)	≯ ½"	86,000	75,000	30,000 (27,000)	30,000 (27,000)	20,000	25,000	23,000 (20,000)	28,750 (26,670)
		> ½"	76,000	66,000						
	Twin* Twisted Bars B.S. No 1144	All sizes	63,000	54,000	27,000	30,000 (27,000)	20,000	25,000	Not suitable for compression reinforcement	
	Hard Drawn Wire B.S. No 785	All sizes	82,880	(70,000)**	30,000 (27,000)	30,000 (27,000)	20,000	25,000		
	Expanded Metal B.S. Nos. 405 & 1221	< ⅛"	44,800 to 62,720	50,000	25,000 in the strand	—	—	—		
		≮ ⅛"	58,240 to 71,680							
Limiting Stresses			C.P. 114 (1957)		$\frac{1}{2}t_y$ (≯ 30,000)	$\frac{5}{8}t_y$ (≯ 30,000)	$\frac{1}{2}t_y$ (≯ 20,000)	$\frac{5}{8}t_y$ (≯ 25,000)	$\frac{1}{2}t_y$ (≯ 23,000)	$\frac{5}{8}t_y$ (≯ 28,750)
			London By-laws (1952)		$\frac{1}{2}t_y$ (≯ 27,000)	$\frac{2}{3}t_y$ (≯ 27,000)	—	—	$\frac{1}{2}t_y$ (≯ 20,000)	$\frac{2}{3}t_y$ (≯ 26,670)

* Not now readily obtainable. ** Per B.S. Code No. 114.

B.S. Code No. 114 (1957).—Tabulated stresses (not in brackets) excluding "Corrosive Conditions". Expanded metal not specifically mentioned in Code.

London By-laws (1952).—Stresses given in brackets; otherwise same as stresses in B.S. Code (excluding "Corrosive conditions").

Corrosive Conditions include external members, internal members under corrosive conditions, and members against earth (not B.S. Code or London By-laws).

Shearing Reinforcement.—B.S. Code No. 114: Tabulated stresses, but not greater than 20,000 lb. per sq. in. (25,000 lb. with wind). London By-laws: 18,000 lb. per sq. in. in mild steel; otherwise $0{\cdot}5t_y$ (or $0{\cdot}67t_y$ with wind), but not greater than 20,000 lb. per sq. in. (26,670 lb. with wind).

Helical Binding in Columns.—B.S. Code No. 114: 13,500 lb. per sq. in. for all reinforcement. London By-laws: 13,500 lb. per sq. in. in mild steel; otherwise $0{\cdot}35t_y$ but not greater than 18,000 lb. per sq. in.

Liquid-containers.—See page 59 for stresses in reinforcement in accordance with B.S. Code No. 2007.

CROSS-SECTIONAL AREA : SQUARE INCH. WEIGHT : LB. PER FOOT (OR AS STATED) PERIMETER : INCH.

PLAIN ROUND BARS OR WIRE	DIAM. D	3/16"	1/4"	5/16"	3/8"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"
	PERIMETER	0·59	0·79	0·98	1·18	1·57	1·96	2·36	2·75	3·14	3·54	3·92	4·31	4·71
	AREA	0·028	0·049	0·077	0·110	0·196	0·307	0·442	0·601	0·785	0·994	1·227	1·484	1·767
	WEIGHT	0·094	0·167	0·261	0·375	0·667	1·043	1·502	2·044	2·670	3·379	4·173	5·049	6·008
	I.W.G. No.	20	18	16	14	12	10	8	6	5	4	3	2	1
	DIAM D	0·036	0·048	0·064	0·080	0·104	0·128	0·160	0·192	0·212	0·232	0·252	0·276	0·300
	AREA	0·0010	0·0018	0·0032	0·0050	0·0085	0·0123	0·0201	0·0289	0·0353	0·0423	0·0499	0·0598	0·0707
	WEIGHT	0·0034	0·0061	0·0109	0·017	0·029	0·044	0·068	0·098	0·120	0·143	0·169	0·203	0·240
TWISTED SQUARE BARS	SIZE D	No. 6g.	No. 5g.	1/4"	5/16"	3/8"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	–
	AREA (MIN)	0 031	0·045	0·063	0·098	0·140	0·250	0·391	0·563	0·766	1·000	1·266	1·565	–
	WEIGHT	0·126	0·153	0·213	0·332	0·478	0·85	1·33	1·91	2·60	3·40	4·304	5·32	–

TWISTED RIBBED BARS	NOMINAL DIAMETER D = DIAM. OF PLAIN ROUND BAR OF SAME WEIGHT PER FOOT. CROSS-SECTIONAL AREA = AREA OF PLAIN ROUND BAR OF DIAMETER D. OVERALL SIZE = 1·1 D APPROX.
NOTES	PERIMETER : PLAIN ROUND BARS = 3·14 D AREAS : PLAIN ROUND BARS = 0·7854 D^2 TWISTED SQUARE BARS = D^2 (MIN) TWISTED RIBBED BARS = 0·7854 D^2 WEIGHTS. STEEL = 0·28 LB. PER CU. IN. 3·4 LB. PER FOOT PER SQ. IN. OF CROSS-SECTION. PLAIN ROUND BARS = 2·67 D^2 LB. PER FOOT.

WEIGHTS OF PLAIN ROUND BARS (LB.)

	DIAM.	3/16"	1/4"	5/16"	3/8	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"
	WEIGHT PER FOOT	0·094	0·167	0·261	0·375	0·511	0·667	1·043	1·502	2·044	2·670	3 379	4·173	5·049	6·008
	FEET PER TON	23830	13413	8582	5973	4384	3358	2148	1491	1096	839	663	537	444	373
TOTAL WT. PER FT. OF SPECIFIED No.	1	0·094	0·167	0·261	0·376	0·51	0·67	1·04	1·50	2·04	2·67	3·38	4·17	5·05	6·01
	2	0·188	0·334	0·522	0·752	1·02	1·34	2·09	3·00	4·09	5·34	6·76	8 34	10·10	12·02
	3	0·282	0·501	0·783	1·128	1·53	2·00	3·13	4·51	6·13	8·01	10·14	12·52	15·15	18·02
	4	0·376	0·668	1·044	1·504	2·04	2·67	4·17	6·01	8·18	10·68	13·52	16·69	20·20	24·03
	5	0·470	0·835	1·305	1·880	2·55	3·34	5·22	7·51	10·22	13·35	16·90	20·86	25·25	30·04
	6	0·564	1·002	1·566	2·256	3 06	4·01	6·26	9·01	12·26	16·02	20·28	25·03	30·29	36·05
	7	0·658	1·169	1·827	2·632	3·57	4·68	7·30	10·51	14·31	18·69	23·66	29·20	35·34	42·06
	8	0·752	1·336	2 088	3·008	4·08	5·34	8·34	12·02	16·35	21·36	27·04	33·38	40·39	48·06
	9	0·846	1·503	2·349	3·384	4·59	6·01	9·39	13·52	18·40	24·03	30·42	37·55	45·44	54·07
	10	0·940	1·670	2·610	3·760	5·11	6·68	10·43	15·02	20·44	26·70	33·80	41·72	50·49	60·08
WT. PER SQ. FT. (BARS AT VARIOUS CENTRES)	3"	0·38	0·67	1·05	1·50	2·04	2·67	4·18	6·01	8·19	10·68	13·52	14·35	20·18	24·03
	4"	0·28	0·50	0·77	1·13	1·53	2·00	3·13	4·51	6·16	8·01	10·14	12·52	15·15	18·02
	4 1/2"	0·25	0·45	0·70	1·00	1·36	1·74	2·78	4·01	5·47	7·12	9·02	11·13	13·47	16·02
	5"	0·23	0·40	0·63	0·90	1·22	1·60	2·50	3·61	4·91	6·41	8·13	10·01	12·12	14·42
	6"	0·19	0·33	0·52	0·76	1·02	1·34	2·09	3·00	4·09	5·35	6·74	8·34	10·10	12·02
	7	0·16	0·29	0·45	0·64	0·87	1·15	1·79	2·57	3·52	4·58	5·80	7·15	8·32	10·32
	7 1/2"	0·15	0·27	0·42	0·60	0·82	1·07	1·67	2·40	3·28	4·28	5·42	6·68	8·08	9·64
	8"	0·14	0·25	0·39	0·56	0·77	1·00	1·56	2·26	3·08	4·01	5·07	6·28	7·57	9·03
	9"	0·13	0·22	0·35	0·50	0·68	0·89	1·39	2·00	2·73	3·56	4·51	5·56	6·73	8·03
	10"	0·11	0·20	0·31	0·45	0·62	0·80	1·25	1·80	2·45	3·20	4·06	5·01	6·06	7·21
	11"	0·10	0·18	0·29	0·41	0·56	0·73	1·14	1·64	2·23	2·91	3·70	4·55	5·51	6·54
	12"	0·09	0·17	0·26	0·38	0·51	0·67	1·04	1·50	2·05	2·67	3·38	4·17	5·05	6·01

TABLE 60.—REINFORCEMENT: CROSS-SECTIONAL AREAS.
PLAIN ROUND BARS.

CROSS-SECTIONAL AREAS FOR SPECIFIED NUMBER OF BARS SQ. IN.

DIAMETER	3/16"	1/4"	5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"
1	0·028	0·049	0·077	0·110	0·150	0·196	0·307	0·442	0·601	0·785	0·994	1·227	1·484	1·767
2	0·055	0·098	0·153	0·221	0·301	0·393	0·614	0·884	1·203	1·571	1·988	2·45	2·97	3·53
3	0·082	0·147	0·230	0·331	0·451	0·589	0·920	1·325	1·804	2·36	2·98	3·68	4·45	5·30
4	0·110	0·196	0·307	0·442	0·601	0·785	1·227	1·767	2·41	3·14	3·98	4·91	5·94	7·07
5	0·138	0·245	0·384	0·552	0·752	0·982	1·534	2·21	3·01	3·93	4·97	6·14	7·42	8·84
6	0·165	0·295	0·460	0·663	0·902	1·178	1·841	2·65	3·61	4·71	5·96	7·36	8·91	10·60
7	0·193	0·344	0·537	0·773	1·052	1·374	2·15	3·09	4·21	5·50	6·96	8·59	10·39	12·37
8	0·221	0·393	0·614	0·884	1·202	1·571	2·45	3·53	4·81	6·28	7·95	9·82	11·88	14·14
9	0·248	0·442	0·690	0·994	1·353	1·767	2·76	3·98	5·41	7·07	8·95	11·04	13·36	15·90
10	0·276	0·491	0·767	1·104	1·503	1·963	3·07	4·42	6·01	7·85	9·94	12·27	14·85	17·67
11	0·304	0·540	0·844	1·215	1·654	2·16	3·37	4·86	6·61	8·64	10·93	13·50	16·33	19·44
12	0·331	0·589	0·920	1·325	1·804	2·36	3·68	5·30	7·22	9·42	11·93	14·73	17·82	21·21
13	0·359	0·638	0·997	1·436	1·954	2·55	3·99	5·74	7·82	10·21	12·92	15·95	19·30	22·97
14	0·387	0·687	1·074	1·547	2·10	2·75	4·30	6·19	8·42	11·00	13·92	17·18	20·79	24·74
15	0·414	0·736	1·151	1·657	2·25	2·95	4·60	6·63	9·02	11·78	14·91	18·41	22·27	26·51
16	0·442	0·785	1·227	1·768	2·41	3·14	4·91	7·07	9·62	12·57	15·90	19·64	23·76	28·27
17	0·469	0·835	1·304	1·878	2·56	3·34	5·22	7·51	10·22	13·35	16·90	20·86	26·24	30·04
18	0·497	0·884	1·381	1·989	2·71	3·53	5·52	7·95	10·82	14·14	17·89	22·09	26·73	31·81
19	0·525	0·933	1·457	2·10	2·86	3·73	5·83	8·39	11·43	14·92	18·89	23·32	28·21	33·57
20	0·552	0·982	1·534	2·21	3·01	3·93	6·14	8·84	12·03	15·71	19·88	24·54	29·70	35·34

CROSS-SECTIONAL AREAS OF BARS AT SPECIFIED SPACINGS SQ. IN. PER FOOT

DIAMETER	3/16"	1/4"	5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"
3"	0·110	0·196	0·307	0·442	0·601	0·785	1·227	1·767	2·405	3·142
3½"	0·095	0·168	0·263	0·379	0·515	0·673	1·052	1·515	2·06	2·69
4"	0·083	0·147	0·230	0·331	0·451	0·589	0·920	1·325	1·804	2·356
4½"	0·074	0·131	0·205	0·295	0·401	0·524	0·818	1·178	1·604	2·09
5"	0·066	0·118	0·184	0·265	0·361	0·471	0·736	1·060	1·443	1·885
5½"	0·060	0·107	0·167	0·241	0·328	0·428	0·669	0·964	1·312	1·714
6"	0·055	0·098	0·153	0·221	0·301	0·393	0·614	0·884	1·203	1·571
6½"	0·051	0·091	0·142	0·204	0·278	0·363	0·566	0·816	1·110	1·450
7"	0·047	0·084	0·131	0·189	0·258	0·337	0·526	0·757	1·031	1·346
7½"	0·044	0·079	0·123	0·177	0·241	0·314	0·491	0·707	0·962	1·257
8"	0·041	0·074	0·115	0·166	0·225	0·295	0·461	0·663	0·902	1·178
8½"	0·039	0·069	0·108	0·156	0·212	0·277	0·433	0·624	0·849	1·109
9"	0·037	0·065	0·102	0·147	0·200	0·262	0·409	0·589	0·802	1·047
9½"	0·035	0·062	0·097	0·140	0·190	0·248	0·388	0·558	0·760	0·992
10"	0·033	0·059	0·092	0·133	0·180	0·236	0·368	0·530	0·722	0·942
10½"	0·032	0·056	0·088	0·126	0·172	0·224	0·351	0·505	0·687	0·898
11"	0·030	0·054	0·084	0·120	0·164	0·214	0·335	0·482	0·656	0·857
12"	0·028	0·049	0·077	0·110	0·150	0·196	0·307	0·442	0·601	0·785
15"	0·022	0·039	0·061	0·088	0·120	0·157	0·245	0·353	0·481	0·628
18"	0·018	0·033	0·051	0·074	0·100	0·131	0·205	0·295	0·401	0·524
24"	0·014	0·025	0·038	0·055	0·075	0·098	0·153	0·221	0·301	0·393

A_{st} = CROSS-SECTIONAL AREA (SQ.IN.)

D = DIAMETER OF BAR (IN)

SPECIFIED NUMBER

N = NUMBER OF BARS

$$A_{st} = \frac{1}{4}\pi D^2 N$$

$$= 0{\cdot}7854\, D^2 N \text{ (SQ.IN.)}$$

SPECIFIED SPACING

S = SPACING (OR PITCH) OF BARS (IN.)

$$A_{st} = \frac{12}{S}(0{\cdot}7854 D^2)$$

$$= \frac{9{\cdot}425}{S} D^2$$

(SQ. IN PER FOOT)

CROSS-SECTIONAL AREA FOR SPECIFIED NUMBER OF BARS
SQUARE INCHES & SQUARE CENTIMETRES

DIAMETER OF BAR		NUMBER OF BARS																			
		1		2		3		4		5		6		7		8		9		10	
INCH	MILLI-METRES	SQ. IN.	SQ CM	SQ IN	SQ CM.	SQ. IN.	SQ. CM.	SQ. IN	SQ. CM.	SQ. IN.	SQ. CM.	SQ. IN.	SQ. CM.	SQ. IN	SQ CM	SQ. IN.	SQ. CM.	SQ. IN.	SQ. CM.	SQ IN.	SQ. CM.
3/16	(4·8)	·028	·178	·055	·362	·082	·529	·110	·71	·138	·89	·165	1·07	·193	1·25	·221	1·43	·248	1·60	·276	1·78
–	6	·044	·283	·088	·566	·131	·848	·175	1·13	·219	1·41	·263	1·70	·307	1·98	·350	2·26	·394	2·55	·440	2·83
1/4	(6·4)	·049	·317	·098	·632	·147	·948	·196	1·26	·245	1·58	·295	1·90	·344	2·22	·395	2·54	·442	2·85	·491	3·17
5/16	(7·9)	·077	·495	·153	·994	·230	1·48	·307	1·98	·384	2·48	·460	2·97	·537	3·46	·614	3·96	·690	4·45	·767	4·95
–	8	·078	·502	·156	1·0	·234	1·51	·313	2·01	·392	2·51	·476	3·02	·546	3·52	·623	4·02	·705	4·52	·784	5·02
3/8	(9·5)	·110	·71	·221	1·42	·331	2·14	·442	2·85	·552	3·56	·663	4·28	·773	4·99	·884	5·70	·994	6·41	1·10	7·10
–	10	·122	·785	·244	1·57	·365	2·35	·487	3·14	·608	3·93	·730	4·71	·852	5·50	·973	6·28	1·10	7·07	1·22	7·85
7/16	(11·1)	·150	·968	·301	1·94	·451	2·91	·601	3·88	·752	4·85	·902	5·82	1·05	7·79	1·20	7·75	1·35	8·73	1·50	9·68
–	12	·175	1·13	·350	2·26	·526	3·39	·701	4·52	·876	5·65	1·05	6·79	1·23	7·92	1·40	9·05	1·57	10·18	1·75	11·31
1/2	(12·7)	·196	1·27	·393	2·53	·589	3·80	·785	5·06	·982	6·34	1·18	8·24	1·37	8·86	1·57	10·13	1·77	11·40	1·96	12·66
–	14	·238	1·54	·476	3·08	·715	4·62	·954	6·16	1·19	7·70	1·43	9·24	1·67	0·77	1·91	12·31	2·15	13·85	2·38	15·39
5/8	(15·9)	·307	1·98	·614	3·96	·920	5·94	1·23	7·92	1·53	9·90	1·84	11·88	2·15	13·87	2·45	15·81	2·76	17·80	3·07	19·81
–	16	·312	2·01	·624	4·02	·935	6·03	1·25	8·04	1·63	10·05	1·87	12·06	2·18	14·07	2·49	16·08	2·80	18·05	3·12	20·10
–	18	·394	2·54	·788	5·08	1·18	7·63	1·57	10·18	1·89	12·22	2·36	15·26	2·76	17·81	3·15	20·35	3·47	22·40	3·79	25·44
3/4	(19·1)	·442	2·85	·884	5·70	1·33	8·55	1·77	11·40	2·21	14·26	2·65	17·09	3·09	19·93	3·53	22·77	3·98	25·68	4·42	28·52
–	20	·487	3·14	·974	6·28	1·43	9·25	1·95	12·57	2·43	15·71	2·92	18·85	3·38	21·82	3·90	25·14	4·38	28·28	4·87	31·42
–	22	·589	3·80	1·18	7·60	1·77	11·40	2·36	15·20	2·94	19·0	3·53	22·81	4·12	26·61	4·71	30·41	5·30	34·21	5·89	38·01
7/8	(22·2)	·601	3·88	1·20	7·75	1·80	11·64	2·41	15·55	3·01	19·41	3·61	23·29	4·21	27·16	4·81	31·03	5·41	34·90	6·01	38·77
–	24	·701	4·52	1·40	9·05	2·09	13·57	2·80	18·09	3·51	22·62	4·21	27·14	4·90	31·66	5·60	36·18	6·13	40·71	7·06	45·24
1	(25·4)	·785	5·07	1·57	10·13	2·36	15·23	3·14	20·26	3·93	25·35	4·71	30·38	5·50	35·48	6·28	40·52	7·07	45·61	7·85	50·65

CROSS-SECTIONAL AREA OF BARS
AT SPECIFIED SPACINGS
SQ. C M. PER METRE WIDTH.

DIAM	SPACING OF BARS (CM.)												
MM.	8	10	12	14	15	16	18	20	22	24	25	28	30
6	3·53	2·83	2·36	2·02	1·88	1·77	1·57	1·41	1·29	1·18	1·13	1·01	0·94
8	6·28	5·03	4·19	3·59	3·35	3·14	2·79	2·51	2·28	2·09	2·01	1·80	1·68
10	9·82	7·85	6·54	5·61	5·24	4·91	4·36	3·93	3·57	3·27	3·14	2·80	2·62
12	14·14	11·31	9·42	8·08	7·54	7·07	6·28	5·65	5·14	4·71	4·52	4·04	3·77
14	19·24	15·39	12·82	10·99	10·26	9·60	8·55	7·70	7·00	6·41	6·16	5·50	5·13
16	25·12	20·10	16·75	14·36	13·40	12·56	11·17	10·05	9·14	8·37	8·04	7·18	6·70
18	31·81	25·45	21·21	18·18	16·97	15·91	14·14	12·72	11·57	10·60	10·18	9·09	8·48
20	39·27	31·42	26·18	22·44	20·95	19·64	17·46	15·71	14·28	13·09	12·57	11·22	10·47
22	47·51	38·01	31·67	27·15	25·34	23·76	21·12	19·00	17·28	15·84	15·20	13·57	12·67
24	56·35	45·24	37·70	32·31	30·16	28·27	25·13	22·62	20·56	18·85	18·10	16·16	15·08
26	66·36	53·09	44·24	37·92	35·39	33·18	29·49	26·54	24·13	22·12	21·24	18·96	17·70
28	76·97	61·58	51·32	43·99	41·05	38·49	34·21	30·79	27·99	25·66	24·63	21·99	20·53
30	88·36	70·69	58·91	50·49	47·13	44·18	39·27	35·34	32·13	29·45	28·28	25·25	23·56

NOTE.—For conversion factors (British units to metric and *vice versa*) see page 338 and *Table* 109.

STRESSES AND BOND.

Modification for Proportions not Tabulated.—The permissible compressive stress in bending, on which other permissible stresses depend, for concrete of proportions intermediate between those tabulated in *Tables* 56 and 57 can be calculated by interpolation, that is

$$p_{cb} = p_{cb_2} + \frac{(V_2 - V)(p_{cb_1} - p_{cb_2})}{V_2 - V_1},$$

where p_{cb} is the compressive stress permissible in concrete for which the sum of the proportions of fine and coarse aggregates is V; p_{cb_1} and p_{cb_2} are the permissible compressive stresses for concretes for which the sum of the proportions of the fine and coarse aggregates are V_1 and V_2 respectively, V being intermediate between V_1 and V_2. For example, in 1 : 1⅔ · 3⅓ ordinary concrete (*Table* 56) the volume of fine and coarse aggregate V is $1\frac{2}{3} + 3\frac{1}{3} = 5$, which is between 1 : 2 : 4 ($V_2 = 6$) and 1 : 1½ : 3 ($V_1 = 4\frac{1}{2}$). $p_{cb} = 750 + \frac{(6 - 5)(850 - 750)}{6 - 4\frac{1}{2}} = 817$ lb. per sq. in., since 750 lb. and 850 lb. per sq. in. are the values of p_{cb_2} and p_{cb_1} for 1 : 2 · 4 and 1 : 1½ :3 ordinary concretes respectively. This method also applies to the stresses in *Table* 57 for the B.S. Code. The other stresses are calculated in a similar manner.

Reduction of Compressive Stresses in Narrow Members.

Narrow Beams —Compressive-stress reduction factors R_B for narrow beams in accordance with the B.S. Code can be expressed by

$$R_B = 1{\cdot}75 - 0{\cdot}025\frac{L}{b}$$

as given in *Table* 57, where L = distance between lateral supports, and b = breadth of the compressive flange. Values of R_B are as follows.

$\frac{L}{b}$ =	30	35	40	45	50	55	60
R_B =	1	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$

The depth of a narrow beam considered effective in resistance to bending should not exceed $8b$ and shearing forces should be resisted by reinforcement.

Beams Subjected to Axial Thrust.—If a beam of overall depth d is subjected to a bending moment M and an axial thrust N, the compressive stress reduction factor should be as follows (B.S. Code).

(i) If $N = \frac{M}{2d}$: Reduction factor R_L as for columns (see page 296 and *Table* 83).

(ii) If $N = 0$: Reduction factor R_B as for narrow beams (see above).

(iii) For intermediate values of N: Reduction factor calculated by linear interpolation between (i) and (ii).

Bond-lengths with Anchorages.—In accordance with the B.S. Code, if an end anchorage is provided, the bond-length can be reduced by the amounts (expressed in terms of D, the diameter of the bar) given in *Table* 62 for each type of anchorage. For more conservative design the lengths ND, when N has the values in *Table* 62, should be considered as minima if a semi-circular hook of value $16D$ is provided; if other forms of anchorage are provided, the length for bond should be ND plus $16D$ minus the value of the anchorage. For example, if the value of N is 37½, the lengths for bond according to the B.S. Code are· $37\frac{1}{2}D$ without an end anchorage; $37\frac{1}{2}D - 8D = 29\frac{1}{2}D$ with a 90-deg. bend; $37\frac{1}{2}D - 12D = 25\frac{1}{2}D$ with a 45-deg. hook; and $37\frac{1}{2}D - 16D = 21\frac{1}{2}D$ with a semi-circular hook. For more conservative design the lengths would be. $37\frac{1}{2}D + 16D = 53\frac{1}{2}D$ without an end anchorage; $53\frac{1}{2}D - 8D = 45\frac{1}{2}D$ with a 90-deg. bend; $53\frac{1}{2}D - 12D = 41\frac{1}{2}D$ with a 45-deg. hook; and $37\frac{1}{2}D$ with a semi-circular hook.

Liquid-containers.—Stresses of p_{st} = 12,000 lb. per sq in. and s_b = 130 lb. per sq in. apply to plain round bars in 1 : 1·6 : 3·2 concrete; for 1 : 2 : 4 concrete with p_{st} = 12,000 lb. per sq. in. and s_b = 120 lb. per sq. in., N = 25.

Local Bond Stress.—The local bond stress s_{b_1} due to variation in the tensile stress in reinforcement in beams is given by

$$\left(Q \pm \frac{M}{d_1}\tan\alpha\right)\frac{1}{l_a o}$$

(*Continued on page* 248.)

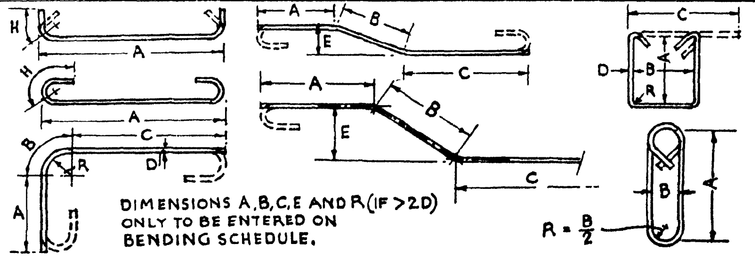
MINIMUM LENGTHS FOR BOND = N × DIAM. OF BAR ($\nless$ 12D) VALUES OF N.

PERMISSIBLE BOND STRESS S_b LB. PER SQ. IN.		90	100	110	120	125	135	150	160	180
TENSILE STRESS IN BAR f_{st} LB. PER SQ. IN. $N = \frac{f_{st}}{4s_b}$	10,000	28	25	23	21	20	19	17	16	14
	12,000	34	30	28	25*	24	23	20	19	17
	14,000	39	35	32	26	28	26	24	22	19
	16,000	45	40	37	34	32	30	27	25	23
	18,000	50	45	41	38	36	34	30	29	25
	20,000	56	50	46	42	40	37	34	32	28
	22,500	63	57	52	47	45	42	38	36	32
	23,000	64	58	53	48	46	43	39	36	32
	24,000	67	60	55	50	48	45	40	38	34
	25,000	70	63	57	52	50	48	42	40	35
	27,000	75	68	62	57	54	50	45	43	38
	30,000	84	75	69	63	60	56	50	47	42

COMPRESSION REINFORCEMENT: $N = \frac{f_{sc}}{5s_b} \nless 12$. BOND LENGTH FOR COMPRESSION BARS = 0·8 × (LENGTH OF TENSION BARS AT SAME STRESS). N = 0·8 × (VALUE OF N AS TABULATED ABOVE)

DIAMETER OF BAR		5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 1/2"
MINIMUM BOND LENGTH = 12 D		(6")	(6")	(6")	6"	7 1/2"	9"	10 1/2"	12"	13 1/2"	15"	18"
BOND LENGTHS FOR COMMON CASE	f_{st} = 20,000 LB./IN.² S_b = 120 " " N = 41 2/3	13"	16"	18"	21"	26"	32"	37"	42"	47"	52"	63"
MINIMUM LAPS	TENSION 30 D	9 1/2"	11 1/2"	13 1/2"	15"	19"	22 1/2"	26 1/2"	30"	34"	37 1/2"	45"
	COMPRESSION 24 D	7 1/2"	9"	10 1/2"	12"	15"	18"	21"	24"	27"	30"	36"
LIQUID* CONTAINERS	MIN. BOND LENGTHS P_{st} = 12,000 S_b = 130 — 24 D APPROX.	7 1/2"	9"	10 1/2"	12"	15"	18"	21"	24"	27"	30"	36"
COMPRESSION BARS P_{sc} = 18,000; S_b = 120 MIN. BOND LENGTH = 30 D		9 1/2"	11 1/2"	13 1/2"	15"	19"	22 1/2"	26 1/2"	30"	34"	37 1/2"	45"
ANCHORAGES.	H = 9D = (4D, 4D)	3"	3 1/2"	4"	4 1/2"	6"	7"	8"	9"	10 1/2"	11 1/2"	13 1/2"
	BOND VALUE = 16D =	5"	6"	7"	8"	10"	12"	14"	16"	18"	22"	24"
	B = 7D = RAD. = 2D (4D)	3"	3"	3 1/2"	3 1/2"	4 1/2"	5 1/2"	6 1/2"	7"	8"	9"	10 1/2"
	BOND VALUE = 8D =	2 1/2"	3"	3 1/2"	4"	5"	6"	7"	8"	9"	11"	12"
	B = 5D = RAD. = 2D (4D)	3"	3"	3"	3"	3 1/2"	4"	4 1/2"	5"	6"	6 1/2"	7 1/2"
	BOND VALUE = 4D =	1 1/2"	1 1/2"	2"	2"	2 1/2	3"	3 1/2"	4"	4 1/2"	5 1/2"	6"

BENDING DIMENSIONS PER B.S. No 1478



DIMENSIONS A, B, C, E AND R (IF > 2D) ONLY TO BE ENTERED ON BENDING SCHEDULE.

STRESSES AND BOND (*continued*).

Local Bond Stress (*continued from page 246*).

where Q = maximum shearing force at the section under consideration (lb.); l_a = lever arm (in.); o = total perimeter of the bars forming the tensile reinforcement at the section (in.); M = bending moment (in.-lb.) at the section; d_1 = effective depth (in.); and α = angle between the top and bottom edges of the beam at the section. The positive sign applies when the bending moment decreases as d_1 increases, as at a haunch at the end of a freely-supported beam. The negative sign applies when the bending moment increases as d_1 increases, as at a haunch at an interior support of a continuous beam.

If the beam is of uniform depth, that is, when the top and bottom edges are parallel,

$$s_{b1} = \frac{Q}{l_a o}.$$

Permissible local bond stresses in accordance with the B.S. Code are given in *Table* 57; the perimeters of round bars are given in *Table* 59.

NOTES ON REINFORCEMENT.

Deformed Bars.—Twisted square bars, twisted ribbed bars, and any bars having projections or indentations, have generally a greater bond value than plain round bars. If the bond value of a deformed bar is not less than 25 per cent. more than that of a plain round bar, the bond stress s_b in *Table* 57 may be increased by 25 per cent. Since many deformed bars are of strength superior to mild steel, and so long as the equivalent yield stress is not less than 60,000 lb. per sq. in., the bond stress s_b is associated with a permissible tensile stress of 30,000 lb. per sq. in. Thus the expression $\frac{p_{st}D}{4s_b}$ for L_0 (see *Table* 63) for high-tensile deformed bars in ordinary 1 : 2 : 4 concrete is $50D$ compared with about $42D$ for plain round bars stressed to 20,000 lb. per sq. in. The greatest permissible compressive stress in high-tensile reinforcement is 23,000 lb. per sq. in.; the corresponding length L_c (*Table* 63) for high-tensile deformed bars is therefore about $31D$, compared with $30D$ for plain round bars stressed to 18,000 lb. per square in. Values of L_0 and L_c for high-tensile deformed bars are given below for s_b = 150 lb. per sq. in.

Diameter (D) of deformed bar (in.)	½	⅝	¾	⅞	1	1⅛	1¼	1½
Minimum bond length (in.) L_0 for bars in tension at 30,000 lb. per sq. in. ($50D$)	25	31½	37½	44	50	56½	62½	75
Minimum bond length (in.) L_c for bars in compression at 23,000 lb. per sq. in. ($30\frac{2}{3}D$)	15½	19½	23	27	31	34½	38½	46

Because of the superior continuous bond value of deformed bars it is not common to provide such bars with an anchorage unless the minimum lengths L_0 or L_c cannot be provided in a straight length; in this case the bar should be hooked round an anchor bar. If a deformed bar is not circular in section, D in the preceding expressions is the diameter of the circle having the same area as the cross-sectional area of the bar.

Cover of Concrete.—The minimum cover of concrete to a reinforcement bar should be as follows, or not less than the diameter of the bar, whichever is smaller.

Buildings and General Structures.—Slabs and walls: ½ in. Beams: 1 in. for main bars; ½ in. for binders. Columns: 1 in. for main ½-in. bars, if least dimension does not exceed 7½ in.; 1½ in. in larger columns; ½ in. for binders. Cover over ends of bars: 1 in. or not less than twice the diameter of the bar. External members, members in contact with ground, corrosive conditions or the like: 1½ in. (These dimensions are in accordance with the B.S. Code No. 114 and also satisfy the London By-laws except that the By-laws require a minimum cover of 3 in. (except in piles) if the concrete is in contact with earth, unless precautions are taken to protect the concrete, and a minimum of 1 in. of concrete over all reinforcement in factory-made precast concrete products.)

Piles: 1½ in. for main bars; 1 in. for binders.

Marine Structures: 2½ in. for main bars; 2 in. for binders.

Liquid Containers: 1½ in. for all bars; 2 in. in sea-water or other corrosive liquids, but this extra ½ in. should not be taken into account when making the resistance calculations. (B.S. Code No. 2007.)

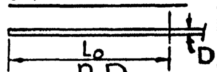
DIA OF BAR D	* MIN 12D	BARS IN TENSION: BOND LENGTH	STRESS IN BAR (LB. PER SQ. IN.) 10,000	12,000	16,000	18,000	20,000	MIN. LAP 30D	BARS IN COMPRESSION: MIN. LAP 24D	STRESS (LB. PER SQ. IN.) 10,000	18,000	20,000
¼"	3	L_o L_2 L_3	5½ 2½ 3½	6½ 3½ 4½	8½ 5½ 6½	9½ 6½ 7½	10½ 7½ 8½	7½	6	4	7½	8½
5/16"	4	L_o L_2 L_3	6½ 3 4	8 4 5½	10½ 6½ 8	12 8 9½	13 9½ 10½	9½	7½	5½	9½	10½
3/8"	4½	L_o L_1 L_2 L_3	8 2 3½ 5	9½ 3½ 5 6½	12½ 6½ 8 9½	14½ 8½ 10 11½	16 10 11½ 13	11½	9	6½	11½	12½
½"	6	L_o L_1 L_2 L_3	10½ 2½ 4½ 6½	12½ 4½ 6½ 8½	17 9 11 13	19 11 13 15	21 13 15 17	15	12	8½	15	17
5/8"	7½	L_o L_1 L_2 L_3	13 3 5½ 8	15½ 6 8 10½	21 11 13½ 16	23½ 13½ 16 18½	26 16 18½ 21	19	15	10½	19	21
¾"	9	L_o L_1	15½ 3½	19 7	25 13	28½ 16½	31½ 19½	22½	18	12½	22½	25
7/8"	10½	L_o L_1	18½ 4½	22 8	29½ 15½	33 19	36½ 22½	26½	21	14½	26½	29
1"	12	L_o L_1	21 5	25 9	33½ 17½	37½ 21½	42 26	30	24	17	30	33½
1⅛"	13½	L_o L_1	23½ 5½	28 10	37½ 19½	42 24	47 29	34	27	19	34	37½
1¼"	15	L_o L_1	26 6	31½ 11½	42 22	47 27	52 32	37½	30	21	37½	42
1⅜"	16½	L_o L_1	29 7	34½ 12½	46 24	52 30	57½ 35½	41½	33	23	41½	46
1½"	18	L_o L_1	31½ 7½	37½ 13½	50 26	56 32	62½ 38½	45	36	25	45	50

NOTES

* BEYOND SECTION WHERE THERE IS NO STRESS.

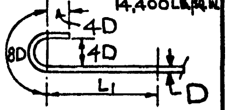
THIS TABLE ONLY APPLIES TO BOND STRESS 120 LB./SQ.IN.

BARS IN TENSION

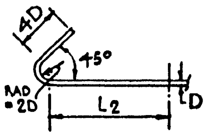


$$L_o = \frac{p_{st} D}{4 s_b} \nless 12D$$

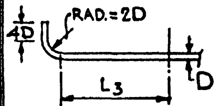
MIN. LAP:— $L_o \nless 30D$ CORRESPONDS TO TENSILE STRESS OF 14,400 LB./SQ.IN.



$$L_1 = L_o - 16D$$

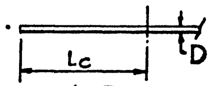


$$L_2 = L_o - 12D$$



$$L_3 = L_o - 8D$$

BARS IN COMPRESSION



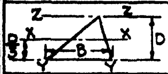
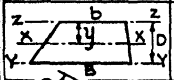
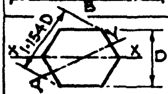
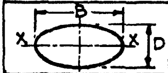
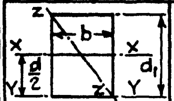
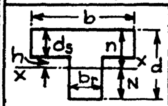
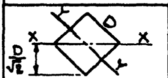
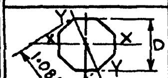
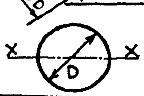
$$L_c = \frac{p_{sc} D}{5 s_b}$$

MIN. LAP:— $L_c \nless 24D$

[24D CORRESPONDS TO A COMPRESSIVE STRESS OF 14,400 LB. PER SQ. IN.]

NOTE.—The data in *Table* 63 is in accordance with B.S. Code No. 114 (1957) for plain round mild steel bars in 1 : 2 : 4 concrete; s_b = 120 lb. per sq. in.

TABLE 64.—GEOMETRICAL PROPERTIES OF SECTIONS.

SECTION	AREA	SECTION MODULUS	(2ND MOMENT OF AREA) MOMENT OF INERTIA	RADIUS OF GYRATION
	TRIANGLE. $\frac{BD}{2}$	XX. TO APEX $\frac{BD^2}{24}$; TO BASE $\frac{BD^2}{12}$	XX. $\frac{BD^3}{36}$ YY. $\frac{BD^3}{12}$ ZZ. $\frac{BD^3}{4}$	ABOUT XX 0·236D
	TRAPEZIUM. $(b+B)\frac{D}{2}=A$ $y=\frac{D}{3}\left[\frac{2B+b}{B+b}\right]$	XX. TO B: $I_{xx}\div(D-y)$ $\frac{(B^2+4Bd+b^2)D^2}{12(B+2b)}$ (To b)	$I_{xx}=\frac{(B^2+4Bb+b^2)}{36(B+b)}D^3$ $I_{YY}=I_{xx}+A(D-y)^2$ $I_{zz}=I_{xx}+A(y)^2$	ABOUT XX $\sqrt{\frac{I_{xx}}{A}}$
	REG. HEXAGON. $0\cdot866D^2$ (SIDE = 0·577D)	XX. $0\cdot12D^3$ YY. $0\cdot104D^3$	XX OR YY. $0\cdot06D^4$	ABOUT XX OR YY 0·263D
	ELLIPSE. $0\cdot25\pi BD$ $=0\cdot7854BD$	XX. $0\cdot098BD^2=\frac{\pi}{32}BD^2$	XX. $\frac{\pi}{64}BD^3=0\cdot049BD^3$	ABOUT XX $\frac{D}{4}$
	RECTANGLE. db	XX. $\frac{bd^2}{6}$ ZZ. $\frac{b^2d^2}{6\sqrt{b^2+d^2}}$	XX. $\frac{bd^3}{12}$ YY. $\frac{bd^3}{3}$ ZZ. $\frac{b^3d^3}{6(\ ^2+\ ^2)}$	ABOUT XX 0·289d
	FLANGED BEAM. $bd_s+b_r(d-d_s)=A$	XX. $\frac{I_{xx}}{n}$ OR $\frac{I_{xx}}{N}$ $N=d-n$ $=\frac{b_rd^2+d_s^2(b-b_r)}{2[bd_s+b_r(d-d_s)]}$	XX. $I_{xx}=\frac{1}{3}\left[bn^3+bN^3-h^3(b-b_r)\right]$ (SEE ALSO BELOW)	ABOUT XX $\sqrt{\frac{I_{xx}}{A}}$
	SQUARE. D^2	XX. $0\cdot118D^3$ YY. $\frac{D^3}{6}$	XX OR YY $\frac{D^4}{12}$	ABOUT XX OR YY 0·289D
	REG. OCTAGON. $0\cdot828D^2$ (SIDE = 0·413D)	XX. $0\cdot109D^3$ YY. $0\cdot1016D^3$	XX. OR YY. $0\cdot055D^4$	ABOUT XX OR YY 0·257D
	CIRCLE $0\cdot7854D^2$ $=\frac{\pi}{4}D^2$	XX. $\frac{\pi}{32}D^3=0\cdot0982D^3$	XX. $\frac{\pi}{64}D^4$ $=0\cdot0491D^4$	ABOUT XX $\frac{D}{4}$

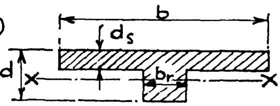
MOMENT OF INERTIA OF TEE SECTION.

(ALSO APPLICABLE TO ELL SECTION AND INVERTED CHANNEL)

MOMENT OF INERTIA ABOUT AXIS X – X THROUGH CENTROID

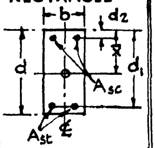
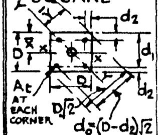
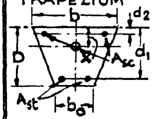
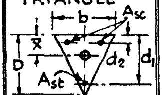
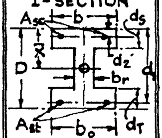
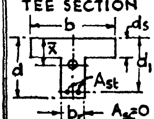
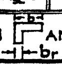
$$I_{xx}=Cb_rd^3$$

VALUES OF C ARE TABULATED.

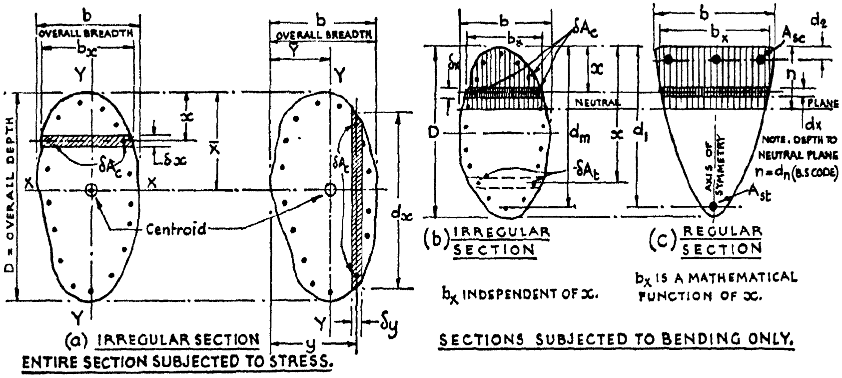


RATIO d_s/d	RATIO b_r/b																
	·12	·14	·16	·18	·20	·22	·24	·26	·28	·30	·35	·40	·45	·50	·60	·70	·80
0·10	·170	·161	·154	·147	·142	·137	·133	·129	·125	·122	·115	·110	·106	·102	·096	·092	·089
0·15	·180	·172	·165	·158	·152	·146	·143	·138	·135	·131	·124	·117	·111	·107	·100	·095	·090
0·20	·183	·175	·169	·163	·157	·152	·147	·143	·140	·137	·128	·121	·116	·111	·103	·097	·091
0·30	·184	·176	·170	·164	·159	·155	·150	·146	·142	·139	·132	·125	·119	·114	·105	·099	·093
0·40	·190	·180	·173	·165	·160	·155	·151	·146	·142	·139	·132	·125	·119	·115	·106	·099	·093
0·45	·197	·185	·176	·169	·162	·156	·152	·147	·143	·140	·132	·125	·119	·115	·106	·099	·093
0·50	·208	·195	·183	·174	·167	·160	·155	·150	·146	·141	·133	·126	·120	·115	·106	·099	·093

RECTILINEAR SECTIONS.

	ENTIRE CROSS-SECTION SUBJECTED TO STRESS	BENDING ONLY CONCRETE INEFFECTIVE IN TENSION COMPRESSION ZONE AT TOP (AS DRAWN)
NOTATION (ADDITIONAL TO DATA ON DIAGRAMS) GEOMETRICAL PROPERTIES ARE EXPRESSED IN EQUIVALENT CONCRETE UNITS	EFFECTIVE AREA: A_e POSITION OF CENTROID ⊕: $\bar{X}$ FROM TOP EDGE MOMENT OF INERTIA ABOUT CENTROIDAL AXIS: I_{xx} MODULUS OF SECTION FOR TOP EDGE: $Z_0 = \frac{I_{xx}}{\bar{X}}$ FOR BOTTOM EDGE: $Z_D = \frac{I_{xx}}{D-\bar{X}}$ OR $\frac{I_{xx}}{d-\bar{X}}$ } UNLESS EXPRESSED OTHERWISE RADIUS OF GYRATION: $g = \sqrt{\frac{I_{xx}}{A_e}}$	DISTANCE OF NEUTRAL PLANE BELOW TOP EDGE: $= n$ ($= d_n$ PER B.S. CODE) RELATED TO MAX. STRESSES: $n\ (= d_n) = \dfrac{d_1}{1 + \dfrac{f_{st}}{m f_{cb}}}$ LEVER ARM: $= a$ ($= \ell_a$ PER B.S. CODE) COMPRESSION-REINFORCEMENT FACTOR: $K = A_{sc}(m-1)\left(\frac{n-d_2}{n}\right)$ MOMENT OF RESISTANCE: M_{rc} (COMPRESSION) $= a C_1 f_{cb}$ M_{rt} (TENSION) $= a f_{st} A_{st}$ } UNLESS EXPRESSED OTHERWISE
RECTANGLE 	$A_e = bd + (m-1)(A_{st}+A_{sc})$ $\bar{X} = \frac{1}{A_e}\left[\frac{bd^2}{2} + (m-1)(A_{st}d_1 + A_{sc}d_2)\right]$ $= 0.5d$ IF $A_{sc} = A_{st}$ $I_{xx} = \frac{b}{3}\left[\bar{X}^3 + (d-\bar{X})^3\right] + (m-1)\left[A_{st}(d_1-\bar{X})^2 + A_{sc}(\bar{X}-d_2)^2\right]$ $= \frac{bd^3}{6} + 2A_{st}\left(\frac{d}{2}-d_2\right)^2$ IF $A_{sc} = A_{st}$	$n = \sqrt{m^2\left(\frac{A_{st}+A_{sc}}{b}\right)^2 + \frac{2d_1}{b}\left(A_{st}+\frac{d_2}{d}A_{sc}\right)m} - \frac{m}{b}(A_{st}+A_{sc})$ $a = \frac{1}{C_1}\left[\frac{bn}{2}\left(d_1-\frac{n}{3}\right) + K(d_1-d_2)\right]$ $C_1 = \frac{bn}{2} + K$ IF $A_{sc} = 0$:— $a = d_1 - \frac{n}{3}$ $M_{rc} = 0.5\, n a b p_{cb}\ (= Q b d_1^2)$
SQUARE 	$A_e = D^2 + 4(m-1)A_t$ ABOUT AXIS X-X. $\bar{X} = 0.5D$ $I_{xx} = \frac{D^4}{12} + 4A_t(m-1)\left(\frac{D}{2}-d_2\right)^2$ $Z_0 = Z_D = \frac{2I_{xx}}{D}$ ABOUT DIAGONAL Y-Y. $\bar{Y} = 0.707D$ $I_{YY} = \frac{D^4}{12} + 2A_t(m-1)\left(\frac{D}{2}-d_2\right)^2$ $Z_0 = Z_D = \frac{I_{YY}2\sqrt{2}}{D}$	ABOUT AXIS X-X:— FORMULÆ AS FOR RECTANGULAR SECTION WITH $A_{st} = A_{sc} = 2A_t$ AND $b = D$. ABOUT DIAGONAL Y-Y:— $n \not> 0.707D$ $a = \frac{1}{C_1}\left[\frac{n^2}{3\sqrt{2}}\left(d_0 - \frac{4}{9}n\right) + A_t(m-1)\left(\frac{n-d_2\sqrt{2}}{n}\right)(D-2d_2)\sqrt{2}\right]$ $C_1 = \frac{n^2}{3\sqrt{2}} + A_t(m-1)\left(\frac{n-d_2\sqrt{2}}{n}\right)$; $A_{st} = A_t$
TRAPEZIUM 	$A_e = \left(\frac{b+b_0}{2}\right)D + (m-1)(A_{st}+A_{sc})$ $\bar{X} = \frac{1}{A_e}\left[\frac{D^2}{6}(b+2b_0) + (m-1)(A_{st}d + A_{sc}d_2)\right] = \left[\frac{b+2b_0}{b+b_0}\right]\frac{D}{3}$ APPROX. $I_{xx} = \left[\frac{(b+b_0)^2 + 2bb_0}{b+b_0}\right]\frac{D^3}{36} + (m-1)\left[A_{st}(d-\bar{X}) + A_{sc}(\bar{X}-d_2)\right]$	$a = \frac{1}{C_1}\left\{\frac{n}{2}\left[b\left(d_1-\frac{n}{3}\right) - \frac{n}{3D}(b-b_0)\left(d_1-\frac{4}{9}n\right)\right] + K(d_1-d_2)\right\}$ $C_1 = \frac{n}{2}\left[b - \frac{n}{3D}(b-b_0)\right] + K$
TRIANGLE 	$A_e = \frac{bD}{2} + (m-1)(A_{st}+A_{sc})$ $\bar{X} = \frac{1}{A_e}\left[\frac{bD^2}{6} + (m-1)(A_{st}d_1 + A_{sc}d_2)\right] = \frac{D}{3}$ APPROX. $I_{xx} = \frac{bD^3}{36} + (m-1)\left[A_{st}(d_1-\bar{X})^2 + A_{sc}(\bar{X}-d_2)^2\right]$	$a = \frac{1}{C_1}\left\{\frac{bn}{2}\left[\left(d_1-\frac{n}{3}\right) - \frac{n}{3D}\left(d_1-\frac{4}{9}n\right)\right] + K(d_1-d_2)\right\}$ $C_1 = \frac{bn}{2}\left(1-\frac{n}{3D}\right) + K$
I-SECTION 	$A_e = b \cdot d_s + b_0 d_T + b_r(D-d_s-d_T) + (m-1)(A_{st}+A_{sc})$ $\bar{X} = \frac{1}{2A_e}\Big[(b-b_r)d_s^2 + (b_0-b_r)(2D-d_T)d_T + b_r D^2 + 2(m-1)(A_{st}d + A_{sc}d_2)\Big]$ $I_{xx} = \frac{1}{3}\Big[b_s\bar{X}^3 - (b_s-b)(\bar{X}-d_s)^3 + b_0(D-\bar{X})^3 - (b_0-b_r)(D-\bar{X}-d_T)^3\Big] + (m-1)\left[A_{st}(\bar{X}-d_1)^2 + A_{sc}(\bar{X}-d_2)^2\right]$	IF $n \not> d_s$ USE FORMULÆ FOR RECTANGLE IF $n > d_s$ $a = \frac{1}{C_1}\Big[\frac{bn}{2}\left(d_1-\frac{n}{3}\right) - \frac{1}{6n}(b-b_r)(n-d_s)(3d_1-2d_s-n) - K(d-d_2)\Big]$ $C_1 = \frac{1}{2}\left[b\,n - \frac{1}{n}(b-b_r)(n-d_s)\right] + K$
TEE SECTION 	$A_e = b\,d_s + b_r(d-d_s) + (m-1)A_{st}$ $\bar{X} = \frac{1}{2A_e}\left[b\,d_s^2 + b_r(d-d_s)(d+d_s) + 2A_{st}d(m-1)\right]$ $I_{xx} = \frac{1}{3}\left[b\,\bar{X}^3 - (b-b_r)(\bar{X}-d_s)^3 + b_r(d-\bar{X})^3\right] + (m-1)A_{st}(d_1-\bar{X})^2$	$n = \frac{0.5b\,d_s^2 + A_{st}m d_1}{b_s d_s + A_T m}$ NOTE: IF $n \not> d_s$ USE FORMULÆ FOR RECTANGLE $a = d_1 - \left(\frac{3n-2d_s}{2n-d_s}\right)\frac{d_s}{3} = d_1 - 0.5d_s$ (APPROX.) $M_{rc} = (2n-d_s)\frac{a p_{cb} b d_s}{2n}$
ELL SECTION  AND INVERTED CHANNEL 		USE FORMULÆ FOR TEE SECTION

PROPERTIES OF REINFORCED CONCRETE SECTIONS.



Formulæ containing summation sign Σ apply to irregular sections only (*Figs. a* and *b*).
Formulæ containing integration sign $\int$ apply only to regular sections (*Fig. c*) in which b_x is a mathematical function of x.

Entire Section subjected to Stress.

Effective area.—$A_e = \sum_0^D [b_x.\delta x + (m - 1)\delta A_C] = \left[\int_0^D b_x.dx\right] + (m - 1)(A_{sc} + A_{st})$.

Position of centroid.—$\bar{X} = \frac{1}{A_e}\left\{\sum_0^D x[b_x.\delta x + (m - 1)\delta A_C]\right\}$

$$= \frac{1}{A_e}\left\{\left[\int_0^D x b_x.dx\right] + (m - 1)(A_{sc}d_2 + A_{st}d_1)\right\}.$$

$$\bar{Y} = \frac{1}{A_e}\left\{\sum_0^b y[d_x.\delta y + (m - 1)\delta A_C]\right\} = \frac{b}{2} \text{ (regular section).}$$

Moment of inertia about axes through centroid.—$I_{XX} = \sum_0^D [b_x.\delta x + (m - 1)\delta A_C](\bar{X} - x)^2$

$$= \left[\int_0^D b_x(\bar{X} - x)^2 dx\right] + (m - 1)[A_{sc}(\bar{X} - d_2)^2 + A_{st}(d_1 - \bar{X})^2]$$

$$I_{YY} = \sum_0^b [d_x.\delta y + (m - 1)\delta A_C](\bar{Y} - y)^2.$$

Radius of gyration.—$gX = \sqrt{\frac{I_{XX}}{A_e}}$; $g_Y = \sqrt{\frac{I_{YY}}{A_e}}$.

Modulus of section.—$Z_{(x=0)} = \frac{I_{XX}}{\bar{X}}$; $Z_{(x=D)} = \frac{I_{XX}}{D - \bar{X}}$; $Z_{(y=0)} = \frac{I_{YY}}{\bar{Y}}$; $Z_{(y=b)} = \frac{I_{YY}}{b - \bar{Y}}$.

Section subjected to Bending only.

$$\delta C = (n - x)[b_x.\delta x + (m - 1)\delta A_C]; \ \delta T = (x - n)\delta A_t$$

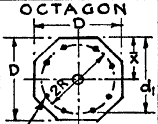
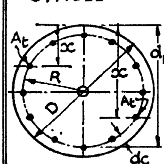
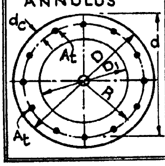
Tota compression factor.—$C' = \sum_0^n \delta C = \left[\int_0^n (n - x)b_x.dx\right] + (m - 1)(n - d_2)A_{sc}$.

Total tension factor.—$T' = \sum_n^{d_m} \delta T = (d_1 - n)A_{st}$.

Position of neutral plane.—General: Value of n ($= d_n$) satisfying formula $C' - mT' = 0$.

In terms of maximum stresses: $n = \dfrac{d_m}{1 + \dfrac{f_{st}}{mf_{cb}}} = \dfrac{d_1}{1 + \dfrac{f_{st}}{mf_{cb}}}$.

(*Continued on page* 253.)

NOTATION (ADDITIONAL TO DATA ON DIAGRAMS). GEOMETRICAL PROPERTIES ARE EXPRESSED IN EQUIVALENT CONCRETE UNITS	ENTIRE CROSS-SECTION SUBJECTED TO STRESS	BENDING ONLY CONCRETE INEFFECTIVE IN TENSION, COMPRESSION ZONE AT TOP (AS DRAWN)
	EFFECTIVE AREA: A_e POSITION OF CENTROID ⊕: $\bar{X}$ FROM TOP EDGE MOMENT OF INERTIA ABOUT CENTROIDAL AXIS: I_{xx} MODULUS OF SECTION FOR TOP EDGE: $Z_0=\frac{I_{xx}}{\bar{X}}$ FOR BOTTOM EDGE: $Z_D=\frac{I_{xx}}{D-\bar{X}}$ (UNLESS EXPRESSED OTHERWISE) RADIUS OF GYRATION: $g=\sqrt{\frac{I_{xx}}{A_e}}$	DISTANCE OF NEUTRAL PLANE BELOW TOP EDGE: $= n$ ($= d_n$ PER B.S. CODE) RELATED TO MAX. STRESSES: $n\ (=d_n)=\frac{d_1}{1+\frac{f_{st}}{mf_{cb}}}$ LEVER ARM: $= a$ ($= l_a$ PER B.S. CODE) MOMENT OF RESISTANCE: M_{rc}(COMPRESSION) $= aC_1f_{cb}$ M_{rt}(TENSION) $= af_{st}A_{st}$ (UNLESS EXPRESSED OTHERWISE)
OCTAGON  A_t AT EACH CORNER $d_1 = D - d_c$ $d_c=\frac{D}{2}-0.924R$	$A_e = 0.828D^2+8A_t(m-1)$ $\bar{X} = 0.5D$ $I_{xx} = 0.055D^4+4A_tR^2(m-1)$ $Z_0=Z_D= 0.109D^3+\frac{4A_tR^2}{D}(m-1)$	$a=\frac{1}{C_1}\left[0.207Dn\left(d-\frac{n}{8}\right)+E+2A_t(m-1)\left(\frac{n-d_c}{n}\right)(d_1-d_c)\right]$ $C_1 = 0.207Dn+F+2A_t(m-1)\left(\frac{n-d_c}{n}\right)$ IF $n \ngtr 0.3D$:— $E=F\left(d-\frac{4}{9}n\right)$ $F=\frac{n^2}{3}$ IF $n > 0.3D$:— $E=F\left[d_1-0.195D\left(\frac{n-0.22D}{n-0.195D}\right)\right]$ $F=\frac{0.086D^2}{n}(n-0.195D)$
CIRCLE 	$A_e= 0.7854D^2+(m-1)\Sigma A_t$ $\bar{X} = 0.5D$ $I_{xx}=\frac{\pi D^4}{64}+(m-1)\frac{R^2}{2}\Sigma A_t$ $Z_0=Z_D=\frac{\pi D^3}{32}+(m-1)\frac{R^2}{D}\Sigma A_t$	$M_{rc}=M_{rt}=\left[C'_c(h_T-h_c)+C'_s(h_T-h_s)\right]f_{cb}$ $C'_c=\left[\frac{D^2}{4n}-\frac{D-n}{3}\right]\sqrt{(D-n)n}+\frac{D^2}{8n}(D-2n)\sin^{-1}\left(\frac{D-2n}{D}\right)$ $C'_s=\frac{m-1}{n}\Sigma_0^n(n-x)A_t$ $h_c=\frac{1}{C_c}\left\{\left[\frac{2}{3}n^2-\frac{7}{12}D\left(n+\frac{D}{4}\right)+\frac{5}{32n}D^3\right]\sqrt{(D-n)n}+\frac{1}{2}\sqrt{(D-n)^3n^3}+\frac{D^3}{8n}\left(\frac{5}{8}D-n\right)\sin^{-1}\left(\frac{D-2n}{D}\right)\right\}$ $h_T=\frac{\Sigma_n^d(x-n)xA_t}{\Sigma_n^d(x-n)A_t}$; $h_s=\frac{\Sigma_0^n(n-x)xA_t}{\Sigma_0^n(n-x)A_t}$
ANNULUS 	$A_e= 0.7854(D^2-D_1^2)+(m-1)\Sigma A_t$ $\bar{X} = 0.5D$ $I_{xx}=\frac{\pi}{64}(D^4-D_1^4)+(m-1)\frac{R^2}{2}\Sigma A_t$ $Z_0=Z_D=\frac{\pi}{32D}(D^4-D_1^4)+(m-1)\frac{R^2}{D}\Sigma A_t$	IF $n \ngtr 0.5(D-D_1)$ USE FORMLÆ FOR CIRCLE. IF $n > 0.5(D-D_1)$ USE GRAPHICAL METHOD.

(*Continued from page* 252.)

Lever arm.—

$$l_a=\frac{\sum_n^{d_m} x.\delta T}{T'}-\frac{\sum_0^n x.\delta C}{C'}=d_1-\frac{\left[\int_0^n x(n-x)b_x.dx\right]+(m-1)(n-d_2)A_{sc}d_2}{C'}$$

Moment of resistance.—$M_{rc}=\frac{p_{cb}l_a}{n}C'$; $M_{rt}=\frac{p_{cb}l_am}{n}T'=p_{st}l_aA_{st}$.

NOTES.—(*a*) For properties of common reinforced concrete sections see *Tables* 65A and 65B.

(*b*) For properties of sections subjected to stress on entire section, but neglecting reinforcement, omit terms δA_t, A_{st}, δA_C and A_{sc} from foregoing formulæ. Properties of some common sections for this condition are given in *Table* 64.

DESIGN OF BEAMS: MODULAR-RATIO METHOD.

Notation.

PROPERTIES OF SECTION.

A_{sc} = area of compression reinforcement.
l_a = lever arm = $a_1 d_1$; l_{ac} (concrete); l_{as} (compression reinforcement).
A_{st} = area of tensile reinforcement.
b = breadth of a rectangular beam or the breadth of slab of flanged beams or breadth of slab assumed to be effective.
b_r = breadth of rib of flanged beam.
d_1 = effective depth of beam, that is the distance from the compression edge of the section to the centroid of the tensile reinforcement.
d_s = thickness of slab of flanged beam; $s_1 = \frac{d_s}{d_1}$.
d_2 = depth to centroid of compression reinforcement from adjacent edge of beam $= f_2 d_1$
M = applied bending moment. M_r = moment of resistance.
d_n = depth to neutral plane $= n_1 d_1$.
r_t = proportion of tensile reinforcement $= \frac{A_{st}}{bd_1}$; r_b for balanced design.
r_c = proportion of compression reinforcement $= \frac{A_{sc}}{bd_1}$.
Q_c = factor in moment of resistance formula, $M_r = Q_c bd_1^2$.

STRESSES.

p_{cb} = maximum permissible compressive stress in concrete.
f_{cb} = actual maximum compressive stress in concrete.
p_{st} = maximum permissible stress in tensile reinforcement.
f_{st} = actual stress in tensile reinforcement.
p_{sc} = maximum permissible stress in compression reinforcement.
f_{sc} = actual stress in compression reinforcement.
m = modular ratio $= \frac{\text{modulus of elasticity of steel}}{\text{modulus of elasticity of concrete}} = \frac{E_s}{E_c}$.
r = ratio of maximum permissible stresses $= \frac{p_{st}}{p_{cb}}$.
r_1 = ratio of actual stresses $= \frac{f_{st}}{f_{cb}}$.

Procedures.

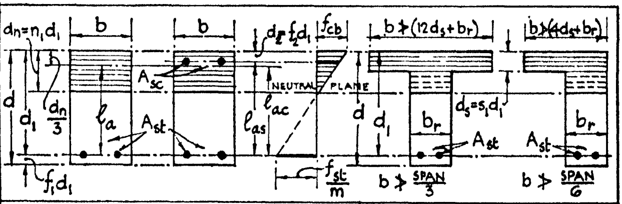
The procedures for the design of rectangular beams and for the determination of stresses in beams make use of the formulæ in *Table* 66 and the data in *Tables* 67, 68, 69 and 70A to 70D The designs are based on the modular-ratio method with $m = 15$.

To Design a Rectangular Beam to Resist a Given Bending Moment with Given Stresses.

Method (*a*).—For the given stresses find Q_c from *Table* 68 and find bd_1^2 required from $\frac{M}{Q_c}$. Select suitable values of b and d_1 from consideration of shearing, or from *Table* 69 to give the required value of bd_1^2. For the ratio of the given stresses find a_1 from *Table* 68 and the amount of the tensile reinforcement A_{st} from formula (10) in *Table* 66 with $l_a = a_1 d_1$.

Method (*b*).—If the permissible stresses are any of those in *Tables* 70A to 70D, assume an effective depth d_1 and read from the appropriate table the moment of resistance corresponding to this depth. The applied bending moment divided by this moment of resistance gives the breadth of beam required. If the relative values of d_1 and b thus derived are unsuitable, select another value of d_1 and repeat. From the same table read the area of reinforcement for the selected value of d_1 and multiply this area by b to give A_{st}.

Method (*c*).—If *Tables* 70A to 70D do not apply, for the permissible stresses read r_1, n_1, a_1 and r_t from the curves in *Table* 67, and calculate Q_c from formula (8) in *Table* 66. Then bd_1^2 is calculated from $\frac{M}{Q_c}$, and b and d_1 are obtained from *Table* 69. The area of tensile reinforcement is found from $A_{st} = r_t bd_1$. Alternatively, the values of n_1, a_1 and Q_c can be obtained from formulæ (1*a*), (5), and (8); then $bd_1^2 = \frac{M}{Q_c}$; find b and d_1 from *Table* 69 and A_{st} from $\frac{M}{a_1 d_1 p_{st}}$. **The values of b and d_1 should be checked to ensure sufficient resistance to the shearing force.**

MODULAR RATIO $= m = \frac{E_s}{E_c}$ f_{st}, f_{cb} = ACTUAL STRESSES IN TENSILE REINFORCEMENT AND CONCRETE p_{st}, p_{cb} = PERMISSIBLE STRESSES IN DITTO $r_t = \frac{A_{st}}{bd_1}$ $\quad r_c = \frac{A_{sc}}{bd_1}$ F_{cc}, F_{cs} = COMPRESSION FORCES IN CONCRETE AND COMPRESSION REINFT.		 RECTANGULAR BEAM	FLANGED BEAM($d_n > d_s$)
POSITION OF NEUTRAL PLANE	$d_n = n_1 d_1$ (1) $n_1 = \frac{1}{1+\frac{f_{st}}{m f_{cb}}}$ OR $n_1 = \frac{1}{1+\frac{p_{st}}{m p_{cb}}}$ (1a)	WITH TENSILE REINFORCEMENT ONLY $n_1 = \sqrt{(mr_t)^2 + 2mr_t} - mr_t$ (2) WITH COMPRESSION REINFORCEMENT $n_1 = \sqrt{m^2(r_t+r_c)^2 + 2m(r_t+f_2 r_c)} - m(r_t+r_c)$ (2a)	$d_n = \frac{m d_1 A_{st} + \frac{1}{2} b d_s^2}{m A_{st} + b d_s}$ (3) $n_1 = \frac{m r_t + \frac{1}{2} s_1^2}{m r_t + s_1}$ (3a)
LEVER ARM	$l_a = a_1 d_1$ (4) CONCRETE ONLY $l_{ac} = a_c d_1$ (4a) COMP. REINF. ONLY (4b) $l_{as} = a_s d_1 = d_1 - d_2$	WITH TENSILE REINFORCEMENT ONLY (5) $l_a = l_{ac} = d_1 - \frac{d_n}{3}$; $a_1 = 1 - \frac{1}{3} n_1$ WITH COMPRESSION REINFORCEMENT (5a) $a_1 = \dfrac{\frac{1}{2} n_1 (1-\frac{1}{3} n_1) + r_c (m-1)\left(\frac{n_1 - f_2}{n_1}\right)(1-f_2)}{\frac{1}{2} n_1 + r_c (m-1)\left(\frac{n_1-f_2}{n_1}\right)}$ $= 1 - \frac{1}{6} n_1 - \frac{1}{2} f_2$ APPROX. (5b)	$l_a = d_1 - \left(\frac{3d_n - 2d_s}{2d_n - d_s}\right)\frac{d_s}{3}$ (6) $a_1 = 1 - \left(\frac{3n_1 - 2s_1}{2n_1 - s_1}\right)\frac{s_1}{3}$ (6a) $l_a = d_1 - \frac{1}{2} d_s$ APPROX. (6b) $a_1 = 1 - \frac{1}{2} s_1$ APPROX. (6c)
MOMENTS OF RESISTANCE	COMPRESSION (7) $M_{rc} = F_{cc} l_{ac} + F_{cs} l_{as}$ TENSION (7a) $M_{rt} = l_a A_{st} f_{st}$	WITH TENSILE REINFORCEMENT ONLY (8) $M_{rc} = Q_c b d_1^2$; $Q_c = \frac{1}{2} n_1 (1-\frac{1}{3} n_1) f_{cb}$ $M_{rt} = a_1 r_t f_{st} b d_1^2$ (8a) WITH COMPRESSION REINFORCEMENT (8b) $M_{rc} = \left[\frac{1}{2} n_1 (1-\frac{1}{3} n_1) + r_c (m-1)\left(\frac{n_1 - f_2}{n_1}\right)(1-f_2)\right] f_{cb} b d_1^2$ $M_{rt} = a_1 r_t f_{st} b d_1^2$ (8c)	$M_{rc} = (2d_n - d_s)(d_1 - \frac{1}{2} d_s)\frac{f_{cb} b d_s}{2d_n}$ (9) $= (2n_1 - s_1)(1 - \frac{1}{2} s_1)\left(\frac{f_{cb} s_1}{2n_1}\right) b d_1^2$ (9a) $= \frac{1}{2}(d_1 - \frac{1}{3} d_s) f_{cb} b d_s$ APPROX. (9b) $M_{rt} = A_{st}(d_1 - \frac{1}{2} d_s) f_{cb}$ APPROX. (9c)
DESIGN	TO RESIST BENDING MOMENT M $M_c = M_r$ OF CONCRETE $M_s = M_r$ REQUIRED FROM COMP. REINFORCEMENT	WITH TENSILE REINFORCEMENT ONLY $d_1 = \sqrt{\frac{M}{Q_c b}}$; $Q_c = \frac{M}{b d_1^2}$; $A_{st} = \frac{M}{l_a p_{st}}$ (10) WITH COMPRESSION REINFORCEMENT $M_c = Q_c b d_1^2$; $M_s = M - M_c$ (10a) $A_{sc} = \dfrac{M_s}{(m-1)\left(\frac{d_n - d_2}{d_n}\right) l_{as} p_{cb}}$; $A_{st} = \left(\frac{M_c}{l_{ac}} + \frac{M_s}{l_{as}}\right)\frac{1}{p_{st}}$	$A_{st} = \frac{M}{a_1 d_1 p_{st}}$ (11) $A_{st} = \frac{M}{(d_1 - \frac{1}{2} d_s) p_{st}}$ (11a) $b \nless \frac{2 M d_n}{(2d_n - d_s)(d_1 - \frac{1}{2} d_s) d_s p_{cb}}$ (11b) OR APPROX. $b \nless \frac{2M}{d_s l_a p_{cb}}$ (11c)
PROPORTION OF TENSILE REINFORCEMENT FOR BALANCED DESIGN $r_b = \frac{A_{st}}{bd_1}$		WITH TENSILE REINFORCEMENT ONLY $r_b = \frac{n_1 p_{cb}}{2 p_{st}}$ (12) WITH COMPRESSION REINFORCEMENT $r_b = \left[\frac{1}{2} n_1 + r_c (m-1)\left(\frac{n_1 - f_2}{n_1}\right)\right]\frac{p_{cb}}{p_{st}}$ (12a)	$r_b = \frac{s_1}{2n_1}(2n_1 - s_1)\frac{p_{cb}}{p_{st}}$ (13)
MAXIMUM STRESSES		$f_{st} = \frac{M}{l_a A_{st}}$ $\quad f_{cb} = \frac{n_1 f_{st}}{m(1-n_1)}$ (14)	RATIO OF STRESSES $r_1 = \frac{f_{st}}{f_{cb}} = m\left(\frac{1}{n_1} - 1\right)$ (15)

DESIGN OF BEAMS: MODULAR-RATIO METHOD

(*continued from page* 254).

To Determine the Stresses in a Rectangular Beam Subjected to a Given Bending Moment.

Method (*a*).—Determine r_t from the given data and from *Table* 68 find the corresponding values of a_1 and r_1. Then the maximum stresses are given by formula (14) in *Table* 66, in which $l_a = a_1 d_1$; or $f_{cb} = \frac{f_{st}}{r_1}$.

Method (*b*).—Determine from *Table* 67 the values of a_1 and n_1 corresponding to the known value of r_t [or calculate n_1 from formula (2) and a_1 from formula (5) in *Table* 66]; also find the corresponding value of r_1 and calculate the stresses as in *Method* (*a*).

To Determine the Moment of Resistance of a Rectangular Beam at Given Maximum Stresses.

Method (*a*).—Determine r_t and from *Table* 68 find the values of a_1 and n_1. The moment of resistance based on the reinforcement is given by formula (8*a*) or, based on the concrete, by formula (8) in *Table* 66. If the two results differ, the maximum safe moment of resistance is the smaller of the two calculated moments.

Method (*b*).—Determine r_t and find the corresponding value of a_1 and n_1 from *Table* 67. The moments of resistance can then be calculated in the same way as in *Method* (*a*). In *Methods* (*a*) and (*b*) it is not necessary to calculate the moments of resistance for the resistance of the reinforcement and the concrete if the ratio of stresses r_1 corresponding to r_t is determined (from *Table* 68 or 67). If r_1 is greater than the ratio of the permissible stresses, the moment of resistance of the reinforcement is the safe moment of resistance, and, if smaller, the moment of resistance of the concrete is the safe value.

To Design a Rectangular Beam with Compression Reinforcement.

Method (*a*).—With given, or selected, values of b and d_1 find Q_c from *Table* 68 for the specified permissible stresses, and from the same table find the value of n_1 for the ratio of these stresses. The required moment of resistance to be provided by the compression reinforcement is $M_S = M - Q_c b d_1^2$. Substitute in formula (10*a*) in *Table* 66 to find A_{sc}, the area of compression reinforcement required. Evaluate the lever arms l_{as} from formula (4*b*) and l_{ac} from formula (5), or approximately from formula (5*b*), and find the area of tensile reinforcement A_{st} required from formula (10*a*) in *Table* 66. If A_{sc} is greater than A_{st}, *Method* (*c*) should be used. If the stresses in *Tables* 70A to 70D apply, these tables can be used directly if $A_{st} = A_{sc}$.

Method (*b*).—Find n_1 and a_1 from *Table* 67 for the given stresses, and calculate Q_c. Then proceed as *Method* (*a*).

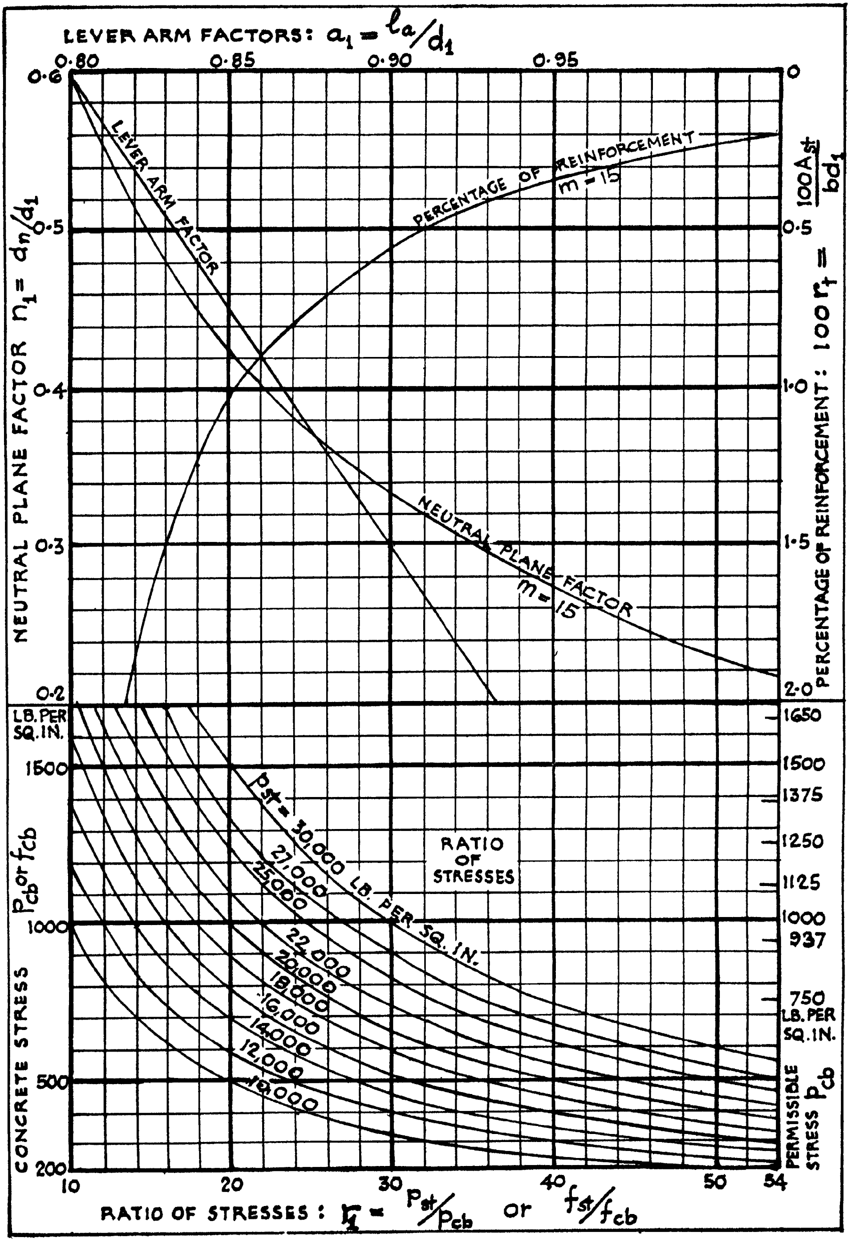
Method (*c*).—If in *Methods* (*a*) or (*b*) A_{sc} exceeds A_{st} the "steel-beam" theory can be applied by substitution in $A_{st} = \frac{M}{l_{as} p_{st}}$ and $A_{sc} = \frac{M}{l_{as} p_{sc}}$.

To Determine the Stresses in a Rectangular Beam with Compression Reinforcement.—Determine n_1 from formula (2*a*) and a_1 from formula (5*a*) [or approximately from formula (5*b*)]. Substitute in formula (14), in which $l_a = a_1 d_1$, to give the maximum stresses.

To Find the Moment of Resistance of a Rectangular Beam with Compression Reinforcement.—Determine n_1 and a_1 from formulæ (2*a*) and (5*a*), or (5*b*) for the known values of r_t and r_c. Substitute with the permissible stresses in formulæ (8*b*) and (8*c*) in *Table* 66; the smaller of the two moments is the safe moment of resistance.

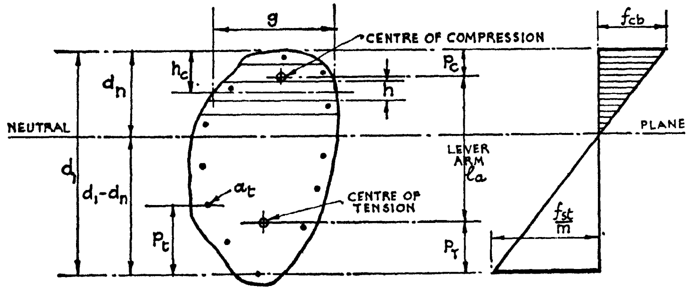
To Design a Flanged Beam for Given Maximum Stresses and a Given Bending Moment.—The thickness of the slab d_s, the effective depth d_1, the breadth of the rib b_r and the maximum effective width of the flange b are generally known since they are determined from considerations other than the bending moment on the beam. Find n_1 for the given stresses from *Tables* 67 or 68 or from formula (1*a*). If $n_1 d_1$ does not exceed d_s, proceed as for a rectangular beam. If $d_n (= n_1 d_1)$ exceeds d_s, find a_1 from formula (6*a*) and A_{st} from formula (11). Check the breadth of flange required from formula (11*b*) in *Table* 66.

For approximate calculations, l_a is given by formula (6*b*), A_{st} by formula (11*a*), and b is checked from formula (11*c*).


LEVER ARM FACTORS: $a_1 = l_a/d_1$
0·80
0·85
0·90
0·95
NEUTRAL PLANE FACTOR $n_1 = d_n/d_1$
0·6
0·5
0·4
0·3
0·2
PERCENTAGE OF REINFORCEMENT: $100 r_t = \frac{100 A_{st}}{bd_1}$
0
0·5
1·0
1·5
2·0
LEVER ARM FACTOR
PERCENTAGE OF REINFORCEMENT m = 15
NEUTRAL PLANE FACTOR m = 15
CONCRETE STRESS p_{cb} or f_{cb}
LB. PER SQ. IN.
1500
1000
500
200
PERMISSIBLE STRESS p_{cb}
1650
1500
1375
1250
1125
1000
937
750
LB. PER SQ. IN.
p_{st} = 30,000 LB. PER SQ. IN.
27,000
25,000
22,000
20,000
18,000
16,000
14,000
12,000
10,000
RATIO OF STRESSES
10
20
30
40
50
54
RATIO OF STRESSES: $r_1 = p_{st}/p_{cb}$ or f_{st}/f_{cb}

DESIGN OF BEAMS: MODULAR-RATIO METHOD

Beams with Irregular Cross-section.—Beams of irregular cross-section can be conveniently designed by the following semi-graphical method.



Draw the cross-section to scale as in the diagram and assume a position for the neutral plane Divide the area above the neutral plane into strips parallel to the neutral plane. For irregular section. the depth h of each strip should be equal, but any regularity in the shape of the cross-section may suggest positions for the boundaries of the strips such that each strip is a common geometrical figures The area a_t is the area of each bar, or of a number of bars each of which is at the same distance p_t from the lowest bar or bars. The measurements to be taken from the diagram are (i) a_t and the corresponding distance p_t; (ii) g, h, and h_c for each of the strips above the neutral plane. The following summations are required, and these can best be made in tabular form: $S_1 = \Sigma a_t(d_1 - d_n - p_t)$; $S_2 = \Sigma a_t(d_1 - d_n - p_t)p_t$; $S_3 = \Sigma(gh)(d_n - h_c)$; and $S_4 = \Sigma(gh)(d_n - h_c)h_c$. The product (gh) is the area of each strip above the neutral plane and, if there is any reinforcement, say, a_c, in any strip, (gh) should be increased by $(m - 1)a_c$.

The position of the centre of tension above the lowest bar or bars is $p_T = \frac{S_2}{S_1}$. The position of the centre of compression below the top edge of the section is $p_c = \frac{S_4}{S_3}$. The lever-arm $l_a = d_1 - p_c - p_T$. Having evaluated numerically the foregoing terms, they are substituted in the following expressions to determine the maximum compressive stress in the concrete and the maximum tensile stress in the steel: $f_{cb} = \frac{Md_n}{l_a S_3}$ and $f_{st} = \frac{f_{cb}(d_1 - d_n)}{d_n} \cdot \frac{S_2}{S_1}$, where M is the applied bending moment.

The assumed depth to the neutral axis is checked by substitution in formula (1) in *Table* 66. If the difference between the assumed and calculated values is greater or less than, say, h the depth of one strip, another value between the two foregoing values should be assumed. The adjustment to the summations is easily made because the distances h_c and p_t are measured from the outer edges of the section, and the adjustment consists of adding or deducting one or more strips and excluding or including one or more areas a_t of reinforcement below the neutral plane.

I-Beams.—The formulæ for the design of a beam of I-section are as follows.

Moment of resistance:

$$Q'bd_1^2 = M$$

Area of tensile reinforcement:

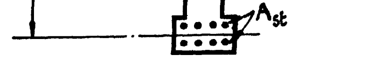
$$A_{st} = \frac{(0{\cdot}5n_1 + C_1 - C_2)bd_1p_{cb}}{p_{st}}$$

in which

$$Q' = Q_c + \left[C_1(1 - f_s) - \frac{C_2}{3}(3 - 2y - n_1)\right]p_{cb}$$

$$C_1 = \frac{14r_c(n_1 - f_s)}{n_1}$$

$$C_2 = \frac{(1 - x)(n_1 - y)^2}{2n_1}$$



The method of designing a beam to resist a bending moment M is first to determine the neutral-plane factor n_1 from *Tables* 66, 67 or 68 for the given permissible stresses. The moment of resistance factor Q_c is obtained from formulæ in *Table* 66 or from *Table* 68. Assume, if they are not given, the dimensions d_1, b, xb, yd_1, and f_sd_1. Calculate C_2 from the formula above. The value of C_1 required is

$$C_1 = \left[\frac{M}{bd_1^2p_{cb}} - \frac{Q_c}{p_{cb}} + \frac{C_2}{3}(3 - 2y - n_1)\right]\frac{1}{1 - f_s}.$$

The cross-sectional area of compression reinforcement required is: $A_{sc} = \frac{C_1n_1bd_1}{14(n_1 - f_s)}$. A_{st} is calculated from the formula given above. The lever-arm l_a is $\frac{M}{A_{st}p_{st}}$. The maximum shearing stress is $\frac{\text{shearing force}}{xbl_a}$.

RATIO OF STRESSES $\frac{p_{st}}{p_{cb}}=r_1$	NEUTRAL AXIS FACTOR n_1	LEVER ARM FACTOR a_1	PERCENTAGE OF TENSILE REINFT. $100r_t$	VALUES OF Q_c ($m = 15$) MAX. CONC. STRESS p_{cb}	MAXIMUM STEEL STRESS = p_{st}					
					12,000	16,000	18,000	20,000	25,000	27,000	30,000
5	0·750	0·750	7·50	500	84	71	66	62	53	50	47
10	0·600	0·800	3·00	550	97	83	77	73	63	59	56
12	0·556	0·815	2·30	600	110	95	89	84	72	69	64
14	0·517	0·828	1·86	650	124	108	101	95	83	78	73
15	0·500	0·833	1·66	700	138	120	113	107	93	89	83
16	0·484	0·839	1·52	750	152	134	126	119	104	99	93
17	0·468	0·844	1·38	760	156	137	128	121	106	102	95
18	0·455	0·848	1·26	800	167	147	139	131	115	109	104
19	0·441	0·853	1·16	817	173	151	143	136	120	110	107
20	0·428	0·857	1·07	850	180	161	152	144	126	122	114
21	0·417	0·861	0·99	880	190	169	159	154	134	128	121
22	0·405	0·865	0·93	900	196	175	166	157	139	133	125
23	0·395	0·869	0·86	950	212	189	179	170	151	145	136
24	0·385	0·872	0·80	975	219	196	186	177	157	151	143
25	0·375	0·875	0·75	1000	226	203	193	184	163	157	148
26	0·366	0·878	0·70	1050	243	218	208	198	176	170	159
27	0·357	0·881	0·66	1100	256	231	221	212	189	182	172
28	0·349	0·884	0·63	1150	271	247	235	226	203	195	184
29	0·341	0·886	0·59	1200	288	262	250	238	216	208	196
30	0·333	0·889	0·56	1250	303	277	264	254	229	221	210
31	0·326	0·891	0·53	1300	319	290	279	268	242	234	222
32	0·319	0·894	0·50	1400	355	322	308	296	271	262	248
33	0·311	0·896	0·47	1500	388	354	340	326	298	289	276
34	0·306	0·898	0·45	1650	431	400	384	374	341	332	316
35	0·300	0·900	0·43								
36	0·294	0·902	0·41								
37	0·288	0·904	0·39								
38	0·283	0·906	0·37								
39	0·277	0·907	0·35								
40	0·273	0·909	0·34								
45	0·250	0·917	0·28								
50	0·231	0·923	0·23								

FORMULÆ:

$$n_1 = \frac{1}{1 + \frac{p_{st}}{m p_{cb}}}$$

$$a_1 = 1 - \frac{n_1}{3} \qquad r_t = \frac{n_1}{2r_1} \qquad Q = \frac{M_r}{bd_1^2} = \frac{n_1 a_1 p_{cb}}{2}$$

FOR ANY MODULAR RATIO (m) AND GIVEN VALUES OF p_{st} AND p_{cb}:—

CALCULATE $r' = \frac{15 p_{st}}{m p_{cb}}$ AND FOR THIS VALUE OF r_1 OBTAIN FROM THE TABLE THE CORRESPONDING VALUES OF n_1 AND a_1. SUBSTITUTE IN $r_t = \frac{n_1}{2r_1}$ AND $Q_c = 0{\cdot}5\, n_1 a_1 p_{cb}$

[For examples of the use of this table, see page facing *Table* 69.]

DESIGN OF BEAMS: MODULAR-RATIO METHOD.

Curtailment of Bars.—The positions at which some of the bars in the bottom of a beam can be terminated or bent up can be estimated from the data in *Table* 69. The positions indicated by the coefficients k_1, k_2 and k_3 are those at which specific bars are no longer required as tensile reinforcement; if they are not bent up, the bar should be continued a distance of not less than twelve times its diameter beyond the position indicated.

Example.—Determine the minimum distance from the supports of the end-span of a continuous beam, carrying a uniformly-distributed load, that two bars can be bent up to assist shearing resistance; the total number of bars of equal diameter in the bottom at mid-span is six; the span of the beam is 20 ft.

From *Table* 69 for the second bar of six, $k_2 = 0{\cdot}30$ and $k_3 = 0{\cdot}18$. Hence the distance from the inner support is $0{\cdot}30 \times 20 = 6$ ft. and the distance from the outer support is $0{\cdot}18 \times 20 = 3$ ft. 7 in.

Examples of Use of Tables 68 and 69.—The designs in the following are to be in accordance with the modular-ratio method with $m = 15$.

(*a*) Design a rectangular beam to resist a bending moment of 500,000 in.-lb. with maximum stresses of 18,000 and 750 lb. per sq. in. From *Table* 68, $Q_c = 126$. Hence $bd_1^2 = \dfrac{500{,}000}{126} = 3968$. From *Table* 69, if $b = 10$ in., $d_1 = 19$ in.; therefore make the total depth 21 in.; $d_1 = 19{\cdot}5$ in., $r_1 = \dfrac{18{,}000}{750} = 24$; from *Table* 68, $a_1 = 0{\cdot}87$.

Hence $A_{st} = \dfrac{500{,}000}{0{\cdot}87 \times 19{\cdot}5 \times 18{,}000} = 1{\cdot}64$ sq. in.; three $\frac{7}{8}$-in. bars (*Table* 60).

(*b*) Calculate the maximum stresses in a rectangular beam 15 in. deep overall (effective depth = $13\frac{1}{2}$ in.) and 9 in. wide, reinforced in tension with two 1-in. bars, and subjected to a bending moment of 200,000 in.-lb. $A_{st} = 1{\cdot}57$ (*Table* 60).

$$100r_t = \frac{1{\cdot}57 \times 100}{9 \times 13{\cdot}5} = 1{\cdot}29.$$

From *Table* 68: $a_1 = 0{\cdot}85$ and $r_1 = 18$. $f_{st} = \dfrac{200{,}000}{0{\cdot}85 \times 13{\cdot}5 \times 1{\cdot}57} = 11{,}100$ lb. per sq. in.; $f_{cb} = \dfrac{11{,}100}{18} = 617$ lb. per sq. in.

(*c*) Calculate the moment of resistance of the beam in (*b*) if stresses are not to exceed 18,000 lb. and 1000 lb. per sq. in. $100r_t = 1{\cdot}26$, $a_1 = 0{\cdot}85$, $n_1 = 0{\cdot}46$ (*Table* 68).

M_r (tension) $= 1{\cdot}57 \times 18{,}000 \times 0{\cdot}85 \times 13{\cdot}5 = 324{,}000$ in.-lb.
M_r (concrete) $= 9 \times 0{\cdot}46 \times (0{\cdot}5 \times 1000) \times 0{\cdot}85 \times 13{\cdot}5^2 = 320{,}000$ in.-lb.

Hence the compressive stress controls, and safe moment of resistance is 320,000 in.-lb.

(*d*) Calculate the amount of tensile reinforcement in a tee-beam, the rib of which is 10 in. wide and extends 18 in. below the soffit of a 6-in. slab. The span is 18 ft. and the distance between adjacent beams 8 ft. Maximum stresses not to exceed 18,000 and 750 lb. per sq. in. Bending moment = 1,800,000 in.-lb.

Effective breadth: Available slab = 96 in., or $\frac{1}{3}$ span = 72 in., or
$12d_s + b = (12 \times 6) + 10 = 82$ in.;

hence maximum $b = 72$ in.; effective depth, say, 20·5 in. (Two layers of bars.) From *Table* 68, d_n (for $r_1 = 24$) $= 0{\cdot}39 \times 20{\cdot}5 = 8$ in.; the neutral plane is below the slab; $l_a - 20{\cdot}5 + 3 = 17{\cdot}5$ in. From formulæ in *Table* 66,

check $M_{rc} = \dfrac{750 \times 72 \times 17{\cdot}5 \times 6}{2 \times 8}[(2 \times 8) - 6] = 3{,}530{,}000$ in.-lb., which is ample.

$$A_{st} = \frac{1{,}800{,}000}{18{,}000 \times 17{\cdot}5} = 5{\cdot}71 \text{ sq. in.; say, six } 1\tfrac{1}{8}\text{-in bars.}$$

(*e*) Calculate the moment of inertia of tee-beam of 20 ft. span with rib 8 in. wide and 16 in. deep below a 4-in. slab. $d_s = 4$ in. $d = 4$ in. + 16 in. = 20 in. $\dfrac{d_s}{d} = \dfrac{4}{20} = 0{\cdot}20$. $b_r = 8$ in. From *Table* 69, the effective width of flange b is the smaller of (i) $12d_s + b_r = (12 \times 4) + 8 = 56$ in., and (ii) $\dfrac{20 \text{ ft.}}{3} = 6$ ft. 8 in. Hence $b = 56$ in.; $\dfrac{b_r}{b} = \dfrac{8}{56} = 0{\cdot}14$. From *Table* 64 with $\dfrac{d_s}{d} = 0{\cdot}20$ and $\dfrac{b_r}{b} = 0{\cdot}14$, $C = 0{\cdot}175$.

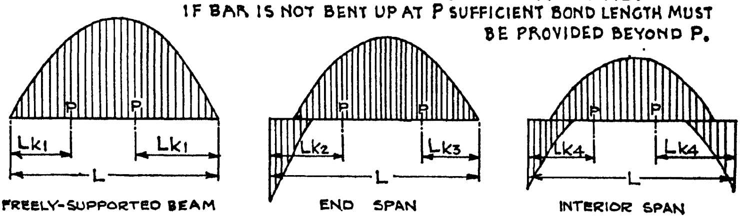
Therefore the moment of inertia is $0{\cdot}175 \times 8 \times 20^3 = 11{,}200$ in.^{4}

VALUES OF bd_1^2

BREADTH b	EFFECTIVE DEPTH d_1 6"	7"	8"	9"	10"	11"	12"	13"	14"	15"	16"	17"	18"	19"	20"	21"	22"	23"	24"
4"	144	196	256	324	400	484	576	676	784	900	1024	1156	1296	1444	1600	1764	1936	2116	2304
5"	180	245	320	405	500	605	720	845	980	1125	1280	1445	1620	1805	2000	2205	2420	2645	2880
6"	216	294	384	486	600	726	864	1014	1176	1350	1536	1734	1944	2166	2400	2646	2904	3174	3456
7"	252	343	448	567	700	847	1008	1183	1372	1575	1792	2023	2268	2527	2800	3087	3388	3703	4032
8"	288	392	512	648	800	968	1152	1352	1568	1800	2048	2312	2592	2888	3200	3528	3872	4232	4608
9"	324	441	576	729	900	1089	1296	1521	1764	2025	2304	2601	2916	3249	3600	3969	4356	4761	5184
10"	360	490	640	810	1000	1200	1440	1690	1960	2250	2560	2890	3240	3610	4000	4410	4840	5290	5760
12"	432	588	768	972	1100	1452	1728	2028	2352	2700	3072	3468	3888	4332	4800	5292	5808	6348	6912
14"	504	686	896	1134	1400	1694	2016	2366	2744	3150	3584	4046	4536	5054	5600	6174	6776	7406	8064
16"	576	784	1024	1296	1600	1936	2304	2704	3136	3600	4096	4624	5184	5776	6400	7056	7744	8464	9216

TERMINATING OR BENDING-UP BARS

POSITION P AT WHICH BARS IN BOTTOM OF BEAMS CAN BE BENT UP OR STOPPED.
MAXIMUM DISTANCE FROM SUPPORT TO P IS KL.
IF BAR IS NOT BENT UP AT P SUFFICIENT BOND LENGTH MUST BE PROVIDED BEYOND P.



FREELY-SUPPORTED BEAM END SPAN INTERIOR SPAN

APPLICABLE TO UNIFORMLY-DISTRIBUTED LOAD ONLY.

No OF BARS AT MID-SPAN	k_1 ORDER OF STOPPING-OFF OR BENDING-UP BARS 1ST	2ND	3RD	4TH	5TH	6TH	k_2 1ST	2ND	3RD	4TH	5TH	6TH	k_3 1ST	2ND	3RD	4TH	5TH	6TH	k_4 1ST	2ND	3RD	4TH	5TH	6TH
1	0	-	-	-	-	-	·11	-	-	-	-	-	0	-	-	-	-	-	·09	-	-	-	-	-
2	·15	0	-	-	-	-	·24	·11	-	-	-	-	·13	0	-	-	-	-	·21	·09	-	-	-	-
3	·21	·09	0	-	-	-	·30	·19	·11	-	-	-	·18	·08	0	-	-	-	·27	·16	·09	-	-	-
4	·25	·15	·07	0	-	-	·33	·24	·17	·11	-	-	·22	·13	·05	0	-	-	·30	·21	·15	·09	-	-
5	·27	·19	·12	·05	0	-	·35	·28	·21	·16	·11	-	·24	·15	·09	·04	0	-	·31	·24	·18	·13	·09	-
6	·30	·21	·15	·09	·04	0	·36	·30	·24	·19	·14	·11	·25	·18	·13	·08	·03	0	·33	·27	·21	·16	·12	·09
7	·31	·23	·17	·12	·08	·04	·39	·32	·26	·22	·18	·14	·29	·20	·15	·11	·07	·03	·34	·29	·23	·19	·15	·12
8	·32	·25	·19	·15	·10	·07	·40	·33	·27	·24	·20	·17	·30	·22	·17	·13	·09	·05	·35	·30	·25	·21	·18	·15

FLANGED BEAMS

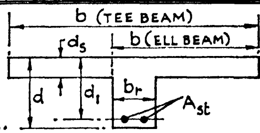
TEE-BEAMS AND ELL-BEAMS

WIDTH OF FLANGE b NOT GREATER THAN THE LEAST OF THE FOLLOWING DIMENSIONS:—

(1) DISTANCE BETWEEN CENTRES OF ADJACENT BEAMS.

(2) $\frac{\text{SPAN}}{3}$ FOR TEE-BEAM. $\frac{\text{SPAN}}{6}$ FOR ELL-BEAM.

(3) $(12d_s + b_r)$ FOR TEE BEAM. $(4d_s + b_r)$ FOR ELL-BEAM.



FORMULÆ FOR BEAMS WHEN CONCRETE EFFECTIVE IN TENSION

	RECTANGULAR BEAMS: TENSILE REINFT. ONLY	RECTANGULAR BEAMS: WITH COMPRESSION REINFT.	FLANGED BEAMS
EFFECTIVE AREA A_e	$bd + (m-1)A_{st}$	$bd + (m-1)(A_{st}+A_{sc})$	$b_r d + (b-b_r)d_s + (m-1)A_{st}$
NEUTRAL PLANE d_n	$\left[\frac{bd^2}{2} + (m-1)A_{st}d_1\right]\frac{1}{A_e}$	$\left\{\frac{bd^2}{2} + (m-1)\left[A_{st}d_1 + A_{sc}d_2\right]\right\}\frac{1}{A_e}$	$\left[b_r d^2 + (b-b_r)d_s^2 + 2(m-1)A_{st}d_1\right]\frac{1}{2A_e}$
MOMENT OF INERTIA I	$\frac{b}{3}\left[d_n^3 + (d-d_n)^3\right] + (m-1)(d-d_n)^2 A_{st}$	$\frac{b}{3}\left[d_n^3 + (d-d_n)^3\right] + (m-1)\left[A_{st}(d_1-d_n)^2 + A_{sc}(d_n-d_2)^2\right]$	$\frac{b_r}{3}\left[d_n^3 + (d-d_n)^3\right] + (m-1)(d_1-d_n)^2 A_{st} + (b-b_r)\left[\frac{1}{12}d_s^2 + (d_n-\frac{1}{2}d_s)^2\right]d_s$

SECTION MODULUS: $Z_o = I/d_n$ $Z_D = I/(d-d_n)$

MAXIMUM STRESSES: $f_{cb} = M/Z_o$ $f_{ct} = M/Z_D$

MOMENT OF RESISTANCE: $M_{rc} = Z_o p_{cb}$ $M_{rt} = Z_c p_{ct}$

DESIGN OF BEAMS: MODULAR-RATIO METHOD.

Examples of Use of Table 70A or 70B (Mild Steel).

Examples (*a*), (*b*) and (*c*) apply to *Table* 70A (p_{st} = 18,000 lb. per sq. in.). The calculations would be similar, but with different numerical values, if p_{st} = 20,000 lb. per sq. in. (*Table* 70B). In examples (*a*) and (*b*) p_{cb} = 1000 lb. per sq. in.; $m = 15$.

(*a*) Design a rectangular beam (tensile reinforcement only) to resist a bending moment of 450,000 in.-lb.

Assume a breadth of 10 in. Moment of resistance required is 45,000 in.-lb. per inch of width, which is given by d_1 = 16 in. and $A_{st} = 0{\cdot}202 \times 10 = 2{\cdot}02$ sq. in. A beam of 18 in. overall depth and 10 in. wide with three 1-in. bars is satisfactory. Other suitable sections can be determined by assuming other widths. The section must have sufficient shearing resistance and should be comparable in size to the span. If there is a difference between the effective depth of the section selected and that in the table, the reinforcement can be reduced thus: With overall depth 18 in. and 1-in. cover, $d_1 = 16{\cdot}5$ in.

$Q_c = \dfrac{450{,}000}{10 \times 16{\cdot}5^2} = 166$; for p_{st} = 18,000 lb., p_{cb}= 900 lb. per sq. in. (from *Table* 68). Hence $r_1 = 20$; $a_1 = 0{\cdot}857$; $A_{st} = \dfrac{450{,}000}{18{,}000 \times 0{\cdot}857 \times 16{\cdot}5} = 1{\cdot}77$ sq. in., that is, three $\frac{7}{8}$-in. bars.

(*b*) Determine the reinforcement required in rectangular beams of the sizes given if the bending moment is 750,000 in.-lb.

(i) Overall depth 18 in. (d_1 = 16 in.); width 10 in.—This beam can resist 49,400 × 10 = 494,000 in.-lb. with $A_{sc} = 0$, and 130,000 × 10 = 1,300,000 in.-lb. if $A_{sc} = A_{st}$. Compression reinforcement is required. The moment of resistance required from the compression reinforcement is 750,000 − 494,000 = 256,000 in.-lb. With $r_1 = 18$, $n_1 = 0{\cdot}455$ (*Table* 68). From the formulæ in *Table* 66 (assuming d_2 = 2 in.) $l_{as} = 16 - 2 = 14$ in.; $d_n = 0{\cdot}455 \times 16 = 7{\cdot}3$ in. $f_{sc} = \dfrac{7{\cdot}3 - 2}{7{\cdot}3} \times 14 \times 1000 = 10{,}170$ lb. per sq. in.; $A_{sc} = \dfrac{256{,}000}{10{,}170 \times 14} = 1{\cdot}80$ sq. in., that is, three $\frac{7}{8}$-in. bars in the top. $A_{st} = (0{\cdot}202 \times 10) + \dfrac{256{,}000}{18{,}000 \times 14} = 3{\cdot}04$ sq. in., that is, three $1\frac{1}{8}$-in. bars in the bottom. With $1\frac{1}{4}$-in. cover, the actual effective depth is $18 - 1\frac{1}{4} - \frac{9}{16} = 16{\cdot}2$ in., $\not< 16$ in.

(ii) Overall depth 15 in. (d_1 = 13 in.); width 9 in.—The bending moment per inch of width is 750,000 ÷ 9 = 83,300 in.-lb. which, if the concrete is taken into account, requires $A_{sc} = A_{st} = 0{\cdot}416 \times 9 = 3{\cdot}74$ sq. in., that is three $1\frac{1}{4}$-in. bars in the top and bottom. Actual effective depth = $15 - 1\frac{1}{4} - \frac{5}{8} = 13{\cdot}13$ in., $\not< 13$ in. $l_{as} = 13{\cdot}13 - 1\frac{1}{4} - \frac{5}{8} = 11{\cdot}25$ in. $= 0{\cdot}86d_1$ compared with the assumed $0{\cdot}9d_1$; there is sufficient margin between the assumed and actual effective depths, and between the moment of resistance and the bending moment to compensate for the difference in the assumed and actual lever-arms.

(iii) Overall depth 15 in.; width 8 in.—Bending moment per inch of width is 93,800 in.-lb., > 85,800 in.-lb. ($A_{sc} = A_{st}$ for d_1 = 13 in. and concrete taken into account); apply "steel-beam" theory. Assume two layers of 1-in. bars and 1-in. cover: $l_{as} = 10{\cdot}5$ in. $A_{st} = A_{sc} = \dfrac{750{,}000}{18{,}000 \times 10{\cdot}5} = 3{\cdot}97$ sq. in., that is five 1-in. bars top and bottom.

(*c*) Determine the reinforcement in a beam 22 in. deep (d_1 = 19 in.) and 12 in. wide, subjected to a bending moment of 2,750,000 in.-lb. if the stresses are not to exceed 18,000 lb. and 1100 lb. per sq. in.

The bending moment per inch of width is 2,750,000 ÷ 12 = 230,000 in.-lb., which requires the maximum (4 per cent.) of compression reinforcement. $A_{sc} = 0{\cdot}04 \times 19 \times 12 = 9{\cdot}13$ sq. in.; $A_{st} = 0{\cdot}792 \times 12 = 9{\cdot}5$ sq. in., that is eight $1\frac{1}{4}$-in. bars in the top and bottom. Since the bars must be in two layers, the actual effective depth with $1\frac{1}{4}$-in. cover is 19·25 in., which is not less than 19 in. Also l_{as} = 16·5 in. = $0{\cdot}87d_1$ compared with the assumed ratio of $0{\cdot}9d_1$; since the area of compression reinforcement provided is 9·82 sq. in. compared with a calculated amount not exceeding 9·13 sq. in., there is a margin to allow for the lower stress.

(*d*) Design a rectangular beam without compression reinforcement to resist a bending moment of 550,000 in.-lb with stresses not exceeding 16,000 lb. and 800 lb. per sq. in. $r_1 = 20$; therefore use *Table* 70B for p_{st} = 20,000 lb. and p_{sc} = 1000 lb. per sq. in., and design for a bending moment of $550{,}000 \times \dfrac{20{,}000}{16{,}000} = 687{,}500$ in.-lb. Assuming a width of 12 in., the moment of resistance required is 57,300 in.-lb. per inch of width, which is given by an effective depth of 18 in. $A_{st} = 0{\cdot}192 \times 12 = 2{\cdot}3$ sq. in., that is four $\frac{7}{8}$-in. bars. With 1-in. cover, d should be 20 in. The remarks in example (*a*) regarding other sections apply.

RECTANGULAR BEAMS: RESISTANCE AND REINFORCEMENT.—TABLE 70A.
p_{st} = 18,000 LB. PER SQ. IN.—MODULAR-RATIO METHOD.

MAXIMUM STRESS IN TENSILE REINFORCEMENT = 18,000 LB. PER SQ. IN. m = 15

EFFECTIVE DEPTH d_1 IN.	p_{cb} = 750 LB PER SQ.IN.				p_{cb} = 1000 LB. PER SQ.IN.				p_{cb} = 1100 LB PER SQ.IN.			
	A_{sc} = 0		A_{sc} = A_{st}		A_{sc} = 0		A_{sc} = A_{st}		A_{sc} = 0		A_{sc} = 4%	
	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}
10	12,600	0·080	22,500	0·142	19,300	0·126	50,700	0·320	22,100	0·147	65,800	0·417
11	15,300	0·088	27,200	0·156	23,400	0·139	61,400	0·352	26,800	0·162	79,700	0·459
12	18,200	0·096	32,400	0·170	27,800	0·152	73,100	0·384	31,900	0·177	94,900	0·500
13	21,300	0·104	37,400	0·185	32,600	0·164	85,800	0·416	37,400	0·191	111,000	0·542
14	24,700	0·112	44,500	0·199	37,800	0·176	99,500	0·448	43,300	0·206	129,000	0·584
15	28,500	0·120	51,200	0·213	43,400	0·189	114,000	0·480	49,800	0·221	148,000	0·626
16	32,300	0·128	58,100	0·227	49,400	0·202	130,000	0·512	56,700	0·235	168,000	0·667
17	36,400	0·136	65,600	0·242	55,700	0·214	147,000	0·544	64,000	0·250	190,000	0·709
18	40,800	0·144	73,600	0·253	62,500	0·227	165,000	0·576	71,700	0·264	213,000	0·751
19	45,500	0·152	81,200	0·270	69,600	0·239	183,000	0·608	80,000	0·279	238,000	0·792
20	50,400	0·160	90,000	0·284	77,200	0·252	203,000	0·690	88,400	0·294	263,000	0·834
21	55,600	0·168	99,000	0·298	85,000	0·265	224,000	0·672	97,500	0·309	290,000	0·876
22	61,000	0·176	109,000	0·312	93,500	0·277	246,000	0·704	107,000	0·323	318,000	0·917
23	65,400	0·184	119,000	0·336	102,000	0·290	268,000	0·736	117,000	0·338	348,000	0·959
24	72,500	0·192	130,000	0·341	111,000	0·302	293,000	0·768	128,000	0·352	379,000	1·00
25	78,800	0·200	141,000	0·355	121,000	0·315	322,000	0·800	138,000	0·368	411,000	1·04
26	85,200	0·208	152,000	0·369	131,000	0·328	343,000	0·832	150,000	0·382	445,000	1·08
27	91,800	0·216	164,000	0·383	141,000	0·340	370,000	0·864	161,000	0·397	479,000	1·13
28	98,800	0·224	176,000	0·397	151,000	0·353	398,000	0·896	173,000	0·412	515,000	1·17
29	106,000	0·232	189,000	0·412	162,000	0·365	427,000	0·928	186,000	0·426	553,000	1·21
30	113,000	0·240	203,000	0·426	174,000	0·378	456,000	0·960	199,000	0·441	592,000	1·25
31	121,000	0·248	216,000	0·440	186,000	0·391	487,000	0·992	213,000	0·456	630,000	1·29
32	129,000	0·256	231,000	0·453	198,000	0·403	520,000	1·02	227,000	0·470	675,000	1·33
33	137,000	0·264	245,000	0·468	210,000	0·416	552,000	1·06	241,000	0·485	716,000	1·38
34	146,000	0·272	260,000	0·483	223,000	0·428	585,000	1·09	256,000	0·500	760,000	1·42
35	155,000	0·280	276,000	0·496	236,000	0·441	620,000	1·12	271,000	0·515	806,000	1·46
36	163,000	0·288	292,000	0·511	250,000	0·454	657,000	1·15	286,000	0·529	853,000	1·50
FACTORS	$126d_1^2$	0·8%	$225d_1^2$	1·42%	$193d_1^2$	1·26%	$507d_1^2$	3·2%	$221d_1^2$	1·47%	$658d_1^2$	4·17%*
STRESS RATIO $\frac{p_{st}}{p_{cb}}$	24				18				16·4		*A_{sc} = 4% MAX.	
	M_r FOR OTHER COMBINATIONS OF STRESSES HAVING THESE RATIOS ARE PROPORTIONAL TO TENSILE STRESS.											

M_r = MOMENT OF RESISTANCE IN INCH-POUNDS PER ONE INCH WIDTH OF BEAM.
A_{st} = AREA OF TENSILE REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM.
A_{sc} = AREA OF COMPRESSIVE REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM.
DISTANCE BETWEEN CENTRES OF TENSILE AND COMPRESSIVE REINFORCEMENT ASSUMED 0·9 d_1; IF ANY OTHER DISTANCE, M_r AND A_{st} FOR A_{st}=A_{sc} MUST BE ADJUSTED.

TABLE 70B.—RECTANGULAR BEAMS: RESISTANCE AND REINFORCEMENT. MODULAR-RATIO METHOD.—p_{st} = 20,000 LB. PER SQ. IN.

MAXIMUM STRESS IN TENSILE REINFORCEMENT = 20,000 LB. PER SQ. IN. $m = 15$

EFFECTIVE DEPTH d_1 IN.	p_{cb} = 750 LB. PER SQ. IN.				p_{cb} = 1000 LB. PER SQ. IN.				p_{cb} = 1100 LB. PER SQ. IN.			
	$A_{sc} = 0$		$A_{sc} = A_{st}$		$A_{sc} = 0$		$A_{sc} = A_{st}$		$A_{sc} = 0$		$A_{sc} = A_{st}$	
	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}
10	12,000	0·067	19,300	0·108	18,400	0·107	40,600	0·231	21,200	0·125	54,900	0·312
11	14,500	0·074	23,400	0·119	22,300	0·118	49,100	0·254	25,600	0·138	66,500	0·343
12	17,200	0·081	27,800	0·130	26,700	0·129	58,600	0·277	30,500	0·150	79,100	0·374
13	20,200	0·087	32,600	0·140	31,100	0·139	68,800	0·300	35,900	0·163	92,900	0·406
14	23,400	0·094	37,800	0·151	36,100	0·150	79,700	0·323	41,600	0·175	108,000	0·437
15	26,900	0·100	43,400	0·162	41,400	0·161	91,500	0·347	47,700	0·188	127,000	0·468
16	30,600	0·107	49,400	0·173	48,900	0·172	104,000	0·370	54,200	0·200	140,000	0·499
17	34,500	0·114	55,700	0·184	53,200	0·182	117,000	0·393	61,200	0·213	159,000	0·530
18	38,800	0·121	62,500	0·194	59,700	0·192	132,000	0·416	68,600	0·225	178,000	0·562
19	43,200	0·127	69,600	0·205	66,500	0·203	147,000	0·439	74,300	0·238	198,000	0·593
20	47,800	0·134	77,200	0·216	73,600	0·214	162,000	0·462	84,800	0·250	220,000	0·624
21	43,100	0·141	85,000	0·227	81,000	0·225	179,000	0·485	93,500	0·263	242,000	0·655
22	52,700	0·148	93,500	0·238	89,000	0·236	197,000	0·508	102,000	0·275	266,000	0·686
23	63,200	0·154	102,000	0·248	97,500	0·246	215,000	0·532	112,000	0·288	290,000	0·718
24	68,800	0·161	111,000	0·259	106,000	0·257	234,000	0·554	122,000	0·300	318,000	0·749
25	74,700	0·168	121,000	0·270	115,000	0·268	254,000	0·578	133,000	0·313	343,000	0·780
26	80,800	0·174	131,000	0·281	125,000	0·279	275,000	0·601	144,000	0·325	371,000	0·811
27	87,000	0·181	141,000	0·292	134,000	0·290	296,000	0·624	154,000	0·338	400,000	0·842
28	93,600	0·188	151,000	0·303	144,000	0·300	318,000	0·647	166,000	0·350	430,000	0·874
29	101,000	0·194	162,000	0·314	155,000	0·311	342,000	0·670	178,000	0·363	462,000	0·905
30	108,000	0·201	174,000	0·324	166,000	0·321	365,000	0·693	191,000	0·375	494,000	0·936
31	115,000	0·208	186,000	0·335	177,000	0·332	390,000	0·716	204,000	0·388	528,000	0·967
32	122,000	0·214	198,000	0·346	188,000	0·343	416,000	0·739	217,000	0·400	562,000	0·998
33	130,000	0·221	210,000	0·356	200,000	0·353	443,000	0·762	231,000	0·413	598,000	1·03
34	138,000	0·228	223,000	0·367	213,000	0·364	470,000	0·785	245,000	0·425	635,000	1·06
35	146,000	0·234	236,000	0·378	225,000	0·375	498,000	0·809	259,000	0·438	673,000	1·09
36	155,000	0·241	250,000	0·389	239,000	0·387	526,000	0·832	264,000	0·450	711,000	1·12
FACTORS	$119{\cdot}5d_1^2$	0·67%	$193d_1^2$	1·08%	$184d_1^2$	1·07%	$406d_1^2$	2·31%	$212d_1^2$	1·25%	$549d_1^2$	3·12%
STRESS RATIO $\frac{p_{st}}{p_{cb}}$	$26\frac{2}{3}$				20				18·2			
	M_r FOR OTHER COMBINATIONS OF STRESSES HAVING THESE RATIOS ARE PROPORTIONAL TO TENSILE STRESS.											

M_r = MOMENT OF RESISTANCE IN INCH-POUNDS PER ONE INCH WIDTH OF BEAM.
A_{st} = AREA OF TENSILE REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM.
A_{sc} = AREA OF COMPRESSIVE REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM.
DISTANCE BETWEEN CENTRES OF TENSILE AND COMPRESSIVE REINFORCEMENT ASSUMED $0{\cdot}9d_1$; IF ANY OTHER DISTANCE, M_r AND A_{st} FOR $A_{st} = A_{sc}$ MUST BE ADJUSTED.

p_{st} = 27,000 LB. PER SQ. IN.—MODULAR-RATIO METHOD.

MAXIMUM STRESS IN TENSILE REINFORCEMENT = 27,000 LB. PER SQ. IN. m = 15

EFFECTIVE DEPTH d_1 IN.	p_{cb} = 750 LB. PER SQ. IN.				p_{cb} = 1000 LB. PER SQ. IN.				p_{cb} = 1100 LB. PER SQ. IN.			
	A_{sc} = 0		A_{sc} = A_{st}		A_{sc} = 0		A_{sc} = A_{st}		A_{sc} = 0		A_{sc} = A_{st}	
	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}
10	9,950	0·041	13,900	0·055	15,700	0·066	25,300	0·106	18,200	0·077	31,800	0·134
11	12,000	0·045	16,800	0·061	19,000	0·073	30,600	0·117	22,000	0·085	38,500	0·147
12	14,300	0·049	20,100	0·066	22,600	0·079	36,400	0·127	26,200	0·092	45,800	0·161
13	16,800	0·053	23,700	0·072	26,500	0·086	42,800	0·138	30,700	0·100	53,800	0·174
14	19,500	0·057	27,300	0·077	30,800	0·092	49,500	0·148	35,700	0·108	62,300	0·188
15	22,400	0·062	31,300	0·083	35,400	0·099	57,000	0·159	40,900	0·116	71,600	0·201
16	25,500	0·066	35,600	0·088	40,200	0·106	64,700	0·170	46,600	0·123	81,400	0·214
17	28,700	0·070	40,100	0·094	45,300	0·112	73,000	0·180	52,500	0·131	92,000	0·228
18	32,300	0·074	45,100	0·099	50,800	0·119	82,000	0·191	59,000	0·139	103,000	0·241
19	35,900	0·078	50,200	0·105	56,700	0·125	91,500	0·201	65,800	0·146	114,000	0·255
20	39,800	0·082	55,600	0·110	62,800	0·132	101,000	0·212	72,800	0·154	127,000	0·268
21	43,800	0·086	61,300	0·116	69,200	0·139	112,000	0·223	80,300	0·162	140,000	0·281
22	48,100	0·090	67,300	0·121	76,000	0·145	123,000	0·233	88,000	0·169	154,000	0·295
23	52,600	0·094	73,500	0·127	83,000	0·152	134,000	0·244	96,200	0·177	168,000	0·308
24	57,400	0·098	80,000	0·132	90,500	0·158	146,000	0·254	105,000	0·185	183,000	0·322
25	62,200	0·103	86,900	0·138	98,100	0·165	158,000	0·265	114,000	0·193	198,000	0·335
26	67,300	0·107	94,000	0·143	106,000	0·172	171,000	0·276	123,000	0·200	215,000	0·348
27	72,500	0·111	101,000	0·149	114,000	0·178	184,000	0·286	133,000	0·208	232,000	0·362
28	78,000	0·115	109,000	0·154	123,000	0·185	198,000	0·297	143,000	0·216	249,000	0·375
29	83,700	0·119	117,000	0·159	132,000	0·191	213,000	0·307	153,000	0·223	267,000	0·389
30	89,600	0·123	125,000	0·165	141,000	0·198	228,000	0·318	164,000	0·231	286,000	0·402
31	95,800	0·127	133,000	0·171	151,000	0·205	243,000	0·329	175,000	0·239	305,000	0·415
32	102,000	0·131	142,000	0·176	161,000	0·211	259,000	0·339	186,000	0·246	326,000	0·429
33	108,000	0·135	151,000	0·182	171,000	0·218	275,000	0·350	198,000	0·254	346,000	0·442
34	115,000	0·139	161,000	0·187	182,000	0·224	292,000	0·360	210,000	0·262	367,000	0·456
35	122,000	0·144	170,000	0·193	192,000	0·231	310,000	0·371	223,000	0·270	390,000	0·469
36	129,000	0·148	180,000	0·198	203,000	0·238	328,000	0·381	235,000	0·277	412,000	0·482
FACTORS	$99\cdot5d_1^2$	0·41%	$139d_1^2$	0·55%	$157d_1^2$	0·66%	$253d_1^2$	1·06%	$182d_1^2$	0·77%	$318d_1^2$	1·34%
STRESS RATIO $\frac{p_{st}}{p_{cb}}$	36				27				24·5			
	M_r FOR OTHER COMBINATIONS OF STRESSES HAVING THESE RATIOS ARE PROPORTIONAL TO TENSILE STRESS.											

M_r = MOMENT OF RESISTANCE IN INCH-POUNDS PER ONE INCH WIDTH OF BEAM.
A_{st} = AREA OF TENSILE REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM.
A_{sc} = AREA OF COMPRESSIVE REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM.
DISTANCE BETWEEN CENTRES OF TENSILE AND COMPRESSIVE REINFORCEMENT ASSUMED $0\cdot9d_1$; IF ANY OTHER DISTANCE, M_r AND A_{st} FOR A_{st} = A_{sc} MUST BE ADJUSTED.

DESIGN OF BEAMS: MODULAR-RATIO METHOD.

Examples of Use of Table 70C or 70D (High-yield-stress Steel).

The following examples apply to *Table* 70D (p_{st} = 30,000 lb. per sq. in.) for the modular-ratio method with $m = 15$. The calculations would be similar, but with different numerical values, if *Table* 70C (p_{st} = 27,000 lb. per sq. in.) were used.

(*a*) A rectangular beam of 1 : 2 : 4 concrete (p_{cb} = 1000 lb.) 24 in. deep (d_1 = 22 in.) and 12 in. wide is to be reinforced with bars having a yield stress not less than 66,000 lb. per sq. in. (p_{st} = 30,000 lb. and p_{sc} = 23,000 lb. per sq. in. from *Table* 58). Determine the reinforcement required to resist the bending moments in the following examples.

(i) Bending moment 840,000 in.-lb. (70,000 in.-lb. per inch of width). With $A_{sc} = 0$, M_r = 71,600 in.-lb. per inch of width. Therefore $A_{st} = 0{\cdot}126 \times 12 = 1{\cdot}51$ sq. in., say, four ⅞-in. twisted ribbed bars (*Tables* 59 and 60) in the bottom.

(ii) Bending moment 1,200,000 in.-lb (100,000 in.-lb. per inch of width). With $A_{st} = A_{sc}$, M_r = 110,000 in.-lb. per inch. $A_{st} = A_{sc} = 0{\cdot}181 \times 12 = 2{\cdot}17$ sq. in., say, three 1-in. twisted ribbed bars (*Tables* 59 and 60) in top and bottom. In this design the compressive resistance of the concrete is taken into account.

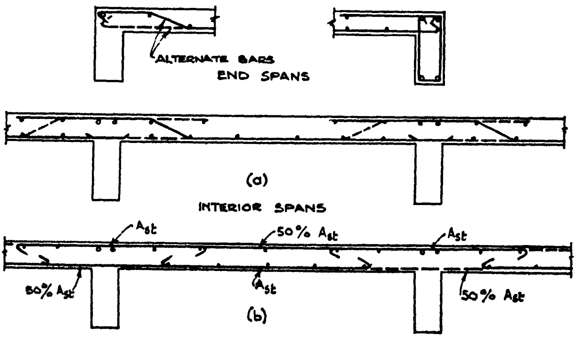
(iii) Bending moment 1,800,000 in.-lb. (150,000 in.-lb. per inch of width). Since this exceeds 110,000 in.-lb. the "steel-beam" theory is applied and the resistance of the concrete in compression is neglected. The lever-arm l_{as} is the distance between the centres of the tensile and compression reinforcement, say, $22 - 2 = 20$ in.

$$A_{st} = \frac{1,800,000}{30,000 \times 20} = 3{\cdot}0 \text{ sq. in.}; \quad A_{sc} = \frac{1,800,000}{23,000 \times 20} = 3{\cdot}9 \text{ sq. in.}$$

Using 1-in. twisted ribbed bars, four bars are required in the bottom and five in the top

REINFORCEMENT IN SOLID SLABS.

Main Bars.—Various arrangements of the main reinforcement in slabs are illustrated in the diagram. Ordinary floor slabs are generally as the design at (*a*). When the live load is so large compared with the dead load that reinforcement against negative bending moments is required in the middle of the span, the design at (*b*) is suitable. Hooks or similar end-anchorages are not generally necessary on bars in slabs, except at the outer supports of end-spans. If the design at (*b*) is adopted hooks might be provided as the bars may terminate in a tensile part of the slab.



The spacing of the main bars in a slab should not exceed three times the effective depth of the slab. For slabs spanning in two directions, the spacing of the bars in either direction should not exceed three times the effective depth, and the bars across the shorter span should, at the midspan, be placed under, and at the supports over, the reinforcement at right-angles to this span.

Minimum Amount of Reinforcement.—Whatever amount of tensile reinforcement is required by calculation to resist the bending moment, the area of steel should be not less than 0·15 per cent. of the gross area of the concrete.

(*Continued on page* 268.)

RECTANGULAR BEAMS: RESISTANCE AND REINFORCEMENT.—TABLE 70D.

p_{st} = 30,000 LB. PER SQ. IN. MODULAR-RATIO METHOD.

MAXIMUM STRESS IN TENSILE REINFORCEMENT = 30,000 LB. PER SQ. IN. m = 15

EFFECTIVE DEPTH d_1 IN.	p_{cb} = 750 LB. PER SQ. IN.				p_{cb} = 1000 LB. PER SQ. IN.				p_{cb} = 1250 LB. PER SQ. IN.			
	A_{sc} = 0		A_{sc} = A_{st}		A_{sc} = 0		A_{sc} = A_{st}		A_{sc} = 0		A_{sc} = A_{st}	
	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}	M_r	A_{st}
10	9,300	0·034	11,900	0·044	14,800	0·056	22,700	0·082	21,000	0·080	37,400	0·141
11	11,250	37	14,400	48	17,900	62	27,400	91	35,400	88	45,300	155
12	13,400	41	17,100	53	21,400	67	32,600	99	30,300	96	53,900	169
13	15,700	44	20,200	57	25,000	73	38,300	107	35,500	104	63,100	183
14	18,200	48	23,400	·61	29,000	78	44,400	115	41,200	112	73,100	197
15	20,900	0·051	26,800	0·066	33,300	0·084	51,000	0·123	47,300	0·120	84,000	0·212
16	23,800	54	30,500	70	37,900	90	58,000	132	53,800	128	96,000	225
17	26,900	58	34,400	75	42,800	95	65,500	140	60,600	136	108,000	240
18	30,200	61	38,600	79	48,000	101	73,500	148	68,000	144	121,000	254
19	33,600	65	43,000	83	53,400	106	82,000	156	75,800	152	135,000	268
20	37,200	0·068	47,600	0·088	59,200	0·112	90,800	0·164	84,000	0·160	149,600	0·282
21	41,000	71	52,500	92	65,300	118	100,000	173	92,500	168	165,000	295
22	45,000	75	57,600	96	71,600	126	110,000	181	102,000	176	181,000	310
23	49,200	78	63,000	101	78,400	129	120,000	189	111,000	184	198,000	324
24	53,500	82	68,600	105	85,300	134	131,000	197	121,000	192	216,000	338
25	58,200	0·085	74,400	0·110	92,500	0·140	142,000	0·205	131,000	0·200	234,000	0·353
26	62,900	89	80,600	114	100,000	145	154,000	214	142,000	208	253,000	366
27	67,800	92	86,600	118	108,000	151	165,000	222	153,000	216	272,000	381
28	73,000	95	93,100	123	116,000	157	178,000	231	166,000	224	293,000	395
29	78,300	99	100,100	127	125,000	162	191,000	239	177,000	232	315,000	408
30	83,700	0·102	107,100	0·132	133,200	0·168	204,300	0·246	189,000	0·240	336,600	0·423
31	89,500	105	115,000	136	142,000	173	218,000	255	202,000	248	360,000	437
32	95,300	109	122,000	140	152,000	179	232,000	263	215,000	256	382,000	451
33	101,000	112	130,000	145	161,000	185	246,000	272	228,000	264	407,000	465
34	107,500	115	138,000	149	171,000	190	262,000	280	242,000	272	435000	479
35	114,000	0·119	146,000	0·154	181,500	0·196	278,000	0·288	258,000	0·280	458,000	0·493
36	120,500	0·122	154,000	0·158	191,000	0·202	294,000	0·296	272,000	0·288	485,000	0·507
FACTORS	$93\,bd_1^2$	0·34%	$119\,bd_1^2$	0·44%	$148\,bd_1^2$	0·56%	$227\,bd_1^2$	0·82%	$210\,bd_1^2$	0·80%	$374\,bd_1^2$	1·41%
STRESS RATIO $\frac{p_{st}}{p_{cb}}$	40				30				24			
	M_r FOR OTHER COMBINATIONS OF STRESSES HAVING THESE RATIOS ARE PROPORTIONAL TO TENSILE STRESS.											

M_r = MOMENT OF RESISTANCE IN INCH-POUNDS PER ONE INCH WIDTH OF BEAM.

A_{st} = AREA OF TENSILE REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM.

A_{sc} = AREA OF COMPRESSION REINFORCEMENT IN SQUARE INCHES PER INCH WIDTH OF BEAM. DISTANCE BETWEEN CENTRES OF TENSILE AND COMPRESSION REINFORCEMENT ASSUMED $0{\cdot}9d_1$; IF ANY OTHER DISTANCE, M_r AND A_{st} FOR $A_{st} = A_{sc}$ MUST BE ADJUSTED.

REINFORCEMENT IN SOLID SLABS (*continued from page* 266).

Distribution (Longitudinal) Bars.—In slabs spanning in one direction bars should be provided at right-angles to the main reinforcement. The B.S. Code recommends that the area of such distribution bars should not be less than 0·15 per cent. of the gross area of the concrete; the spacing of the distribution bars should be not greater than five times the effective depth, although this spacing may be excessive in slabs exceeding 6 in. in thickness. The London By-laws require 10 per cent. of the area of the main reinforcement; in bridge decks the Ministry of Transport requires 40 per cent. for spans of 4 ft., 50 per cent. for 6 ft., 55 per cent. for 8 ft., and 60 per cent. for 10 ft., but in no case need the area exceed 0·5 sq. in. per foot of width. B.S. No. 153 (Part 3A) requires 50 per cent. in reinforced concrete deck slabs for steel bridges. Fabricated meshes do not always have sufficient transverse bars to provide the foregoing amounts and the deficiency has to be made up by an additional sheet or by wiring bars to the main fabric reinforcement.

Where slabs span in one direction there is generally a negative bending moment over main beams parallel to the span. Reinforcement in the top of the slab at right-angles to the main reinforcement is therefore required. The area of the reinforcement provided for this purpose should be 0·3 per cent. of the gross cross-sectional area of the slab. The spacing of these bars should not exceed three times the effective depth, and the bars should extend on each side of the beam an average distance of six times the thickness of the slab. The ends of the bars should be staggered.

DESIGN OF SLABS: MODULAR-RATIO METHOD.

Table 71.—The data in *Table* 71 are based on the principle that the effective depth of, and reinforcement in, a solid slab required to resist a given bending moment M are direct functions of $\sqrt{M}$. The effective depth can be expressed by $d_1 = k_1\sqrt{M'}$, and the area of reinforcement per foot of width by $A_{st} = k_2\sqrt{M'}$. Values of k_1 and k_2 are given in *Table* 71 for various stresses, and for these values M' must be in foot-pounds. The method is to find from the table the value of k_1 for the permissible stresses and to determine the effective depth from the formula. If this depth is adopted, the value of k_2 applicable to the permissible stresses should be used to determine the area of reinforcement per foot of width of the slab. If a depth greater than that determined is adopted find the value of

$$k_1 = \frac{\text{effective depth adopted}}{\sqrt{M'}},$$

and find from the table the corresponding value of k_2 for the permissible stress in the reinforcement. Multiply this value of k_2 by $\sqrt{M'}$ to give A_{st}. If a depth less than that first calculated is adopted, find the value of $k_1 = \dfrac{\text{effective depth adopted}}{\sqrt{M'}}$, and find from the table the corresponding value of k_2 for the permissible stress in the concrete, which should then be multiplied by $\sqrt{M'}$ to give the area of reinforcement required.

Example.—Design a slab to resist a bending moment of 2000 ft.-lb. per ft. width with stresses not exceeding 16,000 per sq. in. in the steel and 700 lb. per sq in. in the concrete.

(i) At maximum concrete and steel stresses:

Effective depth required $= 0·091\sqrt{2000} = 4·07$ in. Total thickness of slab $= 5$ in.

Area of reinforcement required $= 0·0095\sqrt{2000} = 0·420$ sq. in. per ft.

(ii) Specified thickness of slab, 6 in.—$d_1 = 5·25$ in.; $k_1 = \dfrac{5·25}{\sqrt{2000}} = 0·118$.

With $p_{st} = 16{,}000$ lb. per sq. in., the corresponding value of k_2 is 0·0071, and the area of reinforcement required is $0·0071\sqrt{2000} = 0·312$ sq. in. per ft.

(iii) Specified thickness of slab, $4\frac{1}{2}$ in.—$d_1 = 3·75$ in.; $k_1 = \dfrac{3·75}{\sqrt{2000}} = 0·084$.

With $p_{cb} = 700$ lb. per sq. in., the corresponding value of k_2 is 0·014 approximately, and the area of reinforcement required is $0·014\sqrt{2000} = 0·625$ sq. in. per ft.

p_{st}	12,000		14,000		16,000		18,000		20,000		27,000		30,000	
p_{cb}	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2
100	0·43	0·002	0·46	0·002	0·49	0·002	0·50	0·001	0·54	0·001	0·62	0·001	0·65	0·001
200	0·23	0·005	0·25	0·004	0·26	0·003	0·27	0·003	0·28	0·002	0·32	0·001	0·34	0·001
300	0·16	0·007	0·17	0·005	0·18	0·005	0·19	0·004	0·20	0·003	0·22	0·002	0·23	0·002
400	0·13	0·009	0·14	0·007	0·14	0·006	0·15	0·005	0·15	0·004	0·17	0·003	0·18	0·002
500	0·118	0·0105	0·114	0·0085	0·118	0·0071	0·123	0·0060	0·127	0·0052	0·141	0·0035	0·146	0·0029
525	0·109	0·0110	0·110	0·0089	0·114	0·0074	0·118	0·0063	0·122	0·0054	0·135	0·0036	0·140	0·0031
550	0·105	0·0114	0·106	0·0093	0·110	0·0077	0·114	0·0066	0·118	0·0057	0·130	0·0037	0·134	0·0032
575	0·098	0·0118	0·102	0·0096	0·106	0·0080	0·110	0·0068	0·113	0·0060	0·125	0·0039	0·129	0·0033
600	0·095	0·0123	0·099	0·0100	0·103	0·0083	0·106	0·0071	0·110	0·0061	0·120	0·0040	0·125	0·0035
625	0·092	0·0127	0·096	0·0103	0·099	0·0086	0·103	0·0073	0·106	0·0063	0·117	0·0042	0·121	0·0036
650	0·090	0·0131	0·093	0·0107	0·097	0·0089	0·100	0·0076	0·103	0·0066	0·113	0·0043	0·117	0·0037
675	0·087	0·0135	0·091	0·0110	0·094	0·0092	0·097	0·0078	0·100	0·0068	0·109	0·0045	0·114	0·0039
700	0·085	0·0139	0·088	0·0113	0·091	0·0095	0·094	0·0081	0·097	0·0070	0·106	0·0046	0·110	0·0040
725	0·083	0·0143	0·086	0·0117	0·089	0·0098	0·092	0·0083	0·094	0·0072	0·103	0·0048	0·107	0·0041
750	0·081	0·0147	0·084	0·0120	0·087	0·0101	0·089	0·0086	0·092	0·0074	0·100	0·0049	0·104	0·0043
775	0·079	0·0151	0·082	0·0123	0·085	0·0103	0·087	0·0088	0·089	0·0077	0·098	0·0051	0·101	0·0044
800	0·078	0·0155	0·080	0·0127	0·083	0·0106	0·085	0·0090	0·087	0·0079	0·096	0·0052	0·097	0·0046
825	0·076	0·0159	0·078	0·0130	0·081	0·0109	0·083	0·0093	0·085	0·0081	0·094	0·0053	0·096	0·0046
850	0·074	0·0163	0·077	0·0133	0·079	0·0112	0·081	0·0095	0·083	0·0083	0·091	0·0055	0·094	0·0048
875	0·073	0·0166	0·075	0·0136	0·077	0·0114	0·079	0·0098	0·082	0·0085	0·089	0·0057	0·092	0·0049
900	0·071	0·0170	0·074	0·0139	0·076	0·0117	0·078	0·0100	0·080	0·0087	0·087	0·0058	0·089	0·0050
925	0·070	0·0174	0·072	0·0142	0·074	0·0120	0·076	0·0102	0·078	0·0089	0·085	0·0059	0·087	0·0051
950	0·069	0·0177	0·071	0·0146	0·073	0·0122	0·075	0·0105	0·077	0·0091	0·083	0·0060	0·086	0·0052
975	0·068	0·0181	0·070	0·0149	0·072	0·0125	0·073	0·0107	0·075	0·0093	0·082	0·0062	0·084	0·0054
1000	0·067	0·0185	0·068	0·0152	0·070	0·0127	0·072	0·0109	0·074	0·0095	0·080	0·0063	0·082	0·0055
1025	0·065	0·0188	0·067	0·0155	0·069	0·0130	0·070	0·0112	0·072	0·0097	0·078	0·0064	0·081	0·0056
1050	0·064	0·0192	0·066	0·0158	0·068	0·0133	0·069	0·0114	0·071	0·0099	0·077	0·0066	0·079	0·0057
1075	0·063	0·0195	0·065	0·0160	0·067	0·0135	0·068	0·0116	0·070	0·0101	0·075	0·0067	0·078	0·0058
1100	0·062	0·0198	0·064	0·0164	0·066	0·0138	0·067	0·0118	0·069	0·0103	0·074	0·0068	0·076	0·0060

p_{st} = TENSILE STRESS (LB. PER SQ. IN.) IN REINFORCEMENT. $k_1 = \sqrt{\dfrac{1}{Q_c}}$.

p_{cb} = COMPRESSIVE STRESS (LB. PER SQ. IN.) IN CONCRETE. $Q_c = \dfrac{n_1 a_1 p_{cb}}{2}$.

M' = BENDING MOMENT (FOOT-LB.) PER FOOT WIDTH OF SLAB. $k_2 = \dfrac{12}{p_{st} a_1 k_1}$.

EFFECTIVE DEPTH (INCH): $d_1 = k_1 \sqrt{M'}$.

TENSILE REINFORCEMENT (SQ. IN. PER FOOT WIDTH): $A_{st} = k_2 \sqrt{M'}$.

APPLICATION TO RECTANGULAR BEAMS.

$$d_1 = k_1 \sqrt{\frac{M''}{b}} \qquad A_{st} = \frac{M''}{a_1 d_1 p_{st}}$$

M'' = BENDING MOMENT (INCH-LB.).
b = BREADTH (INCH).
a_1 = LEVER-ARM FACTOR.*

$m = 15$

* For values of lever-arm factor a_1, see *Table* 68.

DESIGN OF SLABS: MODULAR-RATIO METHOD.

Table 72.—The data in *Table* 72 relating to moments of resistance and areas of reinforcement for slabs of various thicknesses from 3 in. to 9 in. are in accordance with the recommendations of B.S. Code No. 114 for the modular-ratio method ($m = 15$), using mild steel or high-yield-stress bars and various grades of 1 : 2 : 4 concrete.

Example.—Select a slab to resist a bending moment of 2000 ft.-lb. (= 24,000 in.-lb.) per foot width. Maximum stresses: $p_{st} = 20{,}000$ lb. per sq. in., and $p_{cb} = 1000$ lb. per sq. in. The moment of resistance of a 4-in. slab with these stresses is 23,300 in.-lb. which may be near enough. The area of reinforcement required for this moment is 0·419 sq. in. per ft. width, which is given by ½-in. bars at 5½-in. centres (see *Table* 60) in a 4-in. slab.

If it is imperative that the moment of resistance must not be less than 24,000 in.-lb., a slab 4½ in. thick should be provided ($M_r = 26{,}000$ in.-lb. per ft.). The effective depth with ½-in. bars and ½-in. cover is 3·75 in.; lever-arm (for $r_1 = 20$ from *Table* 68) is $0{\cdot}857 \times 3{\cdot}75 = 3{\cdot}2$ in.

$$A_{st} = \frac{24{,}000}{20{,}000 \times 3{\cdot}2} = 0{\cdot}375 \text{ sq. in. per ft.; } \tfrac{1}{2}\text{-in. bars at 6-in. centres are satisfactory.}$$

SLABS FOR LIQUID-CONTAINING STRUCTURES (B.S. CODE NO. 2007).

Notation.

d, d_1, b, l_a: Thickness, effective depth, breadth and lever-arm respectively of slab.

M'_{rc}, M'_{rs}: Moment of resistance when not cracked and cracked respectively based on the thickness ($= R_c d^2$ and $R_s d^2$ respectively).

Q_c: Moment-of-resistance factor (no compression reinforcement) based on the effective depth.

$r_0 = \dfrac{A_{st}}{bd}$ ($\nless 0{\cdot}003$ with mild steel bars or $0{\cdot}0025$ with deformed bars).

Slabs subjected to Bending only. Cases IA and IB, *Table* 73.

RESISTANCE TO CRACKING.

$$M'_{rc} = R_c d^2 = \frac{p_{ct}}{1 - n_0}\left[\frac{1}{3} - (1 - n_0)n_0 + r_0(m - 1)\left(\frac{d_1}{d} - n_0\right)^2\right]bd^2,$$

in which
$$n_0 = \frac{\frac{1}{2} + r_0\left(\frac{d_1}{d}\right)(m - 1)}{1 + r_0(m - 1)}.$$

RESISTANCE WHEN CRACKED. $M'_{rs} = R_s d^2 = p_{st} r_0\left(1 - \dfrac{n_1}{3}\right)\left(\dfrac{d_1}{d}\right)bd^2$

in which
$$n_1 = \sqrt{\left[mr_0\left(\frac{d}{d_1}\right)\right]^2 + 2mr_0\left(\frac{d}{d_1}\right)} - mr_0\left(\frac{d}{d_1}\right).$$

When the permissible tensile stresses in the concrete (p_{ct}) and reinforcement (p_{st}) are 270 lb. and 12,000 lb. per sq. in. respectively the expressions are as in *Table* 73 for $b = 12$ in. The curves give values of R_c and R_s for various values of $\dfrac{d_1}{d}$ and r_0. For each value of $\dfrac{d_1}{d}$ there is a proportion r_{0E} of reinforcement which gives equal values of R_c and R_s, say R_E, the adoption of which gives the most economical design. If R_E is substituted in $d = \sqrt{\dfrac{M}{R_E}}$, M being the bending moment to be resisted, the resulting value of d is the thickness of slab complying with the two design requirements if A_{st} in a strip 1 ft. wide is not less than $12r_{0E}d$ sq. in.; corresponding values of r_{0E} and R_E are given in *Table* 73.

The resistance of a slab with the proportion of reinforcement r_{0E} is obtained by substitution in $M_r = R_E d^2$; the resistances and reinforcement of slabs of various thicknesses taking into account probable values of $\dfrac{d_1}{d}$ are given in *Table* 73.

If in a slab not less than 9 in. thick the tensile strain is at the face remote from the liquid, design as for an ordinary slab with $p_{st} \ngtr 18{,}000$ lb. per sq. in. in plain bars or 20,000 lb. per sq. in. in deformed bars, and the compressive stress in the concrete not greater than 1200 lb.

(*Continued on page* 272.)

MODULAR-RATIO METHOD.

MAXIMUM WORKING STRESSES LB. PER SQ. IN. $m = 15$			THICKNESS (IN)	3	3½	4	4½	5	6	7	7½	8	9
			COVER (IN.) (ASSUMED)	½	½	½	¾	¾	¾	¾	¾	¾	1
			BAR DIAMETER (ASSUMED)	⅜	½	½	⅝	⅝	¾	¾	¾	¾	⅞
			EFFECTIVE DEPTH	2·31″	2·75″	3·25″	3·44″	3·94″	4·87″	5·87″	6·37″	6·87″	7·56″
TENSILE	COMPRESSIVE	r_1	BASIC FACTORS										
p_{st}	p_{cb} 750	24	$M_r = 1510 d_1^2$	8,050	11,400	16,000	17,800	23,400	35,000	52,000	61,200	71,100	86,500
			$A_{st} = 0·096 d_1$	0·222	0·264	0·312	0·330	0·378	0·467	0·564	0·612	0·660	0·726
18,000	1000	18	$M_r = 2310 d_1^2$	12,300	17,500	24,400	27,300	35,800	54,700	79,500	93,500	109,000	132,000
			$A_{st} = 0·152 d_1$	0·350	0·418	0·494	0·523	0·598	0·740	0·890	0·970	1·04	1·15
	1250	14·4	$M_r = 3168 d_1^2$	16,900	24,000	33,600	37,400	49,100	72,000	109,000	129,000	150,000	181,000
			$A_{st} = 0·212 d_1$	0·490	0·584	0·690	0·730	0·835	1·03	1·24	1·35	1·45	1·60
	750	$26\frac{2}{3}$	$M_r = 1428 d_1^2$	7,600	10,800	15,100	16,700	22,100	33,800	49,000	57,800	67,200	81,500
			$A_{st} = 0·081 d_1$	0·187	0·222	0·262	0·278	0·319	0·394	0·475	0·516	0·556	0·612
20,000	1000	20	$M_r = 2200 d_1^2$	11,700	16,600	23,300	26,000	34,100	52,100	75,600	89,000	104,000	126,000
			$A_{st} = 0·129 d_1$	0·297	0·354	0·419	0·444	0·508	0·628	0·755	0·820	0·885	0·975
	1250	16	$M_r = 3042 d_1^2$	16,200	23,000	32,200	35,900	47,200	72,100	105,000	128,000	144,000	174,000
			$A_{st} = 0·184 d_1$	0·425	0·506	0·598	0·633	0·725	0·896	1·08	1·17	1·26	1·39
	750	36	$M_r = 1188 d_1^2$	6,350	9,000	12,600	14,000	18,400	28,100	49800	48,100	56,000	68,000
			$A_{st} = 0·049 d_1$	0·113	0·135	0·159	0·168	0·193	0·238	0·288	0·312	0·336	0·370
27,000	1000	27	$M_r\ 1884 d_1^2$	10,000	14,200	20,000	22,200	29,200	44,700	64,800	76,200	89,000	108,000
			$A_{st} = 0·079 d_1$	0·183	0·218	0·257	0·272	0·311	0·384	0·464	0·504	0·543	0·598
	1250	21·6	$M_r = 2652 d_1^2$	14,100	20,000	28,100	31,300	41,000	62,700	91,000	107,000	125,000	151,500
			$A_{st} = 0·114 d_1$	0·263	0·314	0·370	0·392	0·449	0·555	0·670	0·727	0·785	0·863
	750	40	$M_r = 1125 d_1^2$	6,000	8,500	11,900	13,300	17,500	26,700	38,700	45,600	53,100	64,500
			$A_{st} = 0·041 d_1$	0·095	0·113	0·133	0·141	0·161	0·200	0·240	0·261	0·282	0·310
30,000	1000	30	$M_r = 1776 d_1^2$	9,480	13,400	18,800	20,900	27,500	42,000	61,000	72,000	83,500	101,000
			$A_{st} = 0·067 d_1$	0·154	0·184	0·217	0·240	0·268	0·326	0·392	0·426	0·460	0·513
	1250	24	$M_r = 2514 d_1^2$	13,400	19,000	26,700	29,700	39,000	59,500	86,500	102,400	118,500	144,000
			$A_{st} = 0·096 d_1$	0·222	0·264	0·312	0·330	0·378	0·467	0·564	0·612	0·660	0·726

M_r = MOMENT OF RESISTANCE IN INCH-POUNDS PER FOOT WIDTH OF SLAB REINFORCED IN TENSION ONLY.
A_{st} = AREA OF TENSILE REINFORCEMENT IN SQUARE INCHES PER FOOT WIDTH OF SLAB.
IF COMBINATION OF ACTUAL COVER AND DIAMETER OF BAR GIVES AN EFFECTIVE DEPTH DIFFERENT FROM THE DEPTH TABULATED, M_r AND A_{st} SHOULD BE CALCULATED FROM BASIC FACTORS.
M_r FOR OTHER COMBINATIONS OF STRESSES HAVING THE SAME RATIOS r_1 AS TABULATED ARE PROPORTIONAL TO THE TENSILE STRESS. $r_1 = p_{st}/p_{cb}$

For example of use of this table, see page 270.

SLABS FOR LIQUID-CONTAINING STRUCTURES (*continued from page* 270).
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per sq. in. (Case IA in *Table* 73); d_1 must be not less than $\sqrt{\dfrac{M}{Q_c b}}$ in which $Q_c = 250$ for $p_{st} = 18{,}000$ lb., and 239 for $p_{st} = 20{,}000$ lb. per sq. in. If the thickness provided is such that $d_1 = \sqrt{\dfrac{M}{Q_c b}}$, $A_{st} = 0{\cdot}017bd$ and $0{\cdot}014bd$ sq. in. in width b if p_{st} is 18,000 lb. and 20,000 lb. per sq. in. respectively. If d_1 exceeds the minimum depth required, A_{st} (in width b) $= \dfrac{M}{p_{st} l_a}$ in which $l_a = 0{\cdot}83d_1$ approximately.

The moment of resistance is $Q_c b d_1^2$ if $A_{st} \nless 0{\cdot}017bd_1$ (or $0{\cdot}014bd_1$ with deformed bars); moments of resistance and corresponding amounts of reinforcement are given in *Table* 73 for probable values of $\dfrac{d_1}{d}$.

Examples (*Table* 73).—Design walls of tanks for the conditions stated (Case IA).

(*a*).—Bending moment of 20,000 in.-lb. per foot.

This design can be taken directly from the table headed " Balanced Design ", since the moment of resistance of a 6-in. slab is approximately 20,000 in.-lb. The reinforcement required is (also from the table) 0·49 sq. in. per foot, which is provided by $\frac{5}{8}$-in. bars at $7\frac{1}{2}$-in. centres with $1\frac{1}{2}$-in. cover.

(*b*).—Bending moment of 24,000 in.-lb. per foot.

The curves in the table can be used directly to obtain a " balanced design." Assume $\dfrac{d_1}{d} = 0{\cdot}7$. (Compare with values on table headed " Balanced Design ".) From curves, $R_c = R_s = R_E = 580$ and $r_0 = 0{\cdot}0067$. $d = \sqrt{\dfrac{24{,}000}{580}} = 6{\cdot}45$ in.; that is, a $6\frac{1}{2}$-in. slab. $A_{st} = 0{\cdot}0067 \times 6\frac{1}{2} \times 12 = 0{\cdot}522$ sq. in ; provide $\frac{5}{8}$-in. bars at 7-in. centres; $1\frac{1}{2}$-in. cover.

(*c*).—Check that an 8-in. slab reinforced with $\frac{5}{8}$-in. bars at 6-in. centres with $1\frac{1}{2}$-in cover and subjected to a bending moment of 36,000 in.-lb. per foot conforms to B.S. Code No. 2007.

$$d_1 = 8 - 1\tfrac{1}{2} - \tfrac{5}{16} = 6{\cdot}19\text{ in.};\ \frac{d_1}{d} = \frac{6{\cdot}19}{8} = 0{\cdot}774;\ \frac{d}{d_1} = \frac{1}{0{\cdot}774} = 1{\cdot}29.\quad A_{st} = 0{\cdot}614\text{ sq. in.};$$

$$r_0 = \frac{0{\cdot}614}{12 \times 8} = 0{\cdot}0064.$$ Substitute in formulæ in *Table* A.

$$n_0 = \frac{\frac{1}{2} + (14 \times 0{\cdot}0064 \times 0{\cdot}774)}{1 + (14 \times 0{\cdot}0064)} = 0{\cdot}523.$$

$$R_c = \frac{3240}{1 - 0{\cdot}523}\{\tfrac{1}{3} - (1 - 0{\cdot}523)\, 0{\cdot}523 + [14 \times 0{\cdot}0064\,(0{\cdot}774 - 0{\cdot}523)^2]\} = 607.$$

$$d \nless \sqrt{\frac{36{,}000}{607}} = 7{\cdot}70 \text{ in., which does not exceed 8 in.}$$

$$r_s = 15 \times 0{\cdot}0064 \times 1{\cdot}29 = 0{\cdot}124;\ n_1 = \sqrt{(0{\cdot}124)^2 + (2 \times 0{\cdot}124)} - 0{\cdot}124 = 0{\cdot}39.$$

$$a_1 = 1 - (\tfrac{1}{3} \times 0{\cdot}39) = 0{\cdot}87;\ A_{st} \nless \frac{36{,}000}{12{,}000 \times 0{\cdot}87 \times 6{\cdot}19} = 0{\cdot}558 \text{ sq. in. per foot.}$$

which does not exceed 0·614 sq. in. Therefore design is satisfactory.

Note.—For designs in Case IB, the procedure is as in the foregoing if $d < 9$ in., and as for ordinary designs if $d \nless 9$ in.

Slabs subjected to Direct Tension (Case II, *Table* 74).

RESISTANCE TO CRACKING.—Determines the thickness of the slab.

$$T = p_{ct}[db + (m - 1)A_{st}].$$

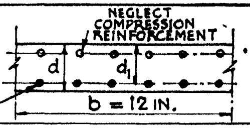
RESISTANCE WHEN CRACKED.—Determines the amount of reinforcement.

If $A_{st} = r_0 bd$, $r_0 = \dfrac{1}{\dfrac{p_{st}}{p_{ct}} - (m - 1)}$ and $d = \dfrac{T}{[1 + (m - 1)r_0]bp_{ct}}$.

The expressions in *Table* 74 give values of r_0 and d for a slab subjected to a tensile force T lb. per ft. if p_{ct} and p_{st} are 190 lb. and 12,000 lb. per sq. in. respectively.

(*Continued on page* 274.)

IN ACCORDANCE WITH B.S. CODE No. 2007.
CONCRETE: NOMINAL PROPORTIONS 1 : 1·6 : 3·2 $m = 15$
MINIMUM PROPORTION OF REINFORCEMENT $r_o = \frac{A_{st}}{bd} \not< 0{\cdot}003$
MINIMUM COVER: $1\frac{1}{2}$ IN.
BENDING MOMENT = M IN.-LB. PER FOOT.



CASE IA BENDING ONLY (TENSILE STRESS AT FACE IN CONTACT WITH LIQUID)

RESISTANCE TO CRACKING.

$M^r_{rc} = R_c d^2$

DETERMINES THICKNESS d. $p_{ct} \not> 270$ LB/SQ.IN.

GIVEN OR ASSUME r_o AND $\frac{d_1}{d}$

$$d \not< \sqrt{\frac{M}{R_c}} \quad \left[\begin{array}{l} \text{WHERE } R_c = \dfrac{3240}{1-n_o}\left[\dfrac{1}{3} - (1-n_o)n_o + 14r_o\left(\dfrac{d_1}{d} - n_o\right)^2\right] \\ \text{AND } n_o = \dfrac{\frac{1}{2} + 14r_o\left(\frac{d_1}{d}\right)}{1+14r_o} \end{array}\right.$$

DESIGN FOR STRENGTH.

$M^r_{RS} = R_s d^2$

DETERMINES REINFORCEMENT A_{st}. $p_{st} \not> 12{,}000$ LB./SQ.IN.

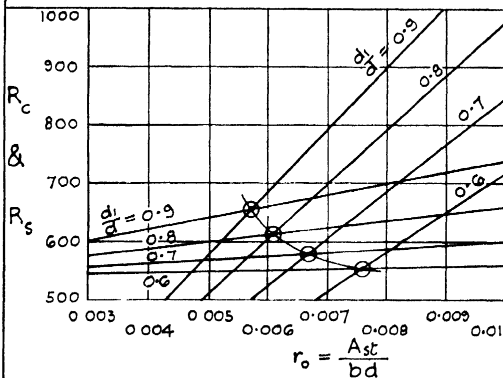
$$A_{st} \not< \frac{M}{12{,}000\, a_1 d_1} \quad \text{WHERE } a_1 = 1 - \tfrac{1}{3}n_1$$

$$\text{AND } n_1 = \sqrt{r_s^2 + 2r_s} - r_s; \quad r_s = 15r_o\left(\frac{d}{d_1}\right)$$

$$[R_s = 144{,}000\, r_o\, a_1 d_1 d]$$

BALANCED DESIGN

POINTS THUS ⊙ INDICATE VALUES OF r_{oE} AND $R_c = R_s = R_E$ FOR BALANCED DESIGN AND ARE APPLIED IN THE TABLE BELOW TO SLABS OF THE THICKNESS STATED.



d IN.	d_1 IN.	$M^r_{rc} = M^r_{rs}$ IN.-LB./FT.	A_{st} ϕ	A_{st} SQ IN/FT
4	2·25	8,800	½"	0·39
4½	2·69	10,300	⅝"	0·41
5	3·19	14,200		0·43
6	4·19	20,800		0·49
7	5·19	29,200		0·54
8	6·19	38,800		0·60
9	7·12	29,400	¾"	0·66
10	8·12	61,800		0·72
12	10·12	91,100		0·85
15	13·06	144,200	⅞"	1·05
18	16·0	211,200	1"	1·25
24	22·0	380,100		1·64

(a) $d < 9$ IN. ADOPT CASE IA.

CASE IB BENDING ONLY (TENSILE STRESS AT FACE REMOTE FROM LIQUID.)

(b) $d \not< 9$ IN. DESIGN FOR STRENGTH ONLY

$p_{st} \not> 18{,}000$ LB./SQ.IN. (PLAIN BARS) OR 20,000 LB./SQ.IN. (DEFORMED BARS)

$p_{cb} \not> 1200$ LB./SQ.IN.

DESIGN.

$$d_1 \not< \sqrt{\frac{M}{3000}} \text{ (PLAIN BARS) OR } \sqrt{\frac{M}{2872}} \text{ (DEFORMED BARS)}$$

$$d = d_1 + 1\tfrac{1}{2}\text{ IN.} + \tfrac{1}{2}(\text{DIAM. OF BAR}).$$

$$A_{st} = \frac{M}{15{,}000\, d_1} \text{ (PLAIN BARS) OR } \frac{M}{16{,}840\, d_1} \text{ (DEFORMED BARS)}. \quad r_o = \frac{A_{st}}{12d}$$

STRESSES. GIVEN d, d_1 AND r_o. n_1 AND a_1 AS CASE IA.

$$f_{st} = \frac{M}{12 r_o a_1 d_1 d} \qquad f_{cb} = \frac{f_{st}\, n_1}{15(1-n_1)}$$

SLABS FOR LIQUID-CONTAINING STRUCTURES (*continued from page 272*).
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Slabs Subjected to Direct Tension and Bending (Cases IIIA and IIIB, *Table* 74).

TENSILE STRAIN AT FACE IN CONTACT WITH LIQUID. $e = \frac{M}{T}$.

Tensile Stresses only.—$e > d_1 - \frac{1}{2}d$.
Resistance to cracking (determines thickness of slab).

$$\frac{T}{bd}\left[\frac{1}{1 + 2r_0(m-1)} + \frac{e}{\left[\frac{1}{6} + 4r_0(m-1)\left(\frac{d_1}{d} - \frac{1}{2}\right)^2\right]}\right] \ngtr p_{ct} \text{ (direct tension)}$$

Resistance when cracked (determines the amount of reinforcement). r_0 = proportion of reinforcement near each face.—$r_0 \nless \frac{T}{2p_{st}bd}\left[1 + \frac{e}{d_1 - \frac{d}{2}}\right]$.

Evaluation of these expressions for $p_{ct} = 190$ lb. and $p_{st} = 12{,}000$ lb. per sq. in. ($b = 12$ in.) are given in *Table* 74.

Tensile and Compressive Stresses.—$e > d_1 - \frac{1}{2}d$. Neglect tension reinforcement.
Resistance to cracking (determines thickness of slab).

$$\frac{T}{bd}\left[\frac{1}{1 + (m-1)r_0} + \frac{e(1 - n_0)}{d\left[\frac{1}{3} - (1 - n_0)n_0 + r_0(m-1)\left(\frac{d_1}{d} - n_0\right)^2\right]}\right] \ngtr p_{ct} \text{ (bending tension)}$$

in which
$$n_0 = \frac{\frac{1}{2} + (m-1)\left(\frac{d_1}{d}\right)r_0}{1 + (m-1)r_0}.$$

Resistance when cracked (determines the amount of tensile reinforcement).

$$r_0 \nless \frac{T}{p_{st}bd}\left(1 + \frac{e + \frac{1}{2}d - d_1}{l_a}\right).$$

The expressions in *Table* 74 are for $p_{ct} = 270$ lb. and $p_{st} = 12{,}000$ lb. per sq. in. ($b = 12$ in.)

TENSILE STRAIN AT FACE REMOTE FROM LIQUID.

Slab less than 9 in. thick.—Design as for Case IIIA, (*a*) or (*b*) as applicable.
Slab not less than 9 in thick.—If d and d_1 are known, or assumed,

$$A_{st} = \frac{T}{p_{st}}\left(1 + \frac{e_s}{l_a}\right) \text{ in which } e_s = e + \tfrac{1}{2}d - d_1. \quad d_1 = \sqrt{\frac{M}{Q_c b}} \text{ approximately.}$$

Q_c is the moment-of-resistance factor for the stresses p_{st} and p_{cb}.

If $e > d_1$, the value of d_1 required to ensure that p_{cb} is not exceeded should not differ much from that calculated from the foregoing expression; if $e < d_1$ a slightly thinner slab may be sufficient. In doubtful cases, calculate by the ordinary methods.

The expressions in *Table* 74 apply to $p_{st} = 18{,}000$ lb. for plain bars or 20,000 lb. per sq. in. for deformed bars, $p_{cb} \ngtr 1200$ lb. per sq. in. ($b = 12$ in.).

Examples (*Table* 74).—Design walls of tanks for the conditions stated.

(*a*).—Direct tensile force of 25,000 lb. per foot (no bending).—Case II.

$$d = \frac{25{,}000}{2930} = 8{\cdot}53 \text{ in.; say } 8\tfrac{1}{2}\text{-in. slab.}$$

$A_{st} = 0{\cdot}244 \times 8{\cdot}5 = 2{\cdot}07$ sq. in. per ft; 1-in. bars at 9-in. centres in each of two rows.

(*b*).—Tensile force of 12,000 lb. per foot, and bending moment of 20,000 in.-lb. per foot. Assume $d = 8$ in. and $d_1 = 8 - 2 = 6$ in.; $d_1 - \frac{1}{2}d = 6 - 4 = 2$ in.

$$e = \frac{20{,}000}{12{,}000} = 1{\cdot}67 \text{ in. } (\ngtr 2 \text{ in.; therefore Case IIIA } a \text{ applies.})$$

$$r_0' = \frac{12{,}000}{288{,}000 \times 8}\left(1 + \frac{1{\cdot}67}{2}\right) = 0{\cdot}0095.$$

$$f_{ct} = \frac{12{,}000}{12 \times 8}\left[\frac{1}{1 + (28 \times 0{\cdot}0095)} + \frac{1{\cdot}67 \times 8}{(\frac{1}{6} \times 8^2) + (56 \times 0{\cdot}0095 \times 2^2)}\right]$$

$= 230$ lb. per square inch (approx.) which exceeds 190 lb. per square inch.

(*Continued on page* 282.)

IN ACCORDANCE WITH B.S. CODE No. 2007.

CONCRETE: NOMINAL PROPORTIONS 1 : 1·6 : 3·2 $m = 15$

MINIMUM PROPORTION OF REINFORCEMENT: $r_0 = \dfrac{A_{st}}{bd} \nless 0{\cdot}003.$

MINIMUM COVER: $1\tfrac{1}{2}$ IN.

DIRECT TENSILE FORCE = T LB. PER FOOT. BENDING MOMENT = M IN.-LB. PER FT.

CASE II — CONCENTRIC TENSILE FORCE ONLY

DESIGN FOR STRENGTH.

DETERMINES REINFORCEMENT A_{st}. $p_{st} \ngtr 12{,}000$ LB./SQ.IN.

$$A_{st} = \frac{T}{12{,}000} \text{ SQ.IN./FT. OR } r_0 = \frac{T}{144{,}000d}.$$

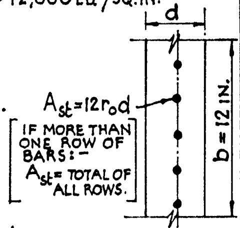
RESISTANCE TO CRACKING

DETERMINES THICKNESS d: $p_{ct} \ngtr 190$ LB./SQ.IN.

$$d \nless \left[\frac{T}{2280} - 1{\cdot}17A_{st}\right] \text{ IN.}$$

BALANCED DESIGN.

$$d = \frac{T}{2930} \text{ IN.} \qquad A_{st} = 0{\cdot}244d \text{ SQ.IN./FT.}$$



CASE IIIA — TENSILE FORCE COMBINED WITH BENDING (TENSILE STRESS AT FACE IN CONTACT WITH LIQUID)

$e = \dfrac{M}{T}$

(a) $e \ngtr (d_1 - \frac{1}{2}d)$. TENSILE STRESSES ONLY. $p_{ct} \ngtr 190$ LB./SQ.IN.

$p_{st} \ngtr 12{,}000$ " "

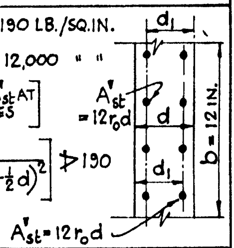
GIVEN OR ASSUME d AND d_1

$$\text{DETERMINE } r_0' = \frac{T}{288{,}000d}\left[1 + \frac{e}{d_1 - \frac{1}{2}d}\right]$$

[PROVIDE A'_{st} AT BOTH SIDES]

$$\text{SUBSTITUTE IN } f_{ct} = \frac{T}{12d}\cdot\left[\frac{1}{1+28r_0'} + \frac{ed}{\frac{1}{6}d^2 + 56r_0'(d_1 - \frac{1}{2}d)^2}\right] \ngtr 190$$

ADJUST d UNTIL EXPRESSION IS SATISFIED.



(b) $e > (d_1 - \frac{1}{2}d)$. TENSILE AND COMPRESSIVE STRESSES. $p_{ct} \ngtr 270$ LB./SQ.IN.

$p_{st} \ngtr 12{,}000$ " "

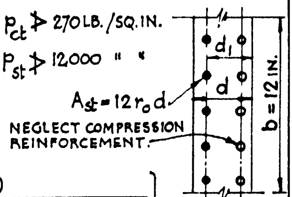
GIVEN OR ASSUME d AND d_1

$$\text{DETERMINE } r_0 = \frac{T}{144{,}000d}\left[1 + \frac{e + \frac{1}{2}d - d_1}{0{\cdot}83d_1}\right]$$

$$n_0 = \frac{\frac{1}{2} + 14r_0\frac{d_1}{d}}{1 + 14r_0}$$

$$\text{SUBSTITUTE IN } \frac{T}{12d}\left\{\frac{1}{1+14r_0} + \frac{e(1-n_0)}{\left[\frac{1}{3} - (1-n_0)n_0 + 14r_0\left(\frac{d_1}{d} - n_0\right)^2\right]d}\right\} \ngtr 270$$

ADJUST d UNTIL EXPRESSION IS SATISFIED.



CASE IIIB — TENSILE FORCE COMBINED WITH BENDING (TENSILE STRESS AT FACE REMOTE FROM LIQUID)

(a) $d < 9$ IN. ADOPT CASE IIIA.

(b) $d \nless 9$ IN. $p_{st} = 18{,}000$ LB./SQ.IN. (20,000 LB./SQ.IN. FOR DEFORMED BARS.)

$p_{cb} \ngtr 1200$ LB./SQ.IN. – NOT CRITICAL). DIAGRAM AS CASE IIIA(b)

GIVEN OR ASSUME d AND d_1 $\left[d_1 \nless \sqrt{\dfrac{M}{3000}}\ \text{(PLAIN BARS)} \text{ OR } \sqrt{\dfrac{M}{2872}}\ \text{(DEFORMED BARS)}\right]$

$$e_s = e + \tfrac{1}{2}d - d_1$$

$$A_{st} = \frac{T}{18{,}000 \text{ OR } 20{,}000}\left[1 + \frac{e_s}{0{\cdot}83d_1}\right] \text{ SQ.IN./FT.}$$

$$\text{IF } d,\ d_1 \text{ AND } A_{st} \text{ GIVEN:– } f_{st} = \frac{T}{A_{st}}\left[1 + \frac{e_s}{0{\cdot}83d_1}\right] \ngtr 18{,}000 \text{ (OR 20,000)}$$

DESIGN OF BEAMS: LOAD-FACTOR METHOD.

Notation.

A_{sc}, A_{st}, areas of reinforcement in compression and tension respectively; A_{sc}', A_{st}', do. in unit width of a rectangular beam.
a_1, lever-arm factor $= l_a/d_1$.
b, breadth of a rectangular beam; breadth of flange of a flanged beam; b_r, breadth of rib of a flanged beam.
d, overall depth of beam or thickness of slab.
d_n, distance from edge in compression to neutral plane $= n_1 d_1$.
d_s, thickness of flange of a flanged beam.
d_1, d_1' actual effective depth and effective depth required respectively.
d_2, distance of reinforcement in compression below top of beam or of reinforcement in tension above bottom of beam.
F, coefficient in formula for A_{st} in rectangular beams with reinforcement in compression.
K, coefficient relating to dimensions of flange of a flanged beam $= \dfrac{4b}{b_r}\gamma$.
l_a, lever arm of compressive resistance of concrete $= a_1 d_1$.
M, applied bending moment; M', applied bending moment per unit width.
M_r, moment of resistance; M_r', moment of resistance of rectangular beam of unit width.
m, modular ratio ($= 15$).
n_1, neutral-plane factor $= d_n/d_1$.
p_{cb}, permissible compressive stress in concrete in bending.
p_{sc}, p_{st}, permissible or actual compressive and tensile stresses respectively in reinforcement. $p_{sc}' = p_{sc}\dfrac{m-1}{m}$.
Q, shearing force; q, shearing stress.
Q_1, bending-moment factor $= \dfrac{M}{bd_1^2}$ or $\dfrac{M}{b_r d_1^2}$.
Q_c, moment-of-resistance factor (compression); Q', moment-of-resistance factor for rectangular beam of unit width.
r_L, limiting proportion of reinforcement in tension for balanced design $= \dfrac{A_{st}}{bd_1}$ or $\dfrac{A_{st}}{b_r d_1}$.
r_t, r_2, proportion of reinforcement $= \dfrac{A_{st}}{bd_1}$ and $\dfrac{A_{sc}}{bd_1}$ in tension and compression respectively.
s', ratio of thickness of flange to effective depth $= \dfrac{d_s}{d_1}$.
γ, factor in B S. Code relating to dimensions of flanged beams.

In the basic formulæ any mutually-consistent units may be used, but in formulæ containing numerical terms the units must be inches and pounds.

Rectangular Beams with Tensile Reinforcement only.—The moment of resistance of a rectangular beam reinforced only in tension is $\left(d_1 - \dfrac{3A_{st}p_{st}}{4bp_{cb}}\right)A_{st}p_{st}$ if based on the tensile resistance and $\frac{1}{4}p_{cb}bd_1^2$ if based on the compressive resistance. If $Q_c = \frac{1}{4}p_{cb}$, the two resulting basic formulæ are as given in series A for this case in *Table* 75.

The safe moment of resistance is the smaller of the two moments calculated by these formulæ. For a balanced design these moments are identical and the corresponding value of A_{st} is $\dfrac{p_{cb}}{3p_{st}}bd_1$, if this amount is expressed as a ratio of bd_1 the limiting proportion r_L is $\dfrac{p_{cb}}{3p_{st}}$. If the proportion of reinforcement in a beam is less than r_L, the tensile resistance controls the strength and if greater the compressive resistance controls. Values of Q_c and r_L are given in *Table* 76 for various stresses.

The expression for the moment of resistance if tension controls, can be written as

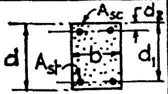
$$M_r = Q_T bd_1^2, \text{ where } Q_T = r_t p_{st}(1 - F_1 r_t);\ F_1 = \frac{3p_{st}}{4p_{cb}}.$$

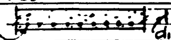
Values of the factor F_1 for various combinations of stresses are given in *Table* 76.

Example.—Calculate the moment of resistance of a rectangular beam 24 in. deep ($d_1 = 22\frac{1}{2}$ in.) and 12 in. wide ($b = 12$ in.) if reinforced with four 1-in. mild steel bars in tension only ($A_{st} = 3{\cdot}14$ sq. in.). The permissible stresses are $p_{st} = 20{,}000$ lb. per sq. in., and $p_{cb} = 1000$ lb. per sq. in. $r_t = \dfrac{3{\cdot}14}{12 \times 22{\cdot}5} = 0{\cdot}0116$.

(*Continued on page* 278.)

RECTANGULAR BEAMS & SOLID SLABS — FORMULÆ FOR LOAD-FACTOR METHOD.





			BASIC FORMULÆ	ORDINARY GRADE 1 : 2 : 4 CONCRETE p_{cb} = 1000 LB. PER SQ.IN.			
			p_{st} = TENSILE STRESS IN BARS OR WIRES; p_{sc} = COMPRESSIVE STRESS IN BARS; p_{cb} = COMPRESSIVE STRESS IN CONCRETE	RECTANGULAR BEAMS — MILD STEEL BARS p_{st} = 20,000 LB./SQ.IN. p_{sc} = 18,000 " " "	RECTANGULAR BEAMS — HIGH-YIELD-STRESS BARS p_{st} = 30,000 LB./SQ.IN. p_{sc} = 23,000 " " "	SOLID SLABS A_{st} PER FOOT, b = 12 IN. — MILD STEEL BARS p_{st} = 20,000 LB./SQ.IN.	SOLID SLABS A_{st} PER FOOT, b = 12 IN. — HIGH-YIELD-STRESS BARS OR WIRE p_{st} = 30,000 LB./SQ.IN.
			SERIES A	SERIES B	SERIES C	SERIES D	SERIES E
MOMENT OF RESISTANCE M_r	TENSILE REINF^t. ONLY	$\frac{A_{st}}{bd_1} < r_L$	$M_r = A_{st}p_{st}\left(d_1 - \frac{3A_{st}p_{st}}{16Q_cb}\right)$	$r_L = 1/60$; $20{,}000A_{st}\left(d_1 - \frac{15A_{st}}{b}\right)$	$r_L = 1/90$; $30{,}000A_{st}\left(d_1 - \frac{22.5A_{st}}{b}\right)$	$20{,}000A_{st}(d_1 - 1.25A_{st})$	$30{,}000A_{st}(d_1 - 1.875A_{st})$
	$r_L = \frac{p_{cb}}{3p_{st}}$	$\not< r_L$	$M_r = Q_cbd_1^2$ $\quad Q_c = \frac{p_{cb}}{4}$	$250bd_1^2$	$250bd_1^2$	$3000d_1^2$	$3000d_1^2$
	WITH COMPRESSION REINFORCEMENT		$M_r = Q_cbd_1^2 + A_{sc}p_{sc}(d_1 - d_2)$ $\left[A_{st} \not< r_Lbd_1 + \frac{p_{sc}}{p_{st}}A_{sc}\right]$	$250[bd_1^2 + 72A_{sc}(d_1 - d_2)]$ $\left[A_{st} \not< \frac{bd_1}{60} + 0.9A_{sc}\right]$	$250[bd_1^2 + 92A_{sc}(d_1 - d_2)]$ $\left[A_{st} \not< \frac{bd_1}{90} + 0.77A_{sc}\right]$	—	—
DESIGN FORMULÆ	EFFECTIVE DEPTH REQUIRED	d_1'	$\sqrt{\frac{M}{Q_cb}}$	$\sqrt{\frac{M}{250b}}$	$\sqrt{\frac{M}{250b}}$	$\sqrt{\frac{M}{3000}}$	$\sqrt{\frac{M}{3000}}$
	EFFECTIVE DEPTH PROVIDED $d_1 < d_1'$	A_{sc}	$\frac{Q_1 - Q_c}{p_{sc}\left(1 - \frac{d_2}{d_1}\right)}bd_1$ $\quad Q_1 = \frac{M}{bd_1^2}$	$\frac{Q_1 - 250}{18{,}000(1 - d_2/d_1)}bd_1$	$\frac{Q_1 - 250}{23{,}000(1 - d_2/d_1)}bd_1$	—	—
			[A_{sc} NOT TO EXCEED 4 PER CENT. OF $b(d_1 + d_2)$]				
		A_{st}	$r_Lbd_1 + \frac{A_{sc}p_{sc}}{p_{st}}$	$\frac{bd_1}{60} + 0.9A_{sc}$	$\frac{bd_1}{90} + 0.77A_{sc}$		
	$d_1 = d_1'$ (A_{sc}=0)	A_{st}	r_Lbd_1	$\frac{bd_1}{60}$	$\frac{bd_1}{90}$	$\frac{d_1}{5}$	$\frac{d_1}{7.5}$
	$d_1 > d_1'$ (A_{sc}=0)	A_{st}	Fr_Lbd_1	$\frac{bd_1}{60}F$	$\frac{bd_1}{90}F$	$\frac{d_1}{5}F$	$\frac{d_1}{7.5}F$
			$F = 2\left(1 - \sqrt{1 - \frac{3Q_1}{p_{cb}}}\right)$	$F = 2\left(1 - \sqrt{1 - \frac{Q_1}{333}}\right)$			

NOTE.—Values of Q_c, r_L and F are given in *Table* 76.

DESIGN OF BEAMS: LOAD-FACTOR METHOD.

Rectangular Beams with Tensile Reinforcement only.

Example (*continued from page* 276).

(i) Using *Table* 75.—Formulæ in series B apply. $r_L = \frac{1}{60} = 0{\cdot}0167$ which exceeds r_t; therefore tension controls and

$$M_r = 20{,}000 \times 3{\cdot}14\left(22{\cdot}5 - \frac{15 \times 3{\cdot}14}{12}\right) = 1{,}170{,}000 \text{ in.-lb.}$$

(ii) Using *Table* 76.—$r_L = 0{\cdot}017$ which exceeds r_t; therefore tension controls. $F_1 = 15$. Substituting in formulæ in foregoing,

$Q_T = 0{\cdot}0116 \times 20{,}000[1 - (15 \times 0{\cdot}0116)] = 193$;

$$M_r = 193 \times 12 \times 22{\cdot}5^2 = 1{,}170{,}000 \text{ in.-lb.}$$

Rectangular Beams with Compression Reinforcement.—If a rectangular beam contains compression reinforcement the moment of resistance is $0{\cdot}25p_{cb}bd_1^2 + A_{sc}p_{sc}(d_1 - d_2)$, which can be rewritten in the form given in *Table* 75 (Series A), which applies if there is sufficient reinforcement in tension to produce at least an equal resistance. The minimum amount of A_{st} for this purpose is $r_Lbd_1 + A_{sc}\frac{p_{sc}}{p_{st}}$; the first term is the amount required to balance the resistance of the concrete and the second term is the amount required to balance the resistance of the compression reinforcement. The compressive stress p_{sc} permissible in the compression reinforcement is in general 18,000 lb. per sq. in. in mild steel bars not greater than $1\frac{1}{8}$ in. diameter, 16,000 lb. per sq. in. in larger mild steel bars, and 23,000 lb. per sq. in. in high-yield-stress bars; the Code recommends also $p_{sc} \not> 50{,}000\left(1 - \frac{d_2}{d_n}\right)$. If the depth to the neutral plane is assumed to be $0{\cdot}5d_1$ (the limiting depth of the compressive zone recommended in the Code) the limiting value of the compressive stress $= 50{,}000\left(1 - 2\frac{d_2}{d_1}\right)$, but this limit may not apply to ordinary designs, since the cover-ratio d_2/d_1 must exceed 0·32 (an uncommonly high ratio) for p_{sc} to be less than 18,000 lb. per sq in.; likewise the cover-ratio for high-yield-stress bars must exceed 0·27 (also a high ratio) for the permissible stress to be less than 23,000 lb. per sq. in.

The Code recommends that the area of reinforcement in compression shall not exceed 4 per cent. of the area bd. With this limiting amount, the moment of resistance is $Q_cbd_1^2 + 0{\cdot}04bdp_{sc}(d_1 - d_2)$. Substituting $Q_c = 0{\cdot}25p_{cb}$ and $d = d_1 + d_2$,

$$M_r = \left\{\frac{p_{cb}}{4} + 0{\cdot}04p_{sc}\left[1 - \left(\frac{d_2}{d_1}\right)^2\right]\right\}bd_1^2.$$

and
$$A_{sc} = 0{\cdot}04\left(1 + \frac{d_2}{d_1}\right)bd_1.$$

Similarly the corresponding area of tensile reinforcement is $r_Lbd_1 + 0{\cdot}04bd\frac{p_{sc}}{p_{st}}$, that is

$$A_{st} = \left[\frac{p_{cb}}{3} + 0{\cdot}04\left(1 + \frac{d_2}{d_1}\right)p_{sc}\right]\frac{bd_1}{p_{st}}.$$

Example.—Calculate the moment of resistance of a rectangular beam 26 in. deep ($d_1 = 22\frac{1}{2}$ in.) and 12 in. wide ($b = 12$ in.) reinforced with six $1\frac{1}{8}$-in. mild steel bars in the bottom and two 1-in. bars in the top. The permissible stresses are $p_{st} = 20{,}000$ lb. per sq. in., $p_{sc} = 18{,}000$ lb. per sq. in., and $p_{cb} = 1000$ lb. per sq. in. $A_{st} = 5{\cdot}96$ sq. in., $A_{sc} = 1{\cdot}57$ sq. in. $d_2 = 1\frac{1}{2}$ in.

Apply formulæ in series B in *Table* 75; assuming compression controls,

$$M_r = 250[(12 \times 22{\cdot}5^2) + (72 \times 1{\cdot}57)(22{\cdot}5 - 1{\cdot}5)] = 2{,}142{,}500 \text{ in.-lb.}$$

Check that there is sufficient tensile reinforcement:

$$\frac{12 \times 22{\cdot}5}{60} + (0{\cdot}9 \times 1{\cdot}57) = 5{\cdot}91 \text{ sq. in.} \not> 5{\cdot}96 \text{ sq. in.}$$

(*Continued on page* 280.)

LOAD-FACTOR METHOD.

PERMISSIBLE COMPRESSIVE STRESS p_{cb} LB PER SQ IN.	FACTOR Q_c	PERMISSIBLE TENSILE STRESS (t LB. PER SQ. IN.) $r_L = p_{cb}/3p_{st}$; $F_1 = \frac{3}{4}\frac{p_{st}}{p_{cb}} = \frac{3}{16}\frac{p_{st}}{Q_c}$											
		16,000		18,000		20,000		25,000		27,000		30,000	
		r_L	F_1	r_L	F_1	r_L	F_1	r_L	F_1	r_L	F_1	r_L	F_1
750	188	0·016	16	0·014	18	0·013	20	0·010	25	0·009	27	0·008	30
800	200	17	15	15	17	13	19	11	23	10	25	09	28
850	213	18	14	16	16	14	18	11	22	11	24	10	27
900	225	19	13	17	15	15	17	12	21	11	23	10	25
950	238	20	13	18	14	16	16	13	20	12	21	11	24
1000	250	0·021	12	0·019	14	0·017	15	0·013	19	0·012	20	0·011	23
1050	263	22	11	19	13	18	14	14	18	13	19	12	21
1100	275	23	11	20	12	18	14	15	17	14	18	12	20
1150	288	24	10	21	12	19	13	15	16	14	18	13	20
1200	300	25	10	22	11	20	13	16	16	15	17	13	19
1250	313	0·026	10	0·023	11	0·021	12	0·017	15	0·015	16	0·014	18
1300	325	27	9·3	24	10	22	12	17	14	16	16	14	17
1350	338	28	8·9	25	10	23	11	18	14	17	15	15	17
1400	350	29	8·6	26	9·7	23	11	19	13	17	14	16	16
1450	368	30	8·3	27	9·3	24	10	19	13	18	14	16	16
1500	375	0·031	8·0	0·028	9·0	0·025	10	0·020	12	0·019	14	0·017	15
1550	388	32	7·7	29	8·7	26	9·7	21	12	19	13	17	15
1600	400	33	7·5	30	8·4	27	9·4	21	12	20	13	18	14
1650	413	34	7·3	31	8·2	28	9·1	22	11	20	12	18	14
1700	425	35	7·1	31	8·0	28	8·8	23	11	21	12	19	13
1750	438	0·037	6·9	0·032	7·7	0·029	8·6	0·023	11	0·022	12	0·019	13
1800	450	38	6·7	33	7·5	30	8·3	24	10	22	11	20	13
1850	463	39	6·5	34	7·3	31	8·1	25	10	23	11	21	12
1900	475	40	6·3	35	7·1	32	7·9	25	9·9	24	11	21	12
1950	488	41	6·2	36	6·9	33	7·7	26	9·6	24	10	22	12
2000	500	42	6·0	37	6·8	33	7·5	27	9·4	25	10	22	11

VALUES OF FACTOR $F = 2\left[1 - \sqrt{1 - \frac{3Q_1}{p_{cb}}}\right]$

B.M. FACTOR $Q_1 = \frac{M}{bd_1^2}$ LB PER SQ. IN		120	144	150	160	170	180	188	200	220	240	250	260	280	300	313
PERMISSIBLE COMPRESSIVE STRESS p_{cb} LB. PER SQ IN	1250	0·31	0·37	0·40	0·43	0·46	0·49	0·52	0·56	0·63	0·70	0·74	0·77	0·86	0·95	1·00
	1000	0·40	0·48	0·52	0·56	0·60	0·64	0·68	0·73	0·83	0·94	1·00	COMP. REINFT. REQUIRED IF $Q_1 > 250$			
	750	0·56	0·67	0·73	0·80	0·87	0·94	1·00	COMPRESSION REINFORCEMENT REQUIRED IF $Q_1 > 188$							

NOTE.—For basic formulæ, see *Table* 75.

DESIGN OF BEAMS: LOAD-FACTOR METHOD

(*continued from page* 278).

Design of Rectangular Beams.—The basic formulæ in series A in *Table* 75 for the moment of resistance can be transposed and simplified to give expressions for determining the size of a beam and the amount of reinforcement to resist a given bending moment. The formula for the effective depth required is given in *Table* 75; formulæ are also given for the amount of reinforcement required for the three cases of when the effective depth provided is equal to, greater than, or less than that required. Compression reinforcement is required in the latter case.

The formulæ in series A are further simplified by substituting permissible stresses assuming the concrete to be 1 : 2 : 4 ordinary grade and the reinforcement to be mild steel bars or high-yield-stress bars; the formulæ in series B and C in *Table* 75 result. Values of the coefficient F for various values of Q_1 and for $p_{cb} = 750$ lb., 1000 lb., and 1250 lb. per sq. in. are given in *Table* 76.

The moments of resistance and areas of reinforcement for rectangular beams of various depths and unit width ($b = 1$ in.) reinforced with mild steel bars not greater than $1\frac{1}{2}$ in. diameter are given in *Table* 77A for 1 : 2 : 4 concrete in which the permissible compressive stresses are 1000 lb. per sq. in. (ordinary grade), 750 lb. per sq. in. (lower grade), and 1250 lb. per sq. in. (higher grade). The data area given for beams without reinforcement in compression, and with the limiting amount of compression reinforcement ($A'_{sc} = 0{\cdot}04d$). The values in *Table* 77A must be multiplied by the width of the beam to give the actual moment of resistance and the corresponding areas of reinforcement in tension and compression. It is assumed in *Table* 77A that the distance from the tensile edge of the beam to the centre of the tensile reinforcement is equal to the distance d_2 from the compressive edge to the centre of the compression reinforcement; a reasonable value for d_2 is assumed for each beam and usually a variation from this value of, say, $\pm\frac{1}{8}$ in. has no significant effect on the tabulated data; if d_2 is much greater than is assumed, the data should be adjusted to conform to the actual value.

The data in *Table* 77B is similar to that in *Table* 77A but relates to rectangular beams reinforced with bars having a yield stress of not less than 60,000 lb. per sq. in.

Examples.—(*a*) Design a beam, reinforced with mild steel bars, to resist a bending moment of 1,800,000 in.-lb., if the stress in the concrete is not to exceed 1000 lb. per sq in. Tensile reinforcement only.

(i) By using *Table* 75; formulæ series B apply. (The method in this example applies to any other stresses by using formulæ series A.)

Assume $b = 15$ in. $d_1' = \sqrt{\dfrac{1{,}800{,}000}{250 \times 15}} = 21{\cdot}9$ in. If d is made 24 in., $d_1 = 22\frac{1}{2}$ in. with 1-in. bars and 1-in. cover; therefore $d_1 > d_1'$ and $A_{st} = \dfrac{bd_1}{60}F$. $Q_1 = \dfrac{1{,}800{,}000}{15 \times 22{\cdot}5^2} = 237$; from *Table* 76, $F = 0{\cdot}93$ approx; $A_{st} = \dfrac{15 \times 22{\cdot}5 \times 0{\cdot}93}{60} = 5{\cdot}24$ sq in , say seven 1-in. bars, which can just be accommodated in one layer in a width of 15 in.

If fewer larger bars, say $1\frac{1}{4}$-in. diameter, are provided with $1\frac{1}{4}$-in. cover, $d_1 = 22\frac{1}{8}$ in. Therefore $d_1 > d_1'$, $Q_1 = 245$, $F = 0{\cdot}96$ approx, and $A_{st} = \dfrac{15 \times 22{\cdot}125 \times 0{\cdot}96}{60} = 5{\cdot}32$ sq. in., say, five $1\frac{1}{4}$-in. bars in a beam 24 in. by 15 in.

(ii) By using *Table* 77A. Assume $b = 15$ in.; therefore $M' = \dfrac{M}{b} = \dfrac{1{,}800{,}000}{15} = 120{,}000$ in.-lb. per inch; from the table, a beam 24 in. deep ($M_r' = 122{,}000$ in.-lb. per inch width) is satisfactory; $A_{st} = 0{\cdot}369 \times 15 = 5{\cdot}54$ sq. in., that is, five $1\frac{1}{4}$-in. bars in a beam 24 in. by 15 in [as in (ii)]. (If high-yield stress bars are provided *Table* 77B would be used in a similar manner.)

(*b*) A beam 20 in. deep and 10 in. wide is required to resist a bending moment of 1,200,000 in.-lb. Determine the amount of high-yield-stress reinforcement ($p_{st} = 30{,}000$ lb. per sq. in.) required if higher-grade 1 : 2 : 4 concrete is used ($p_{cb} = 1250$ lb. per sq. in.).

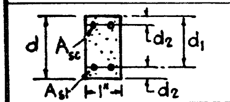
$d = 20$ in.; $d_1 =$ say 18 in.; $d_2 =$ say 2 in.; $b = 10$ in.

From *Table* 76, $Q_c = 313 \left(\text{or} = \dfrac{1250}{4}\right)$; $r_L = 0{\cdot}014$.

(*Continued on page* 282.)

RECTANGULAR BEAMS — MOMENT OF RESISTANCE AND REINFORCEMENT — LOAD-FACTOR METHOD.

MILD STEEL BARS p_{st} = 20,000 LB. PER SQ. IN. p_{sc} = 18,000 LB. PER SQ. IN.



OVERALL DEPTH d IN.	d_2 (ASSUMED) COVER	d_2 (ASSUMED) BAR DIAM.	EFFECTIVE DEPTH (APPROX) d_1 IN.	LOWER GRADE 1:2:4 CONCRETE p_{cb} = 750 LB. PER SQ. IN. — TENSILE ONLY M'_r $187{\cdot}5d_1^2$ IN.-LB.	TENSILE ONLY A'_{st} $d_1/80$ SQ. IN.	TENS. & COMP. M'_r $Q'd_1^2$ IN.-LB.	TENS. & COMP. A'_{st} $\frac{d_1}{80}+\frac{3{\cdot}6d}{100}$ SQ. IN.	ORDINARY GRADE 1:2:4 CONCRETE p_{cb} = 1000 LB PER SQ IN. — TENSILE ONLY M'_r $250d_1^2$ IN.-LB.	TENSILE ONLY A'_{st} $d_1/60$ SQ. IN.	TENS. & COMP. M'_r $Q'd_1^2$ IN.-LB.	TENS. & COMP. A'_{st} $\frac{d_1}{60}+\frac{3{\cdot}6d}{100}$ SQ. IN.	HIGHER GRADE 1:2:4 CONCRETE p_{cb} = 1250 LB. PER SQ. IN. — TENSILE ONLY M'_r $312{\cdot}5d_1^2$ IN.-LB.	TENSILE ONLY A'_{st} $d_1/48$ SQ. IN.	TENS. & COMP. M'_r $Q'd_1^2$ IN.-LB.	TENS. & COMP. A'_{st} $\frac{d_1}{48}+\frac{3{\cdot}6d}{100}$ SQ. IN.	COMPRESSION REINFORCEMENT [MAXIMUM AMOUNT FOR ALL CASES] A'_{sc} = 0·04d SQ. IN.
12			10·5	20,700	0·131	98,900	0·563	27,600	0·175	106,000	0·607	34,500	0·219	112,000	0·651	0·48
14	1 IN.	≯1 IN.	12·5	29,200	0·156	140,000	0·661	39,000	0·208	150,000	0·713	48,800	0·260	160,000	0·765	0·56
15	(MIN.)		13·5	34,100	0·169	163,000	0·709	45,500	0·225	175,000	0·765	57,000	0·281	186,000	0·821	0·60
16	$d_2 = 1\frac{1}{2}$ IN.		14·5	39,400	0·181	189,000	0·757	52,500	0·242	202,000	0·818	65,700	0·302	215,000	0·878	0·64
18			16·5	51,000	0·206	245,000	0·853	68,000	0·275	262,500	0·922	85,000	0·344	279,000	0·991	0·72
20			18·13	61,600	0·227	297,000	0·947	81,300	0·302	316,000	1·022	103,000	0·378	338,000	1·098	0·80
21			19·13	68,600	0·238	330,000	0·994	91,500	0·319	352,000	1·075	114,000	0·399	376,000	1·155	0·84
22	$1\frac{1}{4}$ IN.	≯$1\frac{1}{4}$ IN.	20·13	75,900	0·252	365,000	1·045	101,000	0·336	390,000	1·129	126,000	0·420	416,000	1·213	0·88
24			22·13	91,800	0·277	442,000	1·132	122,000	0·369	467,000	1·234	153,000	0·461	503,000	1·326	0·96
26	$d_2 = 1\frac{7}{8}$ IN.		24·13	109,000	0·302	525,000	1·237	145,000	0·402	562,000	1·337	182,000	0·503	598,000	1·438	1·04
27			25·13	118,000	0·314	571,000	1·285	158,000	0·419	611,000	1·390	198,000	0·523	650,000	1·494	1·08
28			26·13	128,000	0·327	614,000	1·347	170,000	0·436	658,000	1·456	213,000	0·545	698,000	1·565	1·12
30			27·75	144,000	0·347	695,000	1·417	192,000	0·463	743,000	1·533	241,000	0·578	791,000	1·448	1·20
32	$1\frac{1}{2}$ IN.	≯$1\frac{1}{2}$ IN.	29·75	165,000	0·372	795,000	1·522	220,000	0·496	850,000	1·646	276,000	0·620	905,000	1·770	1·28
33			30·75	177,000	0·384	852,000	1·572	236,000	0·513	911,000	1·701	295,000	0·640	970,000	1·820	1·32
34	$d_2 = 2\frac{1}{4}$ IN.		31·75	189,000	0·397	910,000	1·622	252,000	0·529	975,000	1·754	315,000	0·661	1,040,000	1·886	1·36
36			33·75	214,000	0·422	1,060,000	1·717	285,000	0·563	1,110,000	1·857	356,000	0·703	1,182,000	1·998	1·44

M'_r = MOMENT OF RESISTANCE PER INCH WIDTH. A'_{st} = AREA OF TENSILE REINFORCEMENT PER INCH WIDTH. } MINIMUM REQUIRED TO PRODUCE TABULATED M'_r

$Q' = 0{\cdot}25p_{cb} + 720\left[1-\left(\frac{d_2}{d_1}\right)^2\right]$ A'_{sc} = AREA OF COMPRESSION REINFORCEMENT PER INCH WIDTH

DESIGN OF BEAMS: LOAD-FACTOR METHOD.

Design of Rectangular Beams

Examples (*continued from page* 280).

From *Table* 75, $d_1' = \sqrt{\dfrac{1,200,000}{313 \times 10}} = 19{\cdot}6$ in.; therefore $d_1 < d_1'$, and compression reinforcement ($p_{sc} = 23,000$ lb. per sq. in.) is required. $Q_1 = \dfrac{1,200,000}{10 \times 18^2} = 370$.

$$A_{sc} = \frac{(370 - 313)(10 \times 18)}{23,000\left(1 - \dfrac{2}{18}\right)} = 0{\cdot}5 \text{ sq. in., say, two } \tfrac{5}{8}\text{-in. bars in the top.}$$

$A_{st} = (0{\cdot}014 \times 10 \times 18) + \dfrac{0{\cdot}5 \times 23,000}{30,000} = 2{\cdot}88$ sq. in., say, three $1\tfrac{1}{8}$-in. bars in the bottom. Twisted ribbed bars or similar bars would be used.

SLABS FOR LIQUID-CONTAINING STRUCTURES.

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Examples (*continued from page* 274).

Recalculate with $d = 9$ in., and $d_1 = 7$ in ; $d_1 - \tfrac{1}{2}d = 2\tfrac{1}{2}$ in.

$r_0' = 0{\cdot}0077$; $f_{ct} = 195$ lb. per square inch ($\triangleq 190$); satisfactory if A_{st} is slightly greater than $12 \times 0{\cdot}0077 \times 9 = 0{\cdot}83$ sq. in. per foot; provide say 1-in. bars at $10\tfrac{1}{2}$-in. centres at each face (total $= 0{\cdot}898$ sq. in. per foot).

(*c*).—Tensile force of 10,000 lb. per foot and bending moment of 48,000 in.-lb. per foot. Assume $d = 10$ in. and $d_1 = 8$ in.; $\dfrac{d_1}{d} = 0{\cdot}8$; $e = \dfrac{48,000}{10,000} = 4{\cdot}8$ in. ($> 8 - 5 = 3$ in.; therefore Case IIIA *b* applies.)

$$r_0 = \frac{10,000}{144,000 \times 10}\left(1 + \frac{4{\cdot}8 + 5 - 8}{0{\cdot}83 \times 8}\right) = 0{\cdot}0088; \quad 14r_0 = 0{\cdot}122.$$

$$n_0 = \frac{0{\cdot}5 + (0{\cdot}122 \times 0{\cdot}8)}{1{\cdot}122} = 0{\cdot}532.$$

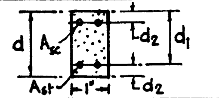
$$f_{ct} = \frac{10,000}{12 \times 10}\left\{\frac{1}{1{\cdot}122} + \frac{4{\cdot}8(1 - 0{\cdot}532)}{[\tfrac{1}{3} - (1 - 0{\cdot}532)0{\cdot}532 + 0{\cdot}122(0{\cdot}8 - 0{\cdot}532)^2]10}\right\}$$

$= 275$ lb. per square inch ($\triangleq 270$); satisfactory if A_{st} is slightly greater than $12 \times 0{\cdot}0088 \times 10 = 1{\cdot}06$ sq. in. per foot; provide 1-in. bars at 8-in. centres ($1{\cdot}178$ sq. in.) at tensile face.

Note.—For designs in Case IIIB in *Table* 74, the procedure is as in the foregoing if $d < 9$ in., and as for ordinary designs if $d \nless 9$ in.

RECTANGULAR BEAMS. MOMENT OF RESISTANCE AND REINFORCEMENT — LOAD-FACTOR METHOD.

HIGH-YIELD-STRESS BARS. p_{st} = 30,000 LB. PER SQ. IN. p_{sc} = 23,000 LB. PER SQ. IN.



OVERALL DEPTH d IN.	d_2 (ASSUMED) COVER	d_2 (ASSUMED) BAR DIAM.	EFFECTIVE DEPTH (APPROX.) d_1 IN.	LOWER GRADE 1:2:4 CONCRETE p_{cb} = 750 LB. PER SQ. IN. — TENSILE ONLY M'_r $187{\cdot}5d_1^2$ IN.-LB.	TENSILE ONLY A'_{st} $d_1/120$ SQ. IN.	TENS. & COMP. M'_r $Q'd_1^2$ IN.-LB.	TENS. & COMP. A'_{st} $\frac{d_1}{120}+\frac{3{\cdot}07d}{100}$ SQ. IN.	ORDINARY GRADE 1:2:4 CONCRETE p_{cb} = 1000 LB. PER SQ. IN. — TENSILE ONLY M'_r $250d_1^2$ IN.-LB.	TENSILE ONLY A'_{st} $d_1/90$ SQ. IN.	TENS. & COMP. M'_r $Q'd_1^2$ IN.-LB.	TENS. & COMP. A'_{st} $\frac{d_1}{90}+\frac{3{\cdot}07d}{100}$ SQ. IN.	HIGHER GRADE 1:2:4 CONCRETE p_{cb} = 1250 LB. PER SQ. IN. — TENSILE ONLY M'_r $312{\cdot}5d_1^2$ IN.-LB.	TENSILE ONLY A'_{st} $d_1/72$ SQ. IN.	TENS. & COMP. M'_r $Q'd_1^2$ IN.-LB.	TENS. & COMP. A'_{st} $\frac{d_1}{72}+\frac{3{\cdot}07d}{100}$ SQ. IN.	COMPRESSION REINFORCEMENT [MAXIMUM AMOUNT FOR ALL CASES] A'_{sc} = 0·04d SQ. IN.
12	1 IN. (MIN.)	3/4 IN.	10·63	21,200	0·089	124,000	0·457	28,200	0·118	130,500	0·486	35,400	0·148	137,500	0·516	0·48
14			12·63	30,000	0·105	175,500	0·535	40,000	0·140	185,000	0·570	50,000	0·175	196,000	0·605	0·56
15	d_2 = 1 3/8 IN.		13·63	34,900	0·114	214,000	0·575	46,500	0·151	216,000	0·612	58,100	0·190	228,000	0·651	0·60
16			14·63	40,300	0·122	236,500	0·612	53,800	0·163	250,000	0·653	67,200	0·203	264,000	0·693	0·64
18			16·63	52,000	0·139	306,000	0·691	69,200	0·185	322,000	0·737	86,500	0·232	340,000	0·784	0·72
20	1 IN.	1 IN.	18·50	64,100	0·154	377,000	0·767	85,500	0·206	398,000	0·819	108,000	0·257	418,000	0·870	0·80
21			19·50	71,200	0·163	419,000	0·808	95,000	0·217	443,000	0·862	119,000	0·271	466,000	0·916	0·84
22			20·50	79,600	0·172	463,000	0·847	105,000	0·228	489,000	0·903	131,000	0·285	515,000	0·960	0·88
24	d_2 = 1 1/2 IN.		22·50	95,000	0·188	559,000	0·823	126,700	0·250	590,000	0·985	158,000	0·312	622,000	1·047	0·96
26			24·50	112,000	0·204	663,000	1·002	150,000	0·272	700,000	1·070	187,000	0·340	737,000	1·138	1·04
27			25·50	122,000	0·213	718,000	1·042	162,000	0·283	757,000	1·112	207,000	0·354	798,000	1·183	1·08
28			26·50	131,000	0·221	774,000	1·080	175,000	0·294	815,000	1·153	219,000	0·368	860,000	1·227	1·12
30	1 1/4 IN.	1 1/4 IN.	28·13	148,000	0·234	872,000	1·154	197,000	0·312	920,000	1·232	248,000	0·391	970,000	1·311	1·20
32			30·13	171,000	0·251	1,005,000	1·232	227,000	0·335	1,060,000	1·316	284,000	0·419	1,118,000	1·400	1·28
33	d_2 = 1 7/8 IN.		31·13	181,000	0·259	1,007,000	1·269	242,000	0·346	1,130,000	1·356	303,000	0·433	1,190,000	1·443	1·32
34			32·13	194,000	0·268	1,145,000	1·310	259,000	0·357	1,210,000	1·399	323,000	0·447	1,275,000	1·489	1·36
36			34·13	222,000	0·284	1,305,000	1·387	270,000	0·379	1,375,000	1·482	370,000	0·475	1,450,000	1·578	1·44

M'_r = MOMENT OF RESISTANCE PER INCH WIDTH. $Q' = 0{\cdot}25p_{cb} + 920\left[1-\left(\frac{d_2}{d_1}\right)^2\right]$

A'_{st} = AREA OF TENSILE REINFORCEMENT PER INCH WIDTH. A'_{sc} = AREA OF COMPRESSION REINFORCEMENT PER INCH WIDTH. } MINIMUM REQUIRED TO PRODUCE TABULATED M'_r

DESIGN OF SOLID SLABS: LOAD-FACTOR METHOD.

Solid Slabs.—Solid slabs can be designed as rectangular beams of 1 ft. width if the applied bending moment is in inch-pounds per foot width of slab and A_{st} is the area of reinforcement in square inches per foot width. The basic formulæ in series A in *Table* 75 apply directly to solid slabs if 12 in. is substituted for b. Similarly the formulæ in series D for mild steel bars and series E for high-yield-stress bars or wire are obtained from formulæ in series B and C respectively by substituting $b = 12$ in. No formulæ are given for solid slabs with compression reinforcement as such slabs are uncommon; any case of such a slab can be designed directly from the basic formulæ.

In *Table* 78 are given the moments of resistance and areas of reinforcement in tension for solid slabs 1 ft. wide and of various thicknesses reinforced with mild steel bars not greater than $1\frac{1}{2}$ in. diameter or with high-yield-stress bars.

Examples.

(*a*) Design a solid slab to resist a bending moment of 27,000 in.-lb. per ft. width; maximum stresses $p_{st} = 18{,}000$ lb. and $p_{cb} = 1000$ lb. per sq. in.

From *Table* 76, $Q_C = 250$. From formula in series D in *Table* 75,

$d_1' = \sqrt{\dfrac{27{,}000}{3000}} = 3{\cdot}0$ in.; that is 4-in. slab ($\frac{1}{2}$-in. cover); $d_1 = 3{\cdot}25$ in. From *Table* 76, $r_L = 0{\cdot}019$; $A_{st} = 0{\cdot}019 \times 3{\cdot}0 \times 12 = 0{\cdot}68$ sq. in.; say, $\frac{1}{2}$-in. mild steel bars at $3\frac{1}{2}$-in. centres

(*b*) Repeat example (*a*) by using *Table* 78 if maximum stresses are $p_{st} = 20{,}000$ lb. and $p_{sc} = 1000$ lb. per sq. in.

For $M_r = 27{,}000$ in.-lb., a 4-in. slab is required with 0·600 sq. in. (say, $\frac{3}{8}$-in. mild steel bars at 6-in. centres).

(*c*) Calculate the moment of resistance of a 4-in. slab reinforced with $\frac{1}{2}$-in. bars at $4\frac{1}{2}$-in. centres. $p_{st} \not> 18{,}000$ lb. and $p_{cb} \not> 1000$ lb. per sq. in.

$d_1 = 3{\cdot}25$ in. $A_{st} = 0{\cdot}524$ sq. in. $r_t = \dfrac{0{\cdot}524}{12 \times 3{\cdot}25} = 0{\cdot}0134$. $r_L = 0{\cdot}019$ (*Table* 76); therefore $r_t < r_L$.

$M_r = Q_T b d_1^2$ (see page facing *Table* 75). F_1 (*Table* 76) $= 14$.

$Q_T = 0{\cdot}0134 \times 18{,}000\,[1 - (14 \times 0{\cdot}0134)] = 196$.

$M_r = 196 \times 12 \times 3{\cdot}25^2 = 24{,}600$ in.-lb. per ft. width.

(*d*) Repeat example (*c*) by using *Table* 75 if $p_{st} \not> 20{,}000$ lb. per sq. in.

As before, $d_1 = 3{\cdot}25$ in., $r_t = 0{\cdot}0134$ with $A_{st} = 0{\cdot}524$ sq. in. per ft.

From *Table* 75, $r_L = \dfrac{1}{60} = 0{\cdot}0167$ which is greater than r_t; therefore tension controls and, from formula in series D,

$$M_r = 20{,}000 \times 0{\cdot}524[3{\cdot}25 - (1{\cdot}25 \times 0{\cdot}524)] = 25{,}900 \text{ in.-lb. per ft.}$$

DESIGN OF BEAMS: LOAD-FACTOR METHOD.

Flanged Beams.—Flanged beams include tee-beams, ell-beams, I-beams, inverted channels, and the like. The moment of resistance of a flanged beam is $A_{st}p_{st}(d_1 - 0{\cdot}5d_s)$ if based on the tensile resistance, and $\gamma p_{cb} b d_1^2$ if based on the compressive resistance, where the coefficient γ is $\dfrac{b_r}{4b} + \dfrac{1}{3}\left(1 - \dfrac{b_r}{b}\right)\left[2\dfrac{d_s}{d_1} - \left(\dfrac{d_s}{d_1}\right)^2\right]$. If the beam contains compression reinforcement the moment of resistance is $\gamma p_{cb} b d_1^2 + A_{sc}p_{sc}(d_1 - d_2)$. As in the case of rectangular beams, these expressions are simplified to give the formulæ in *Table* 79 (corresponding to those for rectangular beams in *Table* 75).

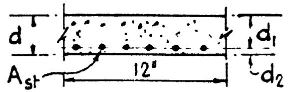
If $4\gamma\dfrac{b}{b_r} = K$, $s' = \dfrac{d_s}{d_1}$, and $Q_c = \dfrac{p_{cb}}{4}$ (as before),

$$K = 1 + \frac{4s'}{3}\left(\frac{b}{b_r} - 1\right)(2 - s')$$

and therefore, with tensile reinforcement only $M_r = KQ_c b_r d_1^2$. This formula applies when the compressive resistance of the concrete controls. If the tensile resistance of the reinforcement controls, $M_r = A_{st}p_{st}d_1(1 - 0{\cdot}5s')$.

(*Continued on page* 286.)

SOLID SLABS. MOMENT OF RESISTANCE AND REINFORCEMENT. LOAD-FACTOR METHOD.



THICKNESS	MILD STEEL BARS		HIGH-YIELD-STRESS BARS (OR WIRE)		LOWER GRADE 1:2:4 CONCRETE p_{cb} = 750 LB PER SQ IN.				ORDINARY GRADE 1:2:4 CONCRETE p_{cb} = 1000 LB PER SQ IN.				HIGHER GRADE 1:2:4 CONCRETE p_{cb} = 1250 LB. PER SQ.IN.			
					MILD STEEL BARS p_{st} = 20,000 LB. PER SQ.IN.		HIGH-YIELD-STRESS BARS OR WIRE p_{st} = 30,000 LB PER SQ IN.		MILD STEEL BARS p_{st} = 20,000 LB. PER SQ.IN.		HIGH-YIELD-STRESS BARS OR WIRE p_{st} = 30,000 LB. PER SQ.IN.		MILD STEEL BARS p_{st} = 20,000 LB. PER SQ IN		HIGH-YIELD-STRESS BARS OR WIRE p_{st} = 30,000 LB. PER SQ.IN.	
d IN.	COVER PLUS $\frac{1}{2}$(BAR DIA.) d_2	EFFECTIVE DEPTH d_1 IN.	COVER PLUS $\frac{1}{2}$(BAR DIA.) d_2	EFFECTIVE DEPTH d_1 IN.	M_r $2250d_1^2$ IN.-LB.	A_{st} $0{\cdot}15d_1$ SQ IN.	M_r $2250d_1^2$ IN.-LB.	A_{st} $0{\cdot}10d_1$ SQ.IN.	M_r $3000d_1^2$ IN.-LB.	A_{st} $0{\cdot}20d_1$ SQ.IN.	M_r $3000d_1^2$ IN.-LB.	A_{st} $0{\cdot}133d_1$ SQ.IN.	M_r $3750d_1^2$ IN.-LB.	A_{st} $0{\cdot}25d_1$ SQ.IN.	M_r $3750d_1^2$ IN.-LB.	A_{st} $0{\cdot}167d_1$ SQ.IN.
3	$\not<\frac{3}{4}$ IN.	2·25	$\not<\frac{3}{4}$ IN.	2·25	11,400	0·338	11,400	0·225	15,200	0·450	15,200	0·300	19,000	0·563	19,000	0·375
$3\frac{1}{2}$		2·75		2·75	17,000	0·413	17,000	0·275	22,600	0·550	22,600	0·367	28,400	0·688	28,400	0·485
4	$\not<$ 1 IN	3·00	$\not<\frac{3}{4}$ IN.	3·25	20,300	0·450	23,800	0·325	27,000	0·600	31,800	0·433	33,800	0·750	39,600	0·543
$4\frac{1}{2}$		3·50		3·75	27,600	0·525	31,600	0·375	36,800	0·700	42,200	0·500	45,900	0·875	52,600	0·625
5		4·00		4·25	36,000	0·600	40,600	0·425	48,000	0·800	54,000	0·567	60,000	1·000	67,500	0·708
$5\frac{1}{2}$		4·50		4·75	46,600	0·675	50,800	0·475	60,800	0·900	67,500	0·634	76,000	1·125	84,500	0·792
6	$\not<1\frac{1}{8}$ IN	4·88	$\not<$ 1 IN.	5·00	54,800	0·731	56,400	0·500	71,600	0·975	75,000	0·667	89,500	1·220	93,600	0·833
7		5·88		6·00	78,100	0·881	81,100	0·600	104,100	1·175	108,000	0·800	130,000	1·470	135,000	1·000
8		6·88		7·00	106,800	1·031	110,500	0·700	142,200	1·375	147,000	0·935	177,500	1·720	184,000	1·167
9	$\not<1\frac{1}{2}$ IN.	7·50	$\not<1\frac{1}{8}$ IN.	7·88	126,300	1·125	139,500	0·788	168,600	1·500	186,000	1·050	210,000	1·875	232,000	1·313
10		8·50		8·88	162,600	1·275	177,500	0·888	216,600	1·700	236,400	1·185	270,000	2·125	295,000	1·479
11		9·50		9·88	204,000	1·425	220,500	0·988	271,500	1·900	293,400	1·320	339,000	2·375	366,000	1·646
12		10·50		10·88	248,000	1·575	268,000	1·088	331,500	2·100	354,000	1·450	413,000	2·625	442,000	1·813
14	$\not<1\frac{7}{8}$ IN.	12·13	$\not<1\frac{1}{2}$ IN.	12·50	332,000	1·819	351,500	1·250	442,500	2·425	468,000	1·667	553,000	3·033	585,000	2·083
15		13·13		13·50	380,000	1·969	410,000	1·350	519,000	2·625	546,000	1·800	649,000	3·283	682,000	2·250
16		14·13		14·50	450,000	2·119	474,000	1·450	600,000	2·825	630,000	1·930	750,000	3·533	786,000	2·417
18		16·13		16·50	588,000	2·419	613,000	1·650	783,000	3·225	816,000	2·200	978,000	4·033	1,020,000	2·750

COVER OF CONCRETE = DIAM. OF BAR (OR WIRE) AND $\not<\frac{1}{2}$ IN.

M_r = MOMENT OF RESISTANCE PER FOOT WIDTH. A_{st} = AREA OF TENSILE REINFORCEMENT PER FOOT WIDTH. (MINIMUM REQUIRED TO PROVIDE TABULATED M_r)

DESIGN OF BEAMS: LOAD-FACTOR METHOD

(continued from page facing Table 78).

The limiting proportion r_L of tensile reinforcement $\left(\dfrac{A_{st}}{b_r d_1}\right)$, which determines which condition applies, is given by $\dfrac{KQ_c}{p_{st}(1 - 0{\cdot}5s')}$

For a flanged beam with compression reinforcement

$$M_r = KQ_c b_r d_1^2 + A_{sc} p_{sc}(d_1 - d_2)$$

which applies if there is sufficient tensile reinforcement to produce at least this resistance; the minimum A_{st} required for this purpose is $\dfrac{KQ_c b_r d_1^2}{p_{st}(d_1 - 0{\cdot}5d_s)} + \dfrac{A_{sc} p_{sc}}{p_{st}}$.

The basic formulæ are given in series F in *Table* 79, in which also are given values of K for various values of the ratio of b to b_r and of s'; intermediate values can be interpolated or can be calculated from the formula given at the bottom of *Table* 79.

The dimensions of a flanged beam are generally determined from considerations other than resistance to bending; resistance to shearing may determine the size of the rib, and the thickness d_s of the slab may be determined by its span and loading. The dimensions d_1, d_s, b and b_r are therefore known, or can be assumed, and the amount of tensile reinforcement required to resist the applied bending moment is calculated from the basic and special formulæ for A_{st} in *Table* 79. The compressive resistance can be checked by comparing the value of K corresponding to the known value of b/b_r and of s' with the minimum value required, that is Q_1/Q_c in which $Q_1 = \dfrac{M}{b_r d_1^2}$; if K is less than this value, compression reinforcement is required or the dimensions of the cross-section of the beam must be increased.

Compression reinforcement may be required in tee-beams and other special beams; if $d_s < \frac{1}{2}d_1$, the amounts of compression reinforcement and the corresponding amount of tensile reinforcement required are given by the formulæ for A_{sc} and A_{st} respectively in *Table* 79.

Examples.—(*a*) Calculate the moment of resistance of a tee-beam the rib of which is 10 in. wide and 18 in. deep below the soffit of a 6-in. slab. The span is 18 ft. and the distance between adjacent supporting beams is 8 ft. The tensile reinforcement comprises four 1-in. mild steel bars ($p_{st} = 20{,}000$ lb. per sq. in.) and the concrete is ordinary grade 1 : 2 : 4 ($p_{cb} = 1000$ lb. per sq. in.). $A_{sc} = 0$.

The special formulæ in series G in *Table* 79 can be used. $A_{st} = 3{\cdot}14$ sq. in.

Effective breadth: Available slab = 96 in., or $\frac{1}{3}$ span = 72 in., or $12d_s + b = (12 \times 6) + 10 = 82$ in.; hence maximum $b_s = 72$ in ; effective depth $d_1 = (18 + 6 - 1\frac{1}{2}) = 22\frac{1}{2}$ in. $\dfrac{b}{b_r} = \dfrac{72}{10} = 7{\cdot}2$; $s' = \dfrac{6}{22{\cdot}5} = 0{\cdot}27$. From *Table* 79, $K = 4\frac{3}{4}$ approx., or by calculation $K = 1 + \dfrac{4 \times 0{\cdot}27}{3}(7{\cdot}2 - 1)(2 - 0{\cdot}27) = 4{\cdot}85$. $Q_c = \dfrac{1000}{4} = 250$.

$r_L = \dfrac{250 \times 4{\cdot}85}{20{,}000[1 - (\frac{1}{2} \times 0{\cdot}27)]} = 0{\cdot}07$. $\dfrac{A_{st}}{b_r d_1} = \dfrac{3{\cdot}14}{10 \times 22{\cdot}5} = 0{\cdot}014$, which is less than r_L; therefore tension controls and

$M_r = 20{,}000 \times 3{\cdot}14(22{\cdot}5 - 3) = 1{,}225{,}000$ in.-lb. (This is the same moment of resistance as if calculated by the modular-ratio method.)

(*b*) Design the tee-beam in (*a*) using high-yield-stress steel ($p_{st} = 30{,}000$ lb. per sq. in.) to resist a bending moment of 3,000,000 in.-lb.

As in (*a*), $\dfrac{b}{b_r} = 7{\cdot}2$ but, since a greater amount of tensile reinforcement may be required, d_1 should be decreased to, say, 22 in., and $s' = \dfrac{6}{22} = 0{\cdot}27$ as before, and $K = 4{\cdot}85$ and $Q_c = 250$. $Q_1 = \dfrac{3{,}000{,}000}{10 \times 22^2} = 620$, $\dfrac{Q_1}{Q_c} = \dfrac{620}{250} = 2{\cdot}5$, which is less than K_1; therefore compression steel is not required. Therefore $A_{st} = \dfrac{3{,}000{,}000}{30{,}000(22 - 3)} = 5{\cdot}26$ sq. in., say, seven 1-in. twisted ribbed bars, in two layers ($d_1 = 21\frac{3}{4}$ in. approx. compared with 22 in. as assumed).

FLANGED BEAMS — FORMULÆ FOR LOAD-FACTOR METHOD.

BASIC FORMULÆ

p_{st} = TENSILE STRESS IN BARS.

p_{sc} = COMPRESSIVE STRESS IN BARS $\not> 50{,}000\left(1-2\frac{d_2}{d_1}\right)$

p_{cb} = COMPRESSIVE STRESS IN CONCRETE.

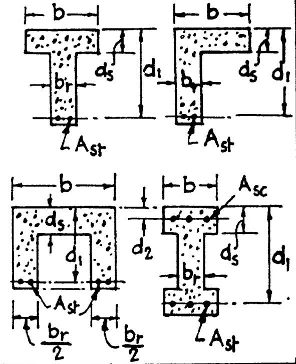
ORDINARY GRADE 1 : 2 : 4 CONCRETE, $p_{cb} = 1000$ LB. PER SQ IN

			SERIES F	SERIES G: MILD STEEL BARS (DIAM NOT GREATER THAN $1\frac{1}{2}$ IN), p_{st} = 20,000 LB./SQ IN., p_{sc} = 18,000 LB./SQ IN.	SERIES H: HIGH-YIELD-STRESS BARS (ALL SIZES), p_{st} = 30,000 LB./SQ.IN., p_{sc} = 23,000 LB./SQ.IN.
MOMENT OF RESISTANCE M_r	TENSILE REINF'T ONLY	$\frac{A_{st}}{b_r d_1} < r_L$	$M_r = A_{st} p_{st} d_1 (1 - s'/2)$	$20{,}000 A_{st}\left(d_1 - \frac{d_s}{2}\right)$	$30{,}000 A_{st}\left(d_1 - \frac{d_s}{2}\right)$
		$> r_L$	$M_r = K Q_c b_r d_1^2$	$250 K b_r d_1^2$	$250 K b_r d_1^2$
	WITH COMPRESSION REINFORCEMENT		$M_r = K Q_c b_r d_1^2 + A_{sc} p_{sc}(d_1 - d_2)$ $\left[A_{st} \not< r_L b_r d_1 + \frac{p_{sc}}{p_{st}} A_{sc}\right]$	$250 K b_r d_1^2 + 18{,}000 A_{sc}(d_1 - d_2)$ $\left[A_{st} \not< r_L b_r d_1 + 0{\cdot}9 A_{sc}\right]$	$250 K b_r d_1^2 + 23{,}000 A_{sc}(d_1 - d_2)$ $\left[A_{st} \not< r_L b_r d_1 + 0{\cdot}77 A_{sc}\right]$
DESIGN FORMULÆ	TENSILE REINF'T ONLY	A_{st}	$\frac{M}{p_{st} d_1 (1 - s'/2)}$	$\frac{M}{20{,}000\left(d_1 - \frac{d_s}{2}\right)}$	$\frac{M}{30{,}000\left(d_1 - \frac{d_s}{2}\right)}$
	COMPRESSION REINF'T REQUIRED IF $K < \frac{Q_1}{Q_c}$	A_{st}	$r_L b_r d_1 + \frac{p_{sc}}{p_{st}} A_{sc}$	$r_L b_r d_1 + 0{\cdot}9 A_{sc}$	$r_L b_r d_1 + 0{\cdot}77 A_{sc}$
		A_{sc}	$\frac{(Q_1 - K Q_c) b_r d_1^2}{p_{sc}(d_1 - d_2)}$	$\frac{(Q_1 - 250K) b_r d_1^2}{18{,}000(d_1 - d_2)}$	$\frac{(Q_1 - 250K) b_r d_1^2}{23{,}000(d_1 - d_2)}$

$$Q_c = \frac{p_{cb}}{4} \qquad Q_1 = \frac{M}{b_r d_1^2}; \qquad \frac{d_s}{d_1} = s'$$

$$r_L = \frac{Q_c K}{p_{st}(1 - s'/2)} \qquad K = 1 + \frac{4s'}{3}\left(\frac{b}{b_r} - 1\right)(2 - s')$$

FORMULÆ IN SERIES (G) AND (H) INCORPORATING p_{sc} = 18,000 OR 23,000 LB. PER SQ.IN. APPLY ONLY IF $\frac{d_2}{d_1} \not> 0{\cdot}32$ FOR MILD STEEL BARS OR $\not> 0{\cdot}17$ FOR HIGH-YIELD-STRESS BARS.



VALUES OF K

$\frac{b}{b_r}$	$s' = d_s/d_1$: 0·1	0·2	0·3	0·4	0·5 OR MORE
1 (RECT.)	(1·0)	(1·0)	(1·0)	(1·0)	1·0
2	1·2	1·5	1·7	1·9	2·0
3	1·5	2·0	2·4	2·7	3·0
4	1·7	2·4	3·0	3·6	4·0
5	2·0	2·9	3·7	4·4	5·0
6	2·2	3·4	4·4	5·3	6·0
8	2·7	4·4	5·8	7·0	8·0
10	3·2	5·3	7·1	8·7	10·0
12	3·8	6·3	8·5	9·4	12·0
15	4·6	7·7	10·5	13·0	15·0

RESISTANCE TO SHEARING FORCE.

Shearing Stresses.—The shearing stress q lb. per sq. in. in a member subject to a bending moment M (in.-lb.) and a shearing force Q (lb.) is determined from

$$q = \frac{Q \pm \frac{M}{d_1}\tan\alpha}{bl_a},$$

where l_a is the lever arm of the member (in.), b is the breadth of a rectangular beam or the breadth of the rib of a flanged beam (in.), d_1 is the effective depth (in.), and α is the angle between the top and bottom edges of a beam. The negative sign applies when M increases as d_1 increases, a condition that occurs at a haunch adjacent to an interior support of a continuous beam. The positive sign applies when M decreases as d_1 increases, a condition that occurs at a haunch adjacent to a support where a beam is freely supported.

When a beam has parallel edges, as in the common case of a beam of uniform depth,

$$q = \frac{Q}{bl_a}.$$

Reinforcement.—The general rule is that if q exceeds the safe shearing stress, such as given in *Tables* 56 and 57, the whole of the shearing force should be resisted by reinforcement in the form of binders or inclined bars or both. If q does not exceed the safe stress, reinforcement to resist shearing is in general not necessary, but the comments on page 72 should be noted.

Binders.—The shearing resistance of vertical binders is given by

$$\frac{A_{st}p_{st}l_a}{s} = Vl_a$$

in which A_{st} = cross-sectional area (sq. in.) of the binder taking into account the number of vertical arms, p_{st} = permissible tensile stress in the binders (lb. per sq. in.), l_a = lever-arm of the member (in.), and s = pitch or spacing of the binders (in.). The factor for the resistance to shearing force, taking into account the diameter of the binders, the allowable stress, and the pitch, is $V = \frac{A_{st}p_{st}}{s}$. Values of V for 18,000 lb. and 20,000 lb. per sq. in. for various spacings and sizes of binders with two arms are given in *Table* 81. The shearing resistance of a system of binders is found by multiplying the appropriate value of V by the lever-arm of the member.

The spacing of binders to resist a shearing force Q is given by

$$s = \frac{A_{st}p_{st}l_a}{Q}$$

and should not exceed the lever arm; if the calculated value of s exceeds l_a, the diameter of the binders can be reduced until a suitable pitch is attained. The minimum diameter of bar suitable for binders is $\frac{3}{16}$ in. for cast-insitu construction although $\frac{1}{8}$-in. steel wire is sometimes used in small precast products; bars over $\frac{1}{2}$ in. diameter are generally difficult and costly to provide satisfactorily. Although binders may not be required to resist shearing force they are always provided except perhaps in simple lintels, and the maximum spacing should not exceed the effective depth. When compression reinforcement equal in area to the tension steel is provided in a beam and the calculation for resistance moment is based on the "steel-beam" theory, the pitch of the binders should not exceed eight times the diameter of the compression bars, and the diameter of the binders should be calculated accordingly. If the concrete is not ignored in calculating the compressive resistance, the binders should be spaced at distances not exceeding twelve times the diameter of the compression bars.

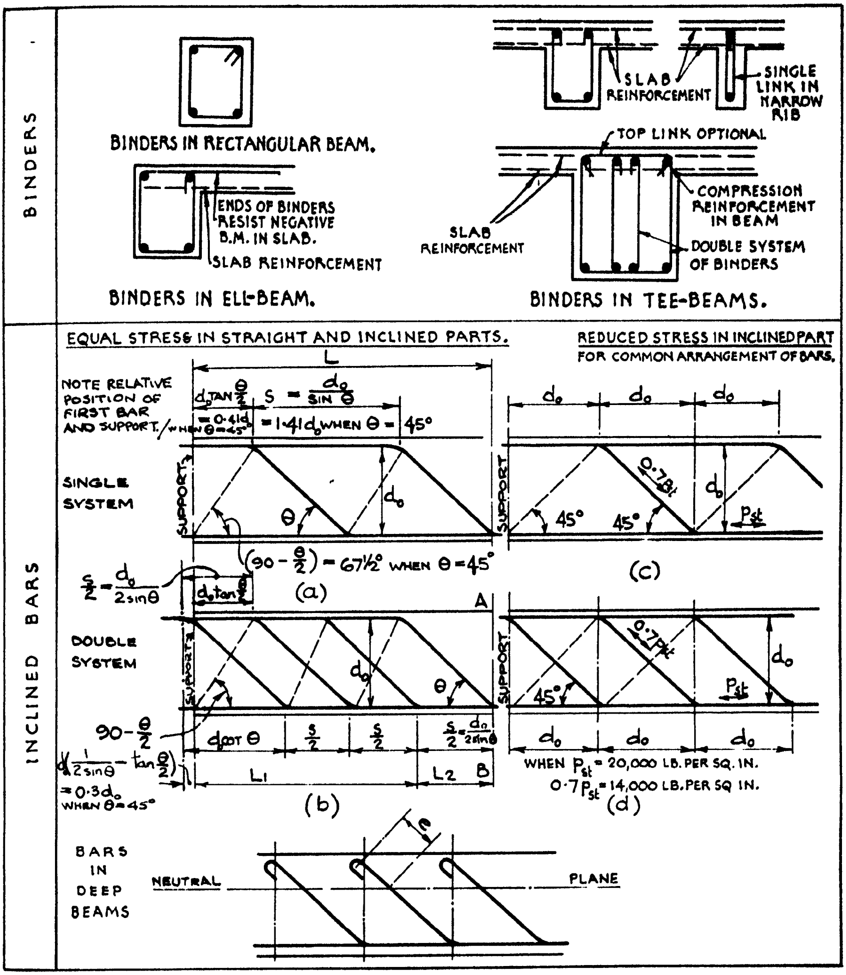
All binding should be effectively anchored to the top bars in a beam. A number of shapes of binders commonly used are illustrated in *Table* 80; the arrangement shown for large tee-beams allows the binders to be open at the top until the reinforcement has been fixed, and, if the top link is provided, is also suitable for heavily-reinforced rectangular beams.

Inclined Bars.—Various arrangements of inclined bars are shown in *Table* 80. With a single system of inclined bars arranged as at (*a*) the shearing resistance Q at any vertical section in the length L is $A_{st}p_{st}\sin\theta$ for a single system; when θ is 45 deg., $Q = 0{\cdot}707A_{st}p_{st}$, where A_{st} = cross-sectional area of the inclined bars (sq. in.), p_{st} = maximum allowable tensile stress (lb. per sq. in.), and θ = angle made by the bar with the horizontal.

For a double system, as (*b*), the shearing resistance of any vertical section within the length L_1 is $2A_{st}p_{st}\sin\theta$ and, when θ is 45 deg. $Q = 1{\cdot}414A_{st}p_{st}$. The same resistance can be considered to be provided for the length L_2 if the shearing stress in the concrete at section AB does not exceed the permissible shearing stress without reinforcement.

(*Continued on page* 289.)

ARRANGEMENT OF BINDERS AND BARS.



RESISTANCE TO SHEARING FORCE (*continued*).

If the inclined bars are spaced in either of the ways shown at (*a*) and (*b*), the stress in the inclined part of the bar is equal to that in the straight bottom part of the bar, but if the common arrangement shown at (*c*) and (*d*) for bars inclined at 45 deg. is adopted, the stress in the inclined part cannot exceed the proportion of the stress in the straight part stated on the diagram. Inclined bars must be anchored sufficiently beyond the intersection with the neutral plane to enable the stress to be developed. If a bar finishes at the top of the bend as at (*e*), the allowable stress should not exceed that capable of being developed by the bond on the length *e*; thus this type of bar is almost valueless except in deep freely-supported beams.

(*Continued on page* 290)

RESISTANCE TO SHEARING FORCE (*continued from page* 289).

In *Table* 81 are tabulated the resistances to shearing force of bars from $\frac{1}{2}$ in. to $1\frac{1}{2}$ in. diameter inclined at 45 deg. and 30 deg. for single and double systems at stresses of 20,000 lb. and 18,000 lb. per sq. in. For other stresses, the resistances to shearing force are proportional to the stresses in the inclined bars.

Design Procedure.—To calculate the shearing resistance at any section, the value of V for the binders is read from *Table* 81 and by multiplying by the lever-arm the shearing resistance of the binders is obtained. The resistance of the inclined bars is also read from the table. An inspection of the arrangement of the inclined bars will indicate whether a single, double, or a more complex system is provided. The total resistance due to the reinforcement is the sum of the resistance of the binders and the inclined bars. The resistance of the concrete is the product of the lever-arm and the breadth multiplied by the safe shearing stress. The higher of the two resistances (that is, the resistance of the reinforcement or of the concrete) represents the safe resistance of the member.

To design a member, first evaluate $\frac{Q}{bl_a}$ and, if less than q in *Table* 56 or 57, no reinforcement is required to resist shearing force, but nominal reinforcement should be provided. If it is greater than q decide from inspection whether the reinforcement shall consist of binders or inclined bars, or both. If binders alone, then $\frac{Q}{l_a}$ gives the value of V required, and a suitable pitch and diameter can be selected from *Table* 81. If inclined bars form the principal reinforcement there would generally be some binders, if only a nominal amount, and the value of V for these binders can be found from the table. Inclined bars should then be provided to resist a shearing force of $Q - Vl_a$. If both inclined bars and binders are provided, the procedure is to combine and adjust the values of Vl_a and $Q - Vl_a$ to suit the member. This is often done by deciding which bars can be bent up from the bottom of the beam and adding extra inclined bars if necessary; binders are then provided to make up the deficiency of shearing resistance.

In important beams it is advisable to plot the shearing-resistance diagram for the entire beam on the same base and to the same scale as the shearing-force diagram, and be assured that the resistance exceeds the force. For most beams it is generally sufficient to determine the point at which no reinforcement to resist shearing force is required and calculate the reinforcement required at the point of maximum shearing force. Between these two points the reinforcement can generally be allocated by judgment.

Examples.—(*a*) Design the reinforcement for the sections specified to resist a shearing force of 30,000 lb. Allowable stresses in inclined bars and binders, 20,000 lb. per sq. in., maximum shearing stress in concrete 100 lb. per sq. in. without reinforcement. Lever-arm of all sections, $0{\cdot}85 \times$ effective depth.

(i) Effective depth = 40 in., breadth = 15 in.—$q = \frac{30{,}000}{0{\cdot}85 \times 40 \times 15} = 59$ lb. per sq. in.

Therefore no reinforcement to resist shearing is required, but nominal binders throughout and one inclined bar at each end of the beam might be advisable.

(ii) Effective depth = 30 in., breadth = 12 in.—$q = \frac{30{,}000}{0{\cdot}85 \times 30 \times 12} = 98$ lb. per sq. in.

This is a border-line case and nominal binders and an inclined bar should be provided as a precaution against accidental overloading, or to offset any other difference between the assumptions in the calculations and actual conditions.

(iii) Effective depth = 20 in., breadth = 10 in.—$q = \frac{30{,}000}{0{\cdot}85 \times 20 \times 10} = 178$ lb. per sq. in.

Therefore all the shearing force should be resisted by reinforcement $l_a = 0{\cdot}85 \times 20 = 17$ in. Nominal binding, say, $\frac{5}{16}$-in. at 12-in. centres ($V = 256$) resists $17 \times 256 = 4350$ lb., leaving 25,650 lb. to be resisted by the inclined bars. If $1\frac{1}{8}$-in. bars are available for bending up, the resistance required is given by one $1\frac{1}{8}$-in. bar bent up at 45 deg. (double system).

(*b*) Calculate the resistance to shearing of the section specified in (iii) if reinforced with one 1-in. bar bent up at 45 deg. (in double system) and $\frac{3}{8}$-in. single binders at 12-in. centres; maximum stress of 20,000 lb. per sq. in. in inclined bars and binders.

Resistance of one 1-in. bar at 45 deg. (double system)	= 22,200 lb.
„ „ $\frac{3}{8}$-in. single binders at 12-in. centres ($V = 368$) $= 17 \times 368$	= 6,250 „
Total	= 28,450 „

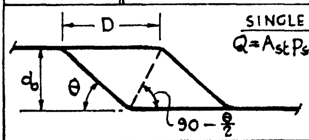
The resistance to shearing of the concrete alone $= 100 \times 17 \times 10 = 17{,}000$ lb.
Hence the safe shearing resistance is that of the reinforcement and is 28,450 lb.

RESISTANCE OF BINDERS AND BARS.

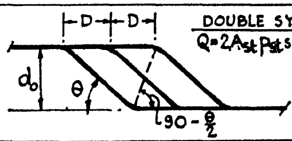
	TENSILE STRESS		18,000 LB. PER SQ.IN.						20,000 LB. PER SQ.IN.					
	DIA. OF BAR FORMING BINDER		3/16"	1/4"	5/16"	3/8"	7/16"	1/2"	3/16"	1/4"	5/16"	3/8"	7/16"	1/2"
BINDERS	VALUES OF V (LB. PER IN.)	2"	496	882	1,386	1,980	2,700	3,530	551	980	1,540	2,210	3,010	3,930
		3"	331	587	924	1,320	1,800	2,350	368	654	1,025	1,474	2,005	2,620
		4"	248	441	693	990	1,350	1,765	280	490	770	1,105	1,505	1,965
		4½"	221	391	616	880	1,200	1,567	245	435	684	982	1,335	1,745
		5"	198	355	554	792	1,080	1,412	220	394	616	880	1,200	1,570
	FOR SINGLE BINDERS (TWO ARMS) AT VARIOUS SPACINGS (S)	6"	166	294	462	660	900	1,175	184	327	513	737	1,003	1,310
		7"	142	252	396	560	772	1,008	158	280	440	622	858	1120
		8"	124	221	347	495	675	883	140	245	385	553	752	983
		9"	110	196	308	440	600	784	123	218	342	491	668	873
		10½"	96	168	264	377	514	671	167	198	282	419	571	746
	$V = \frac{Q}{l_a} = \frac{A_{st}p_{st}}{s}$	12"	82	147	231	330	450	588	92	163	256	368	501	655
		14"	72	126	198	283	·386	504	79	140	220	311	429	560
		16"	63	110	173	248	338	441	70	122	192	277	376	491
		18"	56	98	154	220	300	392	62	109	171	245	334	436
		24"	41	74	115	165	225	294	46	82	128	184	250	327
	SHEARING RESISTANCE (LB) WHEN $s = l_a$		990	1,765	2,770	3,960	5,400	7,060	1,100	1,960	3,080	4,420	6,020	7,860
	OTHER STESSES		SHEARING RESISTANCE AND V OF BINDERS PROPORTIONAL TO TENSILE STRESS.											
	MAXIMUM SPACING OF BINDERS		NORMALLY $s \not> l_a$ BEAMS WITH COMPRESSION REINFORCEMENT:- $s \not>$ 12 × DIAM. OF BAR DITTO STEEL-BEAM THEORY :- $s \not>$ 8 × DIAM. OF BAR											

	TENSILE STRESS		18,000 LB. PER SQ.IN.				20,000 LB. PER SQ.IN.			
	ANGLE θ		45 DEG.		30 DEG.		45 DEG.		30 DEG.	
	SYSTEM		SINGLE	DOUBLE	SINGLE	DOUBLE	SINGLE	DOUBLE	SINGLE	DOUBLE
INCLINED BARS	SHEARING RESISTANCE Q LB.	DIA. OF BAR								
		½"	2,490	4,990	1,170	3,540	2,770	5,540	1,960	3,930
		5/8"	3,900	7,800	2,765	5,530	4,340	8,670	3,070	6,140
		¾"	5,630	11,250	3,980	7,960	6,250	12,500	4,420	8,840
		7/8"	7,650	15,300	5,420	10,840	8,500	17,000	6,010	12,030
		1"	9,990	19,980	7,070	14,140	11,100	22,200	7,850	15,710
		1⅛"	12,650	25,290	8,950	17,890	14,050	28,100	9,940	19,880
		1¼"	15,620	31,230	11,050	22,090	17,350	34,700	12,270	24,540
		1⅜"	18,860	37,710	13,360	26,710	20,950	41,900	14,840	29,680
		1½"	22410	44,820	15,910	31,810	24,900	49,800	17,670	35,340
	MAX VALUE OF D		$1{\cdot}41 d_o$	$0{\cdot}71 d_o$	$2 d_o$	d_o	$1{\cdot}41 d_o$	$0{\cdot}71 d_o$	$2 d_o$	d_o
	OTHER STRESSES		SHEARING RESISTANCE OF INCLINED BARS PROPORTIONAL TO TENSILE STRESS.							

SINGLE SYSTEM
$Q = A_{st} p_{st} \sin\theta$



DOUBLE SYSTEM
$Q = 2A_{st} p_{st} \sin\theta$



TORSIONAL RESISTANCE AND ARCATE BEAMS (BOW GIRDERS).

Examples.

(*a*) A bow girder is 18 in. deep and 18 in. wide, has a radius of 12 ft., and subtends an angle of 90 deg. The ends are rigidly fixed and the total load is 45,000 lb. uniformly distributed. Find the maximum negative bending moment, the maximum positive bending moment, and the maximum twisting moment.

$\theta = 90$ deg.; $R = 12$ ft.; from *Table* 82 $G = 1{\cdot}12$;

$$w = \frac{45{,}000}{\pi \times 2 \times 12 \times 0{\cdot}25} = 2390 \text{ lb. per ft.}; \; K_W = 2390 \times 12 = 28{,}700 \text{ lb.}$$

Maximum negative bending moment (at the support): $C_1 = 4{\cdot}5$; $C_2 = 1{\cdot}00$; $C = 0{\cdot}24$.

$$M_{neg.} = \frac{(1{\cdot}00 \times 1{\cdot}12) + 0{\cdot}24}{(4{\cdot}5 \times 1{\cdot}12) + 1} \times 28{,}700 \times 12 = 77{,}400 \text{ ft.-lb.}$$

Maximum positive bending moment (at midspan): $C_8 = 0{\cdot}32$; $C_9 = 0{\cdot}05$.

$$M_{pos.} = \frac{(0{\cdot}32 \times 1{\cdot}12) + 0{\cdot}05}{6{\cdot}04} \times 28{,}700 \times 12 = 23{,}200 \text{ ft.-lb.}$$

Maximum twisting moment (at point of contraflexure; $\beta = 22$ deg.): $C_6 = 0{\cdot}13$; $C_7 = 0{\cdot}02$.

$$T_{max.} = \frac{(0{\cdot}13 \times 1{\cdot}12) + 0{\cdot}02}{6{\cdot}04} \times 28{,}700 \times 12 = 9360 \text{ ft.-lb.}$$

(*b*) Determine the reinforcement for an 18-in. square beam, circular in plan, radius 12 ft., supported at equal intervals so that the angle subtended at the centre by each arc is 90 deg., the negative bending moment over the support being 890,000 in.-lb., the positive bending moment at mid-span 450,000 in.-lb., and the maximum shearing force at the support 22,500 lb. At the point of contraflexure ($M = 0$) the twisting moment is a maximum and is 136,000 in.-lb. The point of contraflexure is approximately at the fifth-point of the span.

Support section.—With stresses of 18,000 lb. per sq. in. and 1000 lb. per sq. in. and effective depth of 16 in., the moment of resistance (compressive) with $A_{sc} = A_{st}$ (but $p_{cb} \not> 1000$) $= 507 \times 18 \times 16^2 = 2{,}340{,}000$ in.-lb. (from *Table* 70A), which is satisfactory. With actual effective depth of 16·5 in., $A_{st}(= A_{sc}) = \dfrac{890{,}000}{18{,}000 \times 0{\cdot}85 \times 16{\cdot}5} = 3{\cdot}53$ sq. in., say, four $1\frac{1}{8}$-in. bars in the top and bottom. These bars can be arranged by providing one $1\frac{1}{8}$-in. bar in each corner throughout the beam, bars from adjacent spans overlapping both in the top and bottom at the supports to resist negative bending moment.

Shearing stress at support $= \dfrac{22{,}500}{18 \times 0{\cdot}85 \times 16{\cdot}5} = 89$ lb. per sq. in.; because of the importance of the beam, the entire shearing force is resisted by binders; V required $= \dfrac{22{,}500}{0{\cdot}85 \times 16{\cdot}5}$ $= 1600$; from *Table* 81 this is given by $\frac{3}{8}$-in. double binders at $4\frac{1}{2}$-in. centres (at 18,000 lb. per sq. in.). The twisting moment at the support section is small and can be ignored.

Midspan section.—Since the bending moment is assumed to be half that at the support, half the support reinforcement is required, that is, two $1\frac{1}{8}$-in. bars in the top and bottom are sufficient. The shearing force and twisting moment at midspan are zero.

Section at fifth-point of span.—Shearing force $= \dfrac{0{\cdot}5 - 0{\cdot}2}{0{\cdot}5} \times 22{,}500 = 13{,}500$ lb. Shearing stress $= 54$ lb. per sq. in., which does not need reinforcement. The bending moment is theoretically zero, but the four bars provided at the support would be continued beyond this section, thus allowing for negative bending moments that may be produced when adjacent spans only are subjected to live load; a margin is also allowed thereby for the longitudinal steel required to resist twisting moment. Twisting moment $= 136{,}000$ in.-lb. For a rectangular section with $\dfrac{D_1}{D} = 1{\cdot}0$ and $D = 18$ in. (since $M = 0$ the entire cross-section is considered to be uncracked), the section-coefficient varies from 0·30 to 0·39, depending on the percentage of reinforcement. Section-coefficient required: $R = \dfrac{T}{qD^3}$; with $q = 100$ lb. per sq. in. (for 1 : 2 : 4 ordinary concrete, *Table* 57), $R = \dfrac{136{,}000}{100 \times 18^3} < 0{\cdot}30$. Hence from *Table* 82, total percentage of torsional reinforcement required is 1·0. This can be divided into two parts: 50 per cent. as longitudinal bars in the corners of the section and 50 per cent. as binding. Assuming that, of the four bars provided at top and bottom, the four corner bars are available for torsional resistance, the percentage is $\dfrac{3{\cdot}98 \times 100}{18^2} = 1{\cdot}23$, which is more than double the amount required. The volume of binding per foot required is $\dfrac{1{\cdot}00}{2} \times \dfrac{18^2 \times 12}{100}$ $= 19{\cdot}4$ cu. in. The volume of one external $\frac{3}{8}$-in. binder $= 0{\cdot}11 \times 4 \times 16 = 7{\cdot}05$ cu. in. The spacing required $= \dfrac{12 \times 7{\cdot}05}{19{\cdot}4} = 4{\cdot}35$, say 4-in. centres.

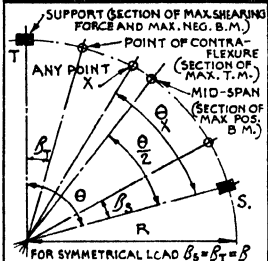
TORSIONAL RESISTANCE SECTION COEFFICIENTS

SHAPE OF SECTION		Circular	Rectangular					Tee			Section	Ell	
$D_1 \div D$		–	1·0	1·5	2·0	3·0	4·0	2·0	3·0	4·0	–	2·0	3·0
TOTAL PERCENTAGE OF TORSIONAL REINFORCEMENT (EQUAL AMOUNTS OF LONGITUDINAL BARS AND BINDERS)	0·75	0·28	0·30	0·61	0·93	1·60	2·44	1·06	1·65	2·44	0·98	0·98	1·61
	1·0	0·29	0·31	0·64	0·98	1·68	2·57	1·12	1·74	2·57	1·03	1·03	1·69
	1·25	0·31	0·33	0·68	1·03	1·76	2·70	1·17	1·83	2·70	1·09	1·09	1·77
	1·5	0·32	0·34	0·71	1·08	1·84	2·82	1·23	1·91	2·82	1·14	1·14	1·86
	1·75	0·34	0·36	0·74	1·12	1·92	2·95	1·28	2·00	2·95	1·19	1·19	1·94
	2·0	0·35	0·38	0·77	1·17	2·01	3·08	1·34	2·09	3·08	1·24	1·24	2·03
	2·25	0·37	0·39	0·81	1·22	2·10	3·21	1·40	2·17	3·21	1·29	1·29	2·11

TWISTING MOMENT OF RESISTANCE $= RqD^3$; R = SECTION COEFFICIENT AS TABULATED.
D = EQUIVALENT DIAMETER OF UNCRACKED SECTION.
q = SAFE SHEARING STRESS ($= 0{\cdot}10\, p_{cb}$ APPROX.)

ARCATE BEAMS (CIRCULAR ARCS) (BOW GIRDERS)

APPLICABLE TO (i) BEAMS FORMING A COMPLETE CIRCULAR SYSTEM WITH SUPPORTS SPACED UNIFORMLY, AND WITH IDENTICAL LOAD ON EACH SPAN.
(ii) BEAM OF SINGLE-SPAN WITH ENDS RIGIDLY FIXED.



FOR SYMMETRICAL LOAD $\beta_S = \beta_T = \beta$

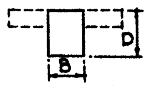
SECTION	BENDING MOMENT	TWISTING MOMENT	SHEARING FORCE
SUPPORT	UK_wR OR $\frac{C_2G+C}{C_1G+1}K_wR$	SK_wR OR $\frac{C_4G+C_5}{C_1G+1}K_wR$	QK_w
POINT OF CONTRAFLEXURE	ZERO	VK_wR OR $\frac{C_6G+C_7}{C_1G+1}K_wR$	NK_w
MID-SPAN	YK_wR OR $\frac{C_8G+C_9}{C_1G+1}K_wR$	ZERO	ZERO

	–	UNIFORMLY-DISTRIBUTED LOAD w LB. PER FOOT $(K_w = wR)$							CENTRAL CONC^D LOAD P. $(K_w = P)$	
	ANGLE θ	30°	45°	60°	72°	90°	120°	180°	90°	180°
	NO. OF SUPPORTS	12	8	6	5	4	3	(2)	4	(2)
	C_1	–	–	10·4	7·5	4·5	2·4	–	4·5	–
SUPPORT	U	0·02 APPROX.	0·053	–	–	–	–	1·00	–	0·5
	C_2	–	–	0·99	1·00	1·00	1·11	–	0·97	–
	C	–	–	0·12	0·14	0·24	0·45	–	0·25	–
	S	NEGLIGIBLE		–	–	–	–	0·30	–	0·18
	C_4			0·025	0·03	0·03	0·027	–	0·04	–
	C_5			0·02	0·02	0·03	0·09	–	0·04	–
POINT OF CONTRAFLEXURE	V	0·001 APPROX.	0·004	–	–	–	–	0·12	–	0·09
	C_6	–	–	0·08	0·10	0·13	0·16	–	0·17	–
	C_7	–	–	ZERO	0·01	0·02	0·03	–	0·26	–
MID-SPAN	Y	0·01 APPROX.	0·023	–	–	–	–	ZERO	–	0·53
	C_8	–	–	0·40	0·38	0·32	0·11	–	0·19	–
	C_9	–	–	0·10	0·08	0·05	0·002	–	ZERO	–
SHEAR	Q	0·28	0·39	0·53	0·63	0·79	1·05	1·57	0·5	0·5
	N	0·20	0·22	0·26	0·35	0·40	0·49	0·66	0·22	0·18
	ANGLE β	6°	10°	15°	18°	22°	36°	52°	25½°	57½°

$G = \frac{\text{TORSIONAL RIGIDITY}}{\text{FLEXURAL RIGIDITY}} = \frac{NJ}{EI}$.

G IS APPROXIMATELY EQUAL FOR RECTANGULAR, TEE, AND ELL SECTIONS WITH THE SAME RATIO OF D : B.

$\frac{D}{B}$	1·0	1·5	2·0	2·5	3·0
G	1·12	0·70	0·47	0·33	0·22



COLUMNS SUPPORTING CONCENTRIC LOADS.

Columns with Separate Binders.—The safe concentric load is calculated in general by the load-factor method. If the modular-ratio method is used, the effective area of a column with separate links or binders is the area of the concrete plus m times the area of the longitudinal reinforcement; for a short rectangular column the safe concentric load is as given by the appropriate formula in *Table* 83. The value of m depends upon the concrete, but generally a value of 15 is assumed. The entire area of the concrete is assumed to be effective in carrying the load, but in some cases, as when the cover is considered to have spalled off in a fire in a building with a high fire risk, the safe direct load should be based on the area of the concrete in the core of the column.

The B.S. Code and London By-laws recommend a load-factor method for the design of columns subjected to concentric load. The compressive stress in the reinforcement is independent of the quality of the concrete and is 18,000 lb. per sq in. in mild steel bars. The safe concentric load is as given by the appropriate formula in *Table* 83.

The amount of longitudinal reinforcement should be not less than 0·8 per cent. of the whole concrete area and not more than 8 per cent. The safe loads, calculated in accordance with the B.S. Code and the London By-laws, on square columns with maximum and minimum percentages of longitudinal reinforcement, are given in *Table* 84; these loads have been calculated with the stresses given in the table, but if other stresses are used the modification to the safe load is not proportional, and the loads should be calculated from the formula.

The pitch of the binders should be not greater than twelve times the diameter of the smallest longitudinal bar or the width of the column, whichever is the smaller. The spacing should not exceed 12 in. and generally need not be less than 6 in. For columns in buildings the diameter of the binder should be not less than $\frac{3}{16}$ in. and not less than one-quarter of the diameter of the largest bar secured by the binder. The largest convenient diameter of binder is $\frac{1}{2}$ in., but larger diameters can be used, but generally a much smaller bar is sufficient; $\frac{1}{4}$ in. or $\frac{5}{16}$ in. are convenient sizes. The arrangement, when the diameter and spacing are known, of separate binders depends on the number of longitudinal bars in the column; a variety of arrangements depending on the number of longitudinal bars is given in *Table* 83

Immediately above and below a junction in the longitudinal reinforcement, the binders are sometimes spaced more closely than elsewhere in the column. Such junctions should be made preferably at floor levels or at beam intersections, as in these positions lateral support is obtained and construction is more convenient. Some types of junctions in the longitudinal reinforcement in a column are indicated in Appendix II. The crank in a bar passing from the lower column into the upper column should be not more acute than 1 in. in a length of 1 ft. The distance which the bars project into the upper column should be such that the bars overlap the bars in the upper column for a length of not less than 24 times the diameter of the upper bars or for a greater length if such is required to develop by bond the stress in the lower bar. If separate splice bars are provided the area of the splice bars should be not less than the area of the bars in the upper column, and the length of each splice bar above and below the joint should be 24 or more times the diameter of the splice bar. If a column is subjected to bending, the bending moment generally attains its greatest value at a floor level, in which case the calculation of the resistance should take into account the positions of any cranked bars.

Columns with Helical Binding.—The following conditions conform to the recommendations in the B.S. Code.

The area of concrete considered as effective in carrying the load is that in the bound core. The maximum safe axial load which a short column with helical binding can carry is given by $P_O = P_C + P_T + P_B$, where P_C is the load carried on the concrete core, P_T is the load carried on the longitudinal reinforcement, and P_B is the increase in load due to the helical binding. The values of these components are given in the formulæ and data in *Table* 83.

The area of the longitudinal reinforcement, which should be not less than 0·8 per cent. or more than 8 per cent. of the gross area of the concrete. The value of P_O must be reduced for a "long" column. The expression 27,000 A_b, for the additional load due to the helical binding, is based on a stress of 13,500 lb. per sq. in. in the binding and, according to the B.S. Code, is the same for all qualities of reinforcement; if high-yield-stress steel is used this stress may be 35 per cent. of the yield stress or not more than 18,000 lb. per sq. in. in accordance with the London By-laws. The stress in the longitudinal reinforcement is as given in *Table* 58. The spacing of the helical binding should be not less than 1 in. and not greater than 3 in. An important limitation is that $P_C + P_B$ should not exceed $\frac{1}{2}Au_w$ where u_w is the specified minimum crushing strength of works-test concrete cubes; reducing this to an expression involving the permissible direct compressive stress $P_C + P_B$ should be not greater than $\frac{1}{2} \times \frac{3p_{cc}}{0{\cdot}76}A = 1{\cdot}975Ap_{cc}$.

(*Continued on page* 296.)

SAFE CONCENTRIC LOAD ON SHORT COLUMNS

P_o = SAFE CONCENTRIC LOAD ON COLUMN (LB.).

A = GROSS CROSS-SECTIONAL AREA = $d'' \times b''$ FOR RECTANGULAR COLUMN (SQ. IN.).

A_{sc} = CROSS-SECTIONAL AREA OF MAIN BARS (SQ. IN.). $r = A_{sc}/A$ ($\ngtr 8\% \nless 0.8\%$).

A_k = CROSS-SECTIONAL AREA OF CORE OF COLUMN WITH HELICAL BINDING = $0.7854 d_k^2$ (SQ. IN.).

A_b = VOLUME OF HELICAL BINDING PER INCH OF COLUMN = $\frac{0.7854}{s} d_b^2 (\pi d_k)$ (SQ. IN.).

d_k = DIAMETER OF CORE OF COLUMN WITH HELICAL BINDING = MEAN DIA. OF HELIX (IN.).

d_b = DIAMETER OF BAR FORMING HELICAL BINDING (IN.). s = PITCH (IN.) OF HELICAL BINDING $\ngtr \frac{d_k}{6}$.

p_{cc} = PERMISSIBLE DIRECT COMPRESSIVE STRESS IN CONCRETE (LB. PER SQ. IN.).

p_{sc} = PERMISSIBLE COMPRESSIVE STRESS IN MAIN BARS (LB. PER SQ. IN.).

MILD STEEL BARS: $\ngtr 1\frac{1}{2}$ IN. DIA. = 18,000 LB./SQ. IN.
$> 1\frac{1}{2}$ IN. DIA. = 16,000 " " "

HIGH-YIELD-STRESS BARS = $\frac{1}{2}$(YIELD-STRESS) AND $\ngtr$ 23,000 LB./SQ. IN.

COLUMNS WITH SEPARATE BINDERS (LATERAL TIES)

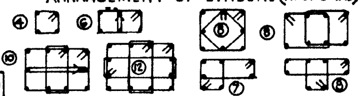
MODULAR-RATIO METHOD.

$$P_o = [A + (m-1)A_{sc}]\,p_{cc}$$

LOAD-FACTOR METHOD.

$$P_o = A p_{cc} + A_{sc}(p_{sc} - p_{cc}) = A[p_{cc} + r(p_{sc} + p_{cc})]$$

ARRANGEMENT OF BINDERS (No. OF BARS)



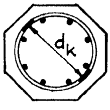
COLUMNS WITH HELICAL BINDING.

LOAD-FACTOR METHOD (B.S. CODE).

$$P_o = A_k p_{cc} + A_{sc} p_{sc} + 27{,}000 A_b$$

$$= 0.7854 d_k^2 p_{cc} + A_{sc}(p_{sc} - p_{cc}) + K_2 d_k$$

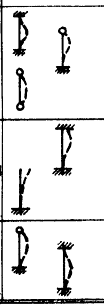
$$K_2 = 66{,}600 \frac{d_b^2}{s}$$



PITCH OF HELICAL BINDING s	VALUES OF K_2 — DIAMETER d_b HELICAL BINDING: 3/16"	1/4"	5/16"	3/8"	1/2"	5/8"	3/4"	7/8"	1"
1"	2340	4140	6510	–	–	–	–	–	–
1½"	1560	2760	4350	6250	11,080	–	–	–	–
2"	1170	2070	3265	4700	8300	13,000	–	–	–
2½"	936	1660	2610	3750	6640	10,400	15,000	–	–
3"	780	1380	2170	3120	5540	8670	12,500	17,000	22,200

LONG COLUMNS

EFFECTIVE LENGTH L_E IN TERMS OF ACTUAL LENGTH L

END CONDITIONS: ONE END	OTHER END	DIAGRAM	EFFECTIVE LENGTH L_E: LONDON BY-LAWS	B.S. CODE
FIXED	FIXED		0·70L	0·75L
FIXED	"HINGED"		0·85L	–
"HINGED"	"HINGED"		1·00L	1·00L
FIXED	PARTIALLY RESTRAINED IN POSITION AND DIRECTION		1·50L	1·00L TO 2·00L DEPENDING ON DEGREE OF PARTIAL RESTRAINT
FIXED	FREE		2·00L	2·00L
PARTIALLY FIXED	"HINGED"		—	$<$ 1·00L TO $\nless$ 0·75L
PARTIALLY FIXED	PARTIALLY FIXED		—	DEPENDING ON DEGREE OF DIRECTIONAL RESTRAINT

REDUCTION FACTORS R_L

BASED ON LEAST LATERAL DIMENSION D_L (RECTANGULAR, ETC., COLUMNS)

$\frac{L_E}{D_L} \ngtr 1.5: \; R_L = 1 \cdot 0$ $\quad \frac{L_E}{D_L} > 1 \ngtr 33: \; R_L = 1 \cdot 5 - \frac{L_E}{30 D_L}$ B. S. CODE ALSO LOND. BY-LAWS FOR ALL VALUES OF $\frac{L_E}{D_L}$

LINEAR INTERPOLATION BETWEEN TABULATED VALUES $\quad \frac{L_E}{D_L} \nless 33 \ngtr 57: \; R_L = 0 \cdot 95 - \frac{L_E}{60 D_L}$ B. S. CODE

L_E/D_L	15	16	17	18	19	20	21	22	24	26	27	30	32	33	57
R_L	1·0	0·97	0·93	0·90	0·87	0·83	0·80	0·77	0·70	0·63	0·60	0·50	0·43	0·40	0* B.S. CODE ONLY

BASED ON LEAST RADIUS OF GYRATION g (GENERAL USE)

$\frac{L_E}{g} \ngtr 50: \; R_L = 1 \cdot 0$ $\quad \frac{L_E}{g} > 50 \ngtr 120: \quad R_L = 1 \cdot 5 - \frac{L_E}{100 g}$

L_E/g	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120
R_L	1·0	0·95	0·90	0·85	0·80	0·75	0·70	0·65	0·60	0·55	0·50	0·45	0·40	0·35	0·30

COLUMNS SUPPORTING CONCENTRIC LOADS

(*continued from page* 294).

Long Columns.—If the ratio of the effective length of a column to the least radius of gyration exceeds about 50, the column is a "long" or slender column and the safe load on the column is less than that on a short column. The reduction factor is R_L given by the formula in *Table* 83 in which L_E is the effective length of the column and g is the least radius of gyration, L_E and g being in the same units. In *Table* 83 are also given values of R_L for slenderness ratios up to the generally accepted limit of 150. The permissible load on a long column is $R_L P_O$, where P_O is the safe concentric load on a short column (that is, a column having a ratio of effective length to least radius of gyration of 50 or less).

For columns with separate binders the least radius of gyration should be calculated on the gross cross-sectional area of the column, and on the core section for columns with helical bindings when the bindings are taken into account in determining the safe load. The radius of gyration may take into account the reinforcement, with a modular ratio of 15. The radii of gyration of some common sections are given in *Tables* 65 (neglecting reinforcement) and 66A and 66B (taking reinforcement into account).

For square and rectangular columns the slenderness ratio can be based on the least lateral dimension D_L. If the ratio exceeds 15, the reduction coefficient R_L is $1{\cdot}5 - \dfrac{L_E}{30D_L}$ up to a ratio of 33 and $0{\cdot}95 - \dfrac{L_E}{60D_L}$ for ratios from 33 to 57. In *Table* 83, values of R_L are given in accordance with these expressions which are based on the B.S. Code for various slenderness ratios up to 57. The London By-laws adopt this basis up to a ratio of 36.

The effective length of a column is the actual length if the end conditions are equivalent to "hinged" at both ends. If a column is "fixed" at both ends, or "fixed" at one end and "free" at the other, or if there are intermediate conditions of restraint, the ratio of effective length to actual length is as given in *Table* 83 for columns designed in accordance with the B.S. Code or the London By-laws. By "hinged" is meant that the end is properly restrained in position but not in direction. By "fixed" is meant that the end is properly restrained both in position and direction. By "free" is meant that the end of the column is unrestrained in position and direction. By "partially fixed" is meant that the end is fixed in position but imperfectly restrained in direction. The actual length of a column in a building is the distance between floors.

Examples.—(*a*) Select from *Table* 84 square columns of 1 : 2 . 4 ordinary concrete, with separate binders lateral ties, to support a safe concentric load of 400,000 lb

Suitable columns are 18 in. square with eight $1\frac{1}{4}$-in. bars; 20 in. square with four $1\frac{3}{8}$-in. bars or eight 1-in. bars; 21 in. square with four $1\frac{1}{8}$-in. bars or eight $\frac{7}{8}$-in. bars.

(*b*) Calculate the safe direct load on an 18-in. octagonal column of 1 : 1 : 2 concrete, with eight $1\frac{1}{2}$-in. vertical bars and $\frac{3}{8}$-in. helical binding at $2\frac{1}{2}$-in. centres.

From *Table* 57, $p_{cc} = 1140$ lb. per sq. in. Diameter of core $d_k = 18 - 2 = 16$ in. Area of longitudinal bars (*Table* 60) $= 14{\cdot}14$ sq. in. ($K_2 = 3750$).

$P_O = (0{\cdot}7854 \times 16^2 \times 1140) + [14{\cdot}14 \times (18{,}000 - 1140)] + (3750 \times 16) = 529{,}000$ lb. (assuming a "short" column).

(*c*) Calculate the safe load on a column 18 in. by 10 in. of 1 : 2 . 4 concrete with six $\frac{7}{8}$-in bars (3·6 sq. in.), if the height of the column extending through several stories is 20 ft. Permissible compressive stresses are 760 lb. per sq. in. and 18,000 lb. per sq. in.

As a "short" column the safe load is $P_O = (18 \times 10 \times 760) + (3{\cdot}6 \times 17{,}240) = 199{,}500$ lb. Since the least lateral dimension is 10 in., and the effective length is the story-height, that is 20 ft., the slenderness-ratio is given by

$\dfrac{L_E}{D_L} = \dfrac{20 \times 12}{10} = 24.$ The reduction factor R_L for this ratio is 0·7 (*Table* 83).

Therefore the safe load is $0{\cdot}7 \times 199{,}500 = 139{,}650$ lb.

SQUARE COLUMNS.—B.S. CODE.*

MIXTURE OF CONCRETE	SIZE OF SQUARE COLUMN	DIAMETER OF MAIN REINFORCEMENT (MILD STEEL) BARS														
		FOUR BARS								EIGHT BARS						
		5/8"	3/4"	7/8"	1"	1⅛"	1¼"	1⅜"	1½"	3/4"	7/8"	1"	1⅛"	1¼"	1⅜"	1½"
1:2:4	10 in.	97	106	118	130									REINFORCEMENT EXCEEDS 8 PER CENT.		
	11 in.	113	122	133	146											
	12 in.	131	140	151	164	178										
	14 in.		179	190	203	218	233	LOADS ARE GIVEN IN UNITS OF 1000 LB.								
	15 in.			213	225	240	256									
	16 in.			236	249	263	279	297								
P_{cc} = 760 LB. PER SQ. IN.	18 in.				300	315	331	349	368	307	329	355	383	415	451	489
	20 in.					373	389	406	426	365	387	412	441	473	508	548
	21 in.					404	420	437	457	396	418	443	472	504	540	579
	24 in.	REINFORCEMENT IS LESS THAN 0·8 PER CENT.									521	546	575	607	642	682
	27 in.											662	691	723	759	798
	30 in.												821	853	888	928
1:1½:3	10 in.	116	125	136	149									REINFORCEMENT EXCEEDS 8 PER CENT.		
	11 in.	136	145	156	169											
	12 in.	158	167	178	190	205										
	14 in.		216	227	240	253	270	LOADS ARE GIVEN IN UNITS OF 1000 LB.								
	15 in.			255	267	282	297									
	16 in.			284	297	311	327	344								
P_{cc} = 950 LB. PER SQ. IN.	18 in.				361	376	392	409	428	368	390	415	444	475	510	549
	20 in.					448	464	481	500	440	462	487	516	548	582	621
	21 in.					487	503	520	539	479	501	526	555	587	621	660
	24 in.	REINFORCEMENT IS LESS THAN 0·8 PER CENT.									629	654	683	715	749	788
	27 in.											800	828	860	895	934
	30 in.												991	1023	1057	1096
1:1:2	10 in.	135	144	155	167									REINFORCEMENT EXCEEDS 8 PER CENT.		
	11 in.	159	168	179	191											
	12 in.	185	194	205	217	231										
	14 in.		253	264	276	291	306	LOADS ARE GIVEN IN UNITS OF 1000 LB.								
	15 in.			297	309	324	339									
	16 in.			332	344	359	375	392								
P_{cc} = 1140 LB. PER SQ. IN.	18 in.				422	436	452	469	488	429	450	475	503	535	569	608
	20 in.					523	539	556	575	516	537	562	590	622	656	694
	21 in.					569	585	603	622	562	584	608	637	668	703	741
	24 in.	REINFORCEMENT IS LESS THAN 0·8 PER CENT.									738	763	791	822	857	895
	27 in.											937	965	997	1031	1069
	30 in.												1160	1192	1226	1265

* This table also applies to concrete Quality A (London By-laws).

COMBINED BENDING AND THRUST (MODULAR-RATIO METHOD).

Example.—Annular Member Subjected to Bending and Direct Thrust.—The method given in *Table* 85 for the determination of the stresses in a member, the cross-section of which is not rectangular, is applied in the following example to the annular member shown at the bottom of the table.

Eccentricity: $e = \frac{1{,}000{,}000}{50{,}000} = 20$ in.; $\frac{e}{D} = \frac{20}{36} = 0{\cdot}556$. Tensile and compressive stresses are likely to occur. Assume $d_n = 13$ in. (about $0{\cdot}38d_1$). The properties of the bars below the neutral plane are considered thus:

δA_t sq. in.	p_a in.	$p_a - d_n$ in.	R	Rp_a	$\delta A_t p_a$
0·614	18	5	3·07	55	11·2
0·614	24	11	6·75	162	14·7
0·614	30	17	10·43	313	18·4
0·614	33	20	12·28	405	20·3
0·307	34	21	6·45	219	10·5
$\Sigma a_t = 2{\cdot}76$			$\Sigma R = 38{\cdot}98$	$\Sigma Rp_a = 1154$	$\Sigma \delta A_t p_a = 75{\cdot}1$

Centre of the tension. $p_t = \frac{1154}{38{\cdot}98} = 29{\cdot}6$ in.

The compressive area above the neutral plane is divided into a series of horizontal strips, the properties of which are tabulated thus:

Strip	b_x in.	δ_x in.	$b_x.\delta_x$ sq in.	$(m - 1)\delta A_c$ sq. in.	a sq. in.	x_1 in.	x_n in.	ax_n	x_a	ax_nx
A	14	3	42	8·6	51	1·5	11·5	586	76	879
B	24	3	72	4·3	76	4·5	8·5	645	342	2900
C	16	3	48	8·6	57	7·5	5·5	314	428	2350
D	12·5	4	50	8·6	59	11	2·0	118	648	1300
					$\Sigma a = 243$	$\Sigma ax_n = 1663$			$\Sigma xa = 1494$	$\Sigma ax_nx = 7449$

Centre of compression. $p_c = \frac{7449}{1663} = 4{\cdot}47$ in.

If $m = 15$, $\bar{x} = \frac{(15 \times 75{\cdot}1) + 1494}{(15 \times 2{\cdot}76) + 243} = 9{\cdot}2$ in.

Stresses $f_{cb} = \frac{50{,}000 \times 13(20 + 29{\cdot}6 - 9{\cdot}2)}{(29{\cdot}6 - 4{\cdot}47)1663} = 628$ lb. per sq. in.

$$f_{st} = \frac{34 - 13}{38{\cdot}98}\left[\frac{628 \times 1663}{13} - 50{,}000\right] = 16{,}400 \text{ lb. per sq. in.}$$

Check trial position of neutral plane: $d_n = \dfrac{34}{1 + \dfrac{16{,}400}{15 \times 483}} = 12{\cdot}35$ in.,

compared with the trial value of 13 in.; a second trial is unnecessary.

If a second value were necessary, say, larger than that assumed, the adjustment to the tabulations could be readily made by adding strip E, $1\frac{1}{2}$ in. deep, the to compressive area and omitting from the reinforcement table any bars now included in the compressive area. For the tensile reinforcement only the values of R and Rp_a are affected. In the compressive area the values of x_n and consequently ax_n and ax_nx are affected. With these modifications the analysis then proceeds as shown for the first trial.

COMPRESSIVE STRESSES ONLY

EQUIVALENT AREA OF STRIP: $a = b_x \delta x + (m-1)\,\delta A_c$

EQUIVALENT AREA OF SECTION

$$A_e = \Sigma a$$

POSITION OF CENTROID

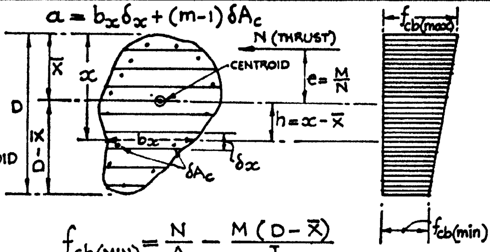
$$\bar{x} = \frac{\Sigma a x}{A_e}$$

MOMENT OF INERTIA ABOUT CENTROID

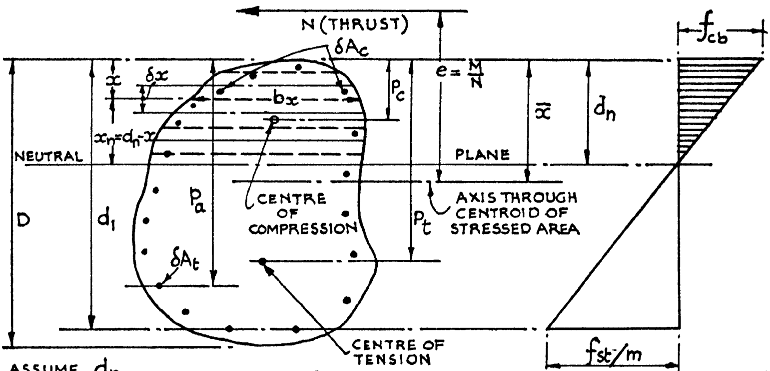
$$I_x = \Sigma a\left[\frac{(\delta x)^2}{12} + h\right]$$

COMPRESSIVE STRESSES

$$f_{cb(MAX.)} = \frac{N}{A} + \frac{M\bar{x}}{I_x} \qquad f_{cb(MIN.)} = \frac{N}{A} - \frac{M(D-\bar{x})}{I_x}$$



COMPRESSIVE AND TENSILE STRESSES



ASSUME d_n.

CENTRE OF TENSION: $p_t = \dfrac{\Sigma R p_a}{\Sigma R}$ $\qquad R = (p_a - d_n)\,\delta A_t$

IF ALL BARS ARE OF SAME SIZE $p_t = \dfrac{\Sigma p_a (p_a - d_n)}{\Sigma (p_a - d_n)}$

EQUIVALENT AREA OF STRIP: $a = b_x \delta x + (m-1)\,\delta A_c$

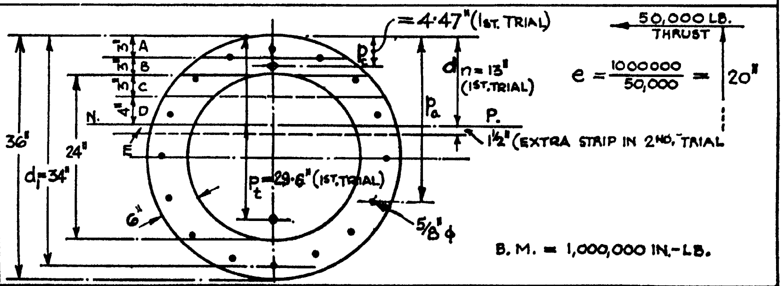
CENTRE OF COMPRESSION: $p_c = \dfrac{\Sigma x_n x a}{\Sigma x_n a}$

POSITION OF CENTROID OF STRESSED AREA: $\bar{x} = \dfrac{m \Sigma \delta A_t p_a + \Sigma a x}{m \Sigma \delta A_t + \Sigma a}$

MAX. STRESSES: $f_{cb} = \dfrac{N d_n (e + p_t - \bar{x})}{(p_t - p_c)\,\Sigma x_n a}$ $\qquad f_{st} = \dfrac{d_1 - d_n}{\Sigma R}\left[\left(\dfrac{f_{cb}}{d_n}\Sigma x_n a\right) - N\right]$

CHECK TRIAL VALUE OF d_n.

EXAMPLE



COMBINED BENDING AND THRUST: RECTANGULAR SECTIONS (MODULAR-RATIO METHOD).

Examples of the Use of Table 86.—A rectangular member is 18 in. deep and 12 in. wide and is reinforced with two 1-in. bars in the top and three 1-in. bars in the bottom. The centres of the bars are $1\frac{1}{2}$ in. from the top and bottom faces respectively. Find the stresses produced in the section by the specified direct thrusts and bending moments.

For all cases $d = 18$ in.; $b = 12$ in.; $d_1 = 16{\cdot}5$ in.; $d_2 = 1\frac{1}{2}$ in.; $f_2 = \frac{1\frac{1}{2}}{16{\cdot}5} = 0{\cdot}091$; $m = 15$; $A_{st} = 2{\cdot}36$ sq. in.; $A_{sc} = 1{\cdot}57$ sq. in.

(*a*) $M = 200{,}000$ in.-lb. $N = 100{,}000$ lb.

$e = \frac{200{,}000}{100{,}000} = 2$ in., which is less than $\frac{d}{6}(= 3$ in.)

$A_c = 12 \times 18 = 216$ sq. in. $A_s = 14(2{\cdot}36 + 1{\cdot}57) = 55$ sq. in.

$$Z = \frac{2 \times 55}{18}(9 - 1{\cdot}5)^2 + (0{\cdot}167 \times 216 \times 18) = 988 \text{ in.}^3$$

$$f_{cb} = \frac{100{,}000}{55 + 216} \pm \frac{200{,}000}{988} = 571 \text{ lb. (max.) and } 165 \text{ lb. (min.) per sq. in.}$$

(*b*) $M = 200{,}000$ in.-lb. $N = 50{,}000$ lb.

$e = \frac{200{,}000}{50{,}000} = 4$ in., which is greater than $\frac{d}{6}$ and less than $\frac{1}{2}d(= 8\frac{1}{2}$ in.).

A_c, A_s and Z are the same as in example (*a*).

$f_{cb(min.)} = \frac{50{,}000}{55 + 216} - \frac{200{,}000}{988} = 184 - 203 = -19$ lb. per sq. in., which is less than $\frac{1}{10}$th of the allowable maximum stress $\left(\text{say, } \frac{1000}{10}\right)$ and therefore the maximum compressive stress is $184 + 203 = 387$ lb. per sq. in.

(*c*) $M = 300{,}000$ in.-lb. $N = 15{,}000$ lb.

$e = \frac{300{,}000}{15{,}000} = 20$ in., which is greater than $\frac{1}{2}d$, and less than $1\frac{1}{2}d_1(=24\frac{3}{4}$ in.).

With $\bar{x} = 0{\cdot}5d$ approx. $= 9$ in., $F = \frac{20 - 9}{16{\cdot}5} + 1 = 1{\cdot}67$. Assume $n_1 = 0{\cdot}55$.

$J = 0{\cdot}55 \times 12 \times 16{\cdot}5 \times 0{\cdot}5 = 55$. From *Table* 86, $G = 0{\cdot}224$ and $H = 11{\cdot}5$ approx.

$$\text{Thus } f_{cb} = \frac{15{,}000 \times 1{\cdot}67}{(0{\cdot}224 \times 12 \times 16{\cdot}5) + (11{\cdot}5 \times 1{\cdot}57 \times 0{\cdot}909)} = 412 \text{ lb. per sq. in.}$$

$$f_{st} = \frac{412[55 + (11{\cdot}5 \times 1{\cdot}57)] - 15{,}000}{2{\cdot}36} = 6440 \text{ lb. per sq. in.}$$

The value of n_1 corresponding to f_{st} and f_{cb} is $\frac{1}{1 + \frac{6440}{412 \times 15}} = 0{\cdot}49$, compared with the assumed value $0{\cdot}55$. Re-calculate with an intermediate value of $n_1 = 0{\cdot}5$, for which $J = 12 \times 16{\cdot}5 \times 0{\cdot}5 \times 0{\cdot}5 = 49{\cdot}5$. The accurate value of $\bar{x}$ is

$$\frac{16{\cdot}5[(49{\cdot}5 \times 0{\cdot}5) + (15 \times 2{\cdot}36) + (14 \times 1{\cdot}57 \times 0{\cdot}091)]}{(2 \times 49{\cdot}5) + (15 \times 2{\cdot}36) + (14 \times 1{\cdot}57)} = 6{\cdot}55 \text{ in.}$$

(It is interesting to compare this value with the assumed approximate value of $0{\cdot}5d = 9$ in.

$F = \frac{20 - 6{\cdot}55}{16{\cdot}5} + 1 = 1{\cdot}815$; $G = 0{\cdot}208$; $H = 11{\cdot}2$ (or $11{\cdot}45$ by calculation with the exact value $f_2 = 0{\cdot}091$).

$$f_{cb} = \frac{15{,}000 \times 1{\cdot}815}{(0{\cdot}208 \times 12 \times 16{\cdot}5) + (11{\cdot}45 \times 1{\cdot}57 \times 0{\cdot}909)} = 475 \text{ lb. per sq. in.}$$

$$f_{st} = \frac{475[49{\cdot}5 + (11{\cdot}45 \times 1{\cdot}57)] - 15{,}000}{2{\cdot}36} = 7240 \text{ lb. per sq. in.}$$

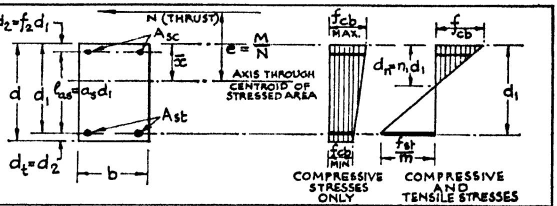
Corresponding value of $n_1 = \frac{1}{1 + \frac{7240}{475 \times 15}} = 0{\cdot}497$, which is practically the same as the trial value ($0{\cdot}50$).

(*Continued on page* 302.)

ECCENTRICITY ABOUT CENTROID OF STRESSED AREA

$$e = \frac{\text{B.M.}}{\text{THRUST}} = \frac{M}{N}$$

METHOD ALLOWS FOR ANY MODULAR RATIO (m) ANY VALUES OF A_{st} & A_{sc} AND $d_2/d_1 (= f_2)$



$e < \frac{d}{6}$

MODULUS OF SECTION: $Z \text{ (APPROX.)} = \frac{A_c d}{6} + \frac{2A_s}{d}\left(\frac{d}{2} - d_2\right)^2$ $\quad A_c = bd$, $\quad A_s = (m-1)(A_{sc} + A_{st})$

STRESSES: $f_{cb(\text{MAX.})}, f_{cb(\text{MIN})} = \frac{N}{A_c + A_s} \pm \frac{M}{Z}$

$e > \frac{d}{6}$, $< \frac{d}{2}$

COMPUTE $f_{cb(\text{MIN})}$ BY METHOD FOR $e < \frac{d}{6}$; IF POSITIVE, METHOD APPLIES. IF NEGATIVE (OR GREATER THAN PERMISSIBLE TENSILE STRESS IN CONCRETE) ADOPT METHOD FOR $e > \frac{d}{2}$.

$e > \frac{1}{2} d_1$

ASSUME d_n; EVALUATE $n_1 = d_n/d_1$ AND $J = \frac{1}{2} b d_n$.

DETERMINE POSITION OF CENTROID OF STRESSED AREA:

$$\bar{x} = \frac{[J n_1 + m A_{st} + (m-1) A_{sc} f_2] d_1}{2J + m A_{st} + (m-1) A_{sc}} \left(= \frac{d}{2} \text{ APPROX.}\right) \qquad \text{CALCULATE } F = \frac{e - \bar{x}}{d_1} + 1.$$

EVALUATE $G = \frac{n_1}{2}\left(1 - \frac{n_1}{3}\right)$ AND $H = \frac{1}{n_1}(m-1)(n_1 - f_2)$ OR OBTAIN FROM TABULATION.

NEUTRAL-PLANE FACTOR n_1			·20	·25	·30	·35	·40	·45	·50	·55	·60	·65	·70	·75
FACTOR $G = \frac{1}{2} n_1 (1 - \frac{1}{3} n_1)$			·093	·114	·135	·156	·173	·191	·208	·224	·240	·254	·268	·281
RATIO OF MAX. STRESSES f_{st}/f_{cb} AND FACTOR $H = 1/n_1 (m-1)(n_1 - f_2)$	$m = 15$	RATIO OF STRESSES	60	45	35	28	22·5	18·3	15	13·7	10	8	6·5	5
		$f_2 = \frac{d_2}{d_1}$ 0·20	-	2·8	4·6	60	7·0	7·7	8·4	9·0	9·4	9·7	9·9	10·1
		0·15	3·5	9·6	7·0	8·0	8·8	9·4	9·8	10·2	10·5	10·7	11·0	11·2
		0·10	7·0	8·4	9·4	10·1	10·5	10·9	11·2	11·5	11·6	11·8	12·0	12·2
		0·05	10·5	11·2	11·6	12·0	12·3	12·4	12·6	12·7	12·9	12·9	13·0	13·0
	$m = 12·5$	RATIO OF STRESSES	50	37·5	29	23	18·8	15·2	12·5	10·2	8·3	6·8	5·4	4·2
		$f_2 = \frac{d_2}{d_1}$ 0·20	-	2·3	3·8	4·9	5·7	6·3	6·9	7·4	7·7	8·0	8·1	8·3
		0·15	2·9	4·6	5·7	6·6	7·2	7·7	8·0	8·4	8·7	8·8	9·0	9·2
		0·10	5·7	6·9	7·7	8·3	8·6	9·0	9·2	9·5	9·6	9·8	9·9	10·0
		0·05	7·8	9·2	9·5	9·9	10·1	10·2	10·4	10·4	10·6	10·6	10·7	10·7
	$m = 10$	RATIO OF STRESSES	40	30	23·3	18·5	15	12·2	10	8·2	6·7	5·4	4·3	3·3
		$f_2 = \frac{d_2}{d_1}$ 0·20	-	1·8	3·0	3·9	4·5	5·0	5·4	5·8	6·1	6·3	6·4	6·5
		0·15	2·3	3·6	4·5	5·1	5·6	6·1	6·3	6·6	6·8	6·9	7·1	7·2
		0·10	4·5	5·4	6·1	6·5	6·8	7·0	7·2	7·4	7·5	7·7	7·7	7·8
		0·05	6·8	7·2	7·5	7·7	7·9	8·0	8·1	8·2	8·3	8·3	8·4	8·4
	$m = 8$	RATIO OF STRESSES	32	24	18·6	14·8	12	9·8	8	6·5	5·4	4·3	3·4	2·6
		$f_2 = \frac{d_2}{d_1}$ 0·20	-	1·4	2·3	3·0	3·5	3·9	4·2	4·5	4·7	4·9	5·0	5·1
		0·15	1·8	2·8	3·5	4·0	4·4	4·7	4·9	5·1	5·3	5·4	5·5	5·6
		0·10	3·5	4·2	4·7	5·1	5·3	5·5	5·6	5·8	5·8	6·0	6·0	6·1
		0·05	5·8	5·6	5·8	6·0	6·2	6·2	6·3	6·4	6·5	6·5	6·5	6·5

TO DETERMINE STRESSES SUBSTITUTE EVALUATED FACTORS IN

$$f_{cb} = \frac{NF}{G b d_1 + H A_{sc}(1 - f_2)} \qquad f_{st} = \frac{f_{cb}(J + H A_{sc}) - N}{A_{st}}$$

CHECK ASSUMED VALUE OF n_1 FROM $n_1 = \frac{1}{1 + \frac{f_{st}}{m f_{cb}}}$; RECALCULATE WITH REVISED VALUE OF n_1 IF DISCREPANCY EXCEEDS 5%

$e > 1\frac{1}{2} d_1$

CALCULATE STRESSES f_{cb} AND f_{st} DUE TO B.M. ONLY, AND DETERMINE $d_n = n_1 d_1$ FOR THESE STRESSES.

EVALUATE $f_c = \frac{N}{b d_n + m A_{st} + (m-1) A_{sc}}$. THEN $f_{cb(\text{MAX})} = f_{cb} + f_c$ AND $f_{st(\text{MAX})} = f_{st} - f_c m$.

COMBINED BENDING AND THRUST: RECTANGULAR SECTIONS (MODULAR-RATIO METHOD) (*continued from page* 300).

Notes on Table 87.—Case I applies to sections subjected to compressive stresses only; Case II is for sections subjected to tensile and compressive stresses. The cross-section of the member is rectangular and there is an equal amount of reinforcement adjacent to each of two opposite faces. The cover ratio is one-tenth of the overall depth.

In a case between Cases I and II, that is, if $f_{cb(min)}$ is a small tensile stress, and if tension in the concrete should be ignored, $f_{cb(max.)}$ is greater than the stress obtained for Case I and is about $\frac{NK_1}{0{\cdot}9bd}$.

The table applies only if the cover-ratio f_2 is 0·1 for A_{st} and A_{sc}, but the table can be used if the following adjustment is made. If $f_2 > 0{\cdot}1$ substitute $(1{\cdot}1 - f_2)d$ for the actual value of d. If $f_2 < 0{\cdot}1$, the maximum stress in the concrete calculated from the table is slightly in excess of the actual stress.

The table is based on the eccentricity about the centroid of the section. It is accurate for Case I and for large values of e_1 in Case II; for small values of e_1 in Case II the stresses are only approximate; for more exact calculated stresses the case $e > 0{\cdot}5d$ in *Table* 86 should be used.

Examples.—Consider a member in which d is 20 in., b is 12 in., A_{st} and A_{sc} both consist of three 1-in. bars (that is, $p = \frac{100 \times 4{\cdot}71}{20 \times 12} = 2{\cdot}0$ per cent.). With $1\frac{1}{2}$-in. cover, $d_1 = 18{\cdot}5$ in., $d_2 = 2$ in.; $\frac{2}{20} = 0{\cdot}1$; therefore *Table* 87 can be used directly for the determination of the maximum stresses in the following examples, in each of which $N = 120{,}000$ lb.

(*a*) $M = 240{,}000$ in.-lb.

$e = \frac{M}{N} = \frac{240{,}000}{120{,}000} = 2$ in.; $e_1 = \frac{e}{d} = \frac{2}{20} = 0{\cdot}10$. This is Case I; for $e_1 = 0{\cdot}1$ and $p = 2$ per cent., $K_1 = 1{\cdot}17$. By substitution

$$f_{cb(max)} = \frac{1{\cdot}17 \times 120{,}000}{12 \times 20} = 585 \text{ lb. per sq. in.}$$

and $f_{cb(min)} = \frac{2N}{(1 + 0{\cdot}14p)bd} - f_{cb(max.)} = \frac{2 \times 120{,}000}{1{\cdot}28 \times 12 \times 20} - 585 = 185$ lb. per sq. in.

(*b*) $M = 720{,}000$ in.-lb.

$e = \frac{720{,}000}{120{,}000} = 6$ in.; $e_1 = \frac{6}{20} = 0{\cdot}3$. This is Case II; for $e_1 = 0{\cdot}3$ and $p = 2{\cdot}0$ per cent., $n_0 = 0{\cdot}80$. For $p = 2$ per cent. and $n_0 = 0{\cdot}80$, $K_2 = 0{\cdot}153$.

$$f_{cb(max.)} = \frac{720{,}000}{0{\cdot}153 \times 12 \times 20^2} = 980 \text{ lb. per sq. in.}$$

$$f_{st} = 15 \times 980\left(\frac{0{\cdot}9}{0{\cdot}8} - 1\right) = 1840 \text{ lb. per sq. in.}$$

COMBINED BENDING AND TENSION.

Example of Use of Table 89.—A rectangular member is 18 in. deep, 12 in. wide, and is reinforced with two 1-in. bars in the top and three 1-in. bars in the bottom. If $d_1 = 16\frac{1}{2}$ in. and $d_2 = 1\frac{1}{2}$ in., determine the stresses if $M = 100{,}000$ in.-lb. and $N =$ direct pull of 20,000 lb.

Table 89 applies. $A_{s1} = 2{\cdot}36$ sq. in.; $A_{s2} = 1{\cdot}57$ sq. in. $\bar{x}' = \frac{2{\cdot}36 \times 15}{3{\cdot}93} = 9$ in.

$$e = \frac{100{,}000}{20{,}000} = 5 \text{ in. which is less than } l_{as} - \bar{x}'(= 6 \text{ in.}).$$

$$f_{st1} \text{ (in } A_{s1}) = \frac{20{,}000(5 + 9)}{2{\cdot}36 \times 15} = 7900 \text{ lb. per sq. in.}$$

$$f_{st2} \text{ (in } A_{s2}) = \frac{20{,}000(15 - 5 - 9)}{1{\cdot}57 \times 15} = 850 \text{ lb. per sq. in.}$$

NOTE:— THIS TABLE APPLIES WHEN $A_{sc} = A_{st}$ WITH $m = 15$, AND $d_2 = 0{\cdot}1d$

e_1	PERCENTAGE OF REINFORCEMENT ($= p$) 0·5	1·0	1·5	2·0	2·5	3·0	4·0	
·04	1·14	1·07	1·00	-	-	-	-	VALUES OF K_1
·06	1·25	1·16	1·09	1·01	-	-	-	
·08	1·36	1·25	1·17	1·09	1·02	-	-	
·10	1·47	1·35	1·25	1·17	1·10	1·04	-	
·12	1·58	1·44	1·33	1·24	1·17	1·10	0·99	
·14	1·69	1·53	1·42	1·32	1·24	1·17	1·05	
·16	1·80	1·62	1·51	1·40	1·31	1·24	1·10	
·18	(1·90)	1·72	1·60	1·48	1·39	1·30	1·17	
·20	(2·00)	(1·82)	(1·69)	1·57	1·45	1·36	1·22	
·30	0·70	0·75	0·77	0·80	0·82	0·84	0·86	VALUES OF n_o
·40	0·54	0·60	0·65	0·67	0·69	0·71	0·74	
·50	0·44	0·52	0·56	0·59	0·62	0·64	0·66	
·60	0·39	0·46	0·51	0·54	0·56	0·59	0·62	
·70	0·35	0·43	0·47	0·50	0·53	0·55	0·59	
·80	0·33	0·41	0·45	0·48	0·51	0·52	0·55	
·90	0·31	0·38	0·43	0·46	0·48	0·50	0·53	
1·0	0·30	0·37	0·41	0·44	0·46	0·48	0·51	
2·0	0·25	0·31	0·35	0·38	0·40	0·42	-	
3·0	0·23	0·29	0·33	0·36	0·40	-	-	
n_o								VALUES OF K_2
·20	0·100	0·162	0·220	0·280	0·340	0·400	-	
·30	0·100	0·140	0·180	0·220	0·260	0·300	0·380	
·40	0·102	0·132	0·164	0·192	0·222	0·252	0·311	
·50	0·108	0·130	0·156	0·180	0·203	0·227	0·273	
·60	0·110	0·130	0·150	0·170	0·190	0·210	0·250	
·70	0·111	0·127	0·144	0·161	0·179	0·196	0·230	
·80	0·109	0·124	0·139	0·153	0·169	0·182	0·213	
·90	0·104	0·118	0·130	0·143	0·158	0·170	0·195	

$$p = \frac{100(A_{st} + A_{sc})}{bd}$$

$$e_1 = \frac{B.M.}{d(\text{THRUST})} = \frac{M}{Nd}$$

CASE I

COMPRESSIVE STRESSES ONLY (FOR VALUES OF K_1 IN BRACKETS A SMALL TENSION IS DEVELOPED).

FIND VALUE OF K_1 FOR GIVEN VALUES OF e_1 AND p AND SUBSTITUTE IN

$$f_{cb\,(max.)} = \frac{K_1 N}{bd}$$

CASE II

COMPRESSIVE AND TENSILE STRESSES

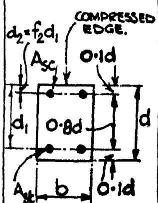
$$n_o = \frac{n_1 d_1}{d}$$

FIND VALUE OF n_o FOR GIVEN VALUES OF e_1 AND p

WITH THIS VALUE OF n_o AND GIVEN VALUE OF p FIND K_2 AND SUBSTITUTE IN

$$f_{cb\,(MAX.)} = \frac{M}{K_2 bd^2}$$

$$f_{st} = f_{cb\,(MAX.)}\, m\left(\frac{0{\cdot}9}{n_o} - 1\right)$$



COLUMNS SUBJECTED TO BENDING: LOAD-FACTOR METHOD.

Examples.—A column of 1 : 2 : 4 ordinary-quality concrete (p_{cc} = 760 lb. per sq. in.) is to be designed for the conditions described in each example. The width d parallel to the plane of bending is 20 in., and the width b at right-angles to this plane is 12 in.; the reinforcement comprises three 1-in. mild steel bars near each of the two 12-in. faces; the cover is 1½ in., and $d_2 = 2$ in. Therefore, $bd = 12 \times 20 = 240$ sq. in.; $f_1 = \frac{d_2}{d} = \frac{2}{20} = 0{\cdot}1$; $A_{sc1} = 2{\cdot}36$ sq. in., and $r = \frac{2A_{sc1}}{bd} = 0{\cdot}02$ (2 per cent.). The effects of wind are not included in the applied load and bending moment; therefore formulæ series (1) and the factors in the lower part of *Table* 88 can be applied since the bars are less than 1½-in. diameter. (p_{sc} = 18,000 lb. per sq. in.; p_{st} = 20,000 lb. per sq. in.)

From *Table* 88 for $100r$ = 2 per cent. and $f_1 = 0{\cdot}10$, K_0 = 1103 lb. per sq. in., K_b = 395 lb. per sq. in. and $\frac{e_b}{d} = 0{\cdot}621$. Therefore $P_0 = 1103 \times 240 = 264{,}720$ lb. $P_b = 395 \times 240 = 94{,}800$ lb.; $e_b = 0{\cdot}621 \times 20 = 12{\cdot}4$ in.

(*a*) Applied load 60,000 lb., which is less than P_b; Case I. Factor K_{I} in *Table* 88 applies.

(i) Bending moment 600,000 in.-lb.; $e = \frac{600{,}000}{60{,}000} = 10$ in. ($< e_b$); $\frac{e}{d} = \frac{10}{20} = 0{\cdot}5$; therefore K_{I} = 484 lb. per sq. in.; safe eccentric load = 484 × 240 = 116,000 lb., which is greater than the applied load; therefore the section is satisfactory.

(ii) Bending moment 1,200,000 in.-lb.; $e = \frac{1{,}200{,}000}{60{,}000} = 20$ in. ($> e_b$). $\frac{e}{d} = \frac{20}{20} = 1{\cdot}0$; K_{I} = 238 lb. per sq. in., and $P_{\mathrm{I}} = 238 \times 240 = 57{,}100$ lb., which is less than the applied load and therefore the section is inadequate; therefore either increase the reinforcement to, say, six 1⅛-in. bars; or increase d to, say, 21 in.; or increase the quality of the concrete to 1 : 1½ : 3.

(*b*) Applied load 94,800 lb., which is P_b; the section must be satisfactory (as this is the borderline of Cases I and II) so long as e is not greater than e_b.

(i) Bending moment 760,000 in.-lb.; $e = \frac{760{,}000}{94{,}800} = 8{\cdot}0$ in ($< e_b$); $\frac{e}{d} = \frac{8{\cdot}0}{20} = 0{\cdot}4$; K_{I} = 555 lb. per sq. in. which, being greater than K_b = 395 lb. per sq. in., shows that the section is satisfactory.

(ii) Bending moment greater than 1,175,000 in.-lb. In this case $e = \frac{1{,}175{,}000}{94{,}800} = 12{\cdot}4$ in. which exceeds e_b; hence K_{I} is less than K_b; the section is inadequate.

(*c*) Applied load 120,000 lb, which is greater than P_b; therefore Case II applies; the appropriate formula is formula (D) for K_{II} in *Table* 88. Safe load (at eccentricity e) = $P_{\mathrm{II}} = K_{\mathrm{II}}bd$. The intensity of the applied load is $\frac{N}{bd} = \frac{120{,}000}{240} = 500$ lb. per sq. in.

(i) Bending moment 1,200,000 in.-lb.; $e = \frac{1{,}200{,}000}{120{,}000} = 10$ in. ($< e_b$). From formula $K_{\mathrm{II}} = \dfrac{1103}{1 + \left(\frac{1103}{395} - 1\right)\frac{10}{12{\cdot}4}} = 450$ lb. per sq. in., which being less than $\frac{N}{bd}$ (= 500), indicates that the section is inadequate although e is less than e_b. The greatest bending moment that can be combined with an applied load of 120,000 lb. can be calculated by transposing formula D and substituting $\frac{N}{bd}$ for K_{II}, thus

$$e_{\mathrm{limit}} = \left(\frac{e_b}{\frac{K_0}{K_b} - 1}\right)\left(\frac{K_0}{\frac{N}{bd}} - 1\right) = \frac{12{\cdot}4}{\left(\frac{1103}{395} - 1\right)}\left(\frac{1103}{500} - 1\right) = 8{\cdot}4 \text{ in.}$$

Max. bending moment with N = 120,000 lb. is Ne_{limit} = 120,000 × 8·4 = 1,008,000 in.-lb.

(ii) Bending moment 1,488,000 in.-lb.; $e = \frac{1{,}488{,}000}{120{,}000} = 12{\cdot}4$ in. ($= e_b$).

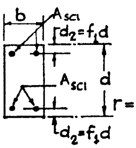
From example (*c*, i) it is seen that the section is inadequate to resist this combination of bending moment and load, although e is equal to e_b. It is obvious that, for $e = e_b$, the greatest load P_b that can be sustained is 94,800 lb.

(iii) Bending moment greater than 1,488,000 in.-lb. For this case, e will exceed e_b and it is obvious from the foregoing, without further calculation, that the section will be inadequate (that is, $K_{\mathrm{II}} > K_b$). If e exceeds e_b the applied load must be less than P_b which is Case I and is dealt with as in example (*a*

TYPE OF REINFORCEMENT		EFFECT OF WIND	SERIES	PERMISSIBLE STRESSES IN REINFORCEMENT LB. PER SQ. IN. TENSILE p_{st}	COMPRESSIVE p_{sc}	LIMITING ECCENTRIC LOAD $P_b = K_b bd$ FORMULÆ (A) FOR K_b	LIMITING ECCENTRICITY e_b FORMULÆ (B) FOR $\frac{e_b}{d}$	MAXIMUM SAFE LOAD AT ECCENTRICITY e CASE I APPLIED LOAD $\not< P_b$ $P_I = K_I bd$ FORMULÆ (C) FOR $\frac{K_I}{p_{cc}}$
MILD STEEL BARS $E_s = 30 \times 10^6$ LB PER SQ IN.	NOT LARGER THAN 1½ IN. DIAMETER	EXCLUDED	1	20,000	18,000	$0{\cdot}607p_{cc}(1-f_1)-1000r$	$\frac{0{\cdot}423p_{cc}(1-f_1)^2+9000r(1-2f_1)}{K_b}-\frac{1}{2}+f_1$	$U+\sqrt{U^2+\frac{2000r}{p_{cc}}\left(10-19f_1-\frac{500r}{p_{cc}}\right)}$; $U=\frac{1}{2}-\frac{e}{d}-\frac{1000r}{p_{cc}}$
		INCLUDED	2	25,000	22,500	$0{\cdot}567p_{cc}(1-f_1)-1250r$	$\frac{0{\cdot}406p_{cc}(1-f_1)^2+11250r(1-2f_1)}{K_b}-\frac{1}{2}+f_1$	$U+\sqrt{U^2+\frac{2500r}{p_{cc}}\left(10-19f_1-\frac{625r}{p_{cc}}\right)}$; $U=\frac{1}{2}-\frac{e}{d}-\frac{1250r}{p_{cc}}$
	LARGER THAN 1½ IN DIAMETER	EXCLUDED	3	18,000	18,000	$0{\cdot}625p_{cc}(1-f_1)-1000r$	$\frac{0{\cdot}430p_{cc}(1-f_1)^2+8000r(1-2f_1)}{K_b}-\frac{1}{2}+f_1$	$U+\sqrt{U^2+\frac{2000r}{p_{cc}}\left(9-17f_1-\frac{500r}{p_{cc}}\right)}$; $U=\frac{1}{2}-\frac{e}{d}-\frac{1000r}{p_{cc}}$
		INCLUDED	4	22,500	20,000	$0{\cdot}585p_{cc}(1-f_1)-1250r$	$\frac{0{\cdot}414p_{cc}(1-f_1)^2+10{,}000r(1-2f_1)}{K_b}-\frac{1}{2}+f_1$	$U+\sqrt{U^2+\frac{2500r}{p_{cc}}\left(9-17f_1-\frac{625r}{p_{cc}}\right)}$; $U=\frac{1}{2}-\frac{e}{d}-\frac{1250r}{p_{cc}}$
HIGH-YIELD-STRESS BARS $t_y \not< 60{,}000$ LB/SQ.IN.	ALL SIZES	EXCLUDED	5	30,000	23,000	$Xp_{cc}(1-f_1)-3500r$	$\frac{V+11{,}500r(1-2f_1)}{K_b}-\frac{1}{2}+f_1$	$U+\sqrt{U^2+\frac{3500r}{p_{cc}}\left(8{\cdot}6-15{\cdot}2f_1-\frac{3500r}{p_{cc}}\right)}$; $U=\frac{1}{2}-\frac{e}{d}-\frac{1250r}{p_{cc}}$
						$X=\frac{0{\cdot}35}{1+\frac{18\times10^6}{E_s}}$	AND $V=(1-f_1)^2(1-0{\cdot}5X)Xp_{cc}$	
		INCLUDED	6	30,000	28,750	$Xp_{cc}(1-f_1)-625r$	$\frac{V+14{,}375r(1-2f_1)}{K_b}-\frac{1}{2}+f_1$	$U+\sqrt{U^2+\frac{1250r}{p_{cc}}\left(24-47f_1-\frac{312r}{p_{cc}}\right)}$; $U=\frac{1}{2}-\frac{e}{d}-\frac{625r}{p_{cc}}$

ALL CONDITIONS

K_b AND e_b CALCULATED FROM FORMULÆ ABOVE



$$r = \frac{2A_{sc1}}{bd}$$

CASE II
APPLIED LOAD $\not> P_b$

$P_{II} = K_{II} bd$

FORMULA D: $K_{II} = \dfrac{K_o}{1+\left(\frac{K_o}{K_b}-1\right)\frac{e}{e_b}}$

WHERE K_o = FACTOR FOR SAFE CONCENTRIC LOAD ON SHORT COLUMN $= p_{cc}+r(p_{sc}-p_{cc})$

MILD STEEL BARS (≯ 1½ IN DIA.) p_{sc} = 18,000 LB/SQ IN. ORDINARY GRADE 1:2:4 CONCRETE p_{cc} = 760 LB/SQ IN.

RATIO OF REINFMT 100r	K_o	COVER RATIO f_1	K_b	$\frac{e_b}{d}$	K_I (LB PER SQ IN) $e/d = 0{\cdot}4$	0·5	1·0	1·5
PER CENT. 0·8	897	0·05	430	0·375	403	324	122	75
		0·10	407	0·381	388	306	110	60
		0·15	384	0·383	372	288	99	47
1·0	932	0·05	428	0·415		406	150	82
		0·10	405	0·420		341	134	74
		0·15	382	0·423		320	121	66
2·0	1103	0·05	418	0·630			261	166
		0·10	395	0·621			238	143
		0·15	372	0·613			214	126
3·0	1277	0·05	408	0·852			359	227
		0·10	385	0·835			322	205
		0·15	362	0·813			293	183
4·0	1446	0·05	398	1·090				291
		0·10	375	1·060				263
		0·15	352	1·025				234
6·0	1794	0·05	378	1·600				
		0·10	355	1·550				
		0·15	332	1·485				
8·0	2131	0·05	358	2·168				
		0·10	335	2·095				
		0·15	312	2·010				

VALUES OF K_I FOR $e < e_b$ NOT APPLICABLE

[FOR PURPOSE OF INTERPOLATION $K_I = K_b$ FOR $e/d = e_b/d$]

FORMULÆ FOR SPECIAL CONDITIONS

$p_{st} \not> 20{,}000$ LB./SQ.IN.

SAFE CONCENTRIC LOAD (SHORT COLUMN):

$P_o = K_o bd$

$K_o = 760 + 17{,}240r$

LIMITING ECCENTRIC LOAD:

$P_b = K_b bd$

$K_b = 460(1-f_1) - 1000r$

LIMITING ECCENTRICITY = e_b:

$$\frac{e_b}{d} = \frac{322(1-f_1)^2+9000r(1-2f_1)}{K_b}-\frac{1}{2}+f_1$$

MAX. SAFE LOAD AT ECCENTRICITY e: (APPLIED LOAD LESS THAN P_b)

$P_I = K_I bd$

$K_I = 760\left[U+\sqrt{U^2+2{\cdot}63r(10-19f_1-0{\cdot}66r)}\right]$

$U = \frac{1}{2} - \frac{e}{d} - 1{\cdot}313r$

TABLE 89.—COMBINED BENDING AND TENSION. (MODULAR-RATIO METHOD.)

TENSILE STRESSES ONLY: $e \not> l_{as} = \bar{x}'$ (CONCRETE INEFFECTIVE IN TENSION)

IRREGULAR GROUPS OF BARS

AVERAGE TENSILE STRESSES IN REINFORCEMENT. [GROUP A_{s1} = BARS IN TENSION DUE TO B.M. ONLY ACTING.]

$$\bar{x}' = \frac{A_{s1} l_{as}}{A_{s1} + A_{s2}}$$

IN GROUP A_{s1}

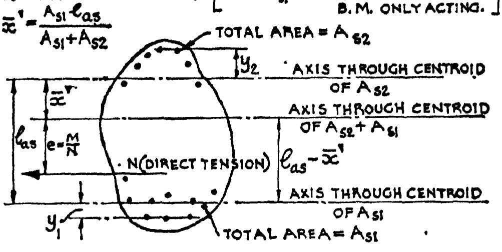
$$f_{st1} = \frac{N(e + \bar{x}')}{l_{as} A_{s1}}$$

IN GROUP A_{s2}

$$f_{st2} = \frac{N(l_{as} - e - \bar{x}')}{l_{as} A_{s2}}$$

MAX. TENSILE STRESSES (IN OUTER BAR OR BARS IN GROUP A_{s1})

$$f_{st1\,\mathrm{MAX.}} = f_{st2} + \left(\frac{l_{as} + y}{l_{as}}\right)(f_{st1} - f_{st2})$$

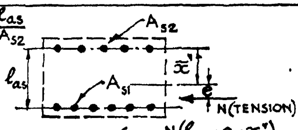


REGULAR GROUPS OF BARS

$A_{s1} \neq A_{s2}$ $\quad \bar{x}' = \frac{A_{s1} l_{as}}{A_{s1} + A_{s2}}$

TENSILE STRESSES IN REINFORCEMENT IN GROUP A_{s1}

$$f_{st1} = \frac{N(e + \bar{x}')}{l_{as} A_{s1}}.$$



IN GROUP A_{s2}: $f_{st2} = \frac{N(l_{as} - e - x')}{l_{as} A_{st}}$

$A_{s1} = A_{s2} = A_s$ [AT BOTH FACES]

IN GROUP A_{s1}

$$f_{st1} = \frac{N(e + \frac{1}{2} l_{as})}{l_{as} A_s}$$

IN GROUP A_{s2}

$$f_{st2} = \frac{N(\frac{1}{2} l_{as} - e)}{l_{as} A_s}$$

COMPRESSIVE AND TENSILE STRESSES (CONCRETE INEFFECTIVE IN TENSION)

NON-RECTANGULAR SECTION

ASSUME d_n

$R = (p_a - d_n)\,\delta A_t$

CENTRE OF TENSION:

$$p_t = \frac{\Sigma R p_a}{\Sigma R}$$

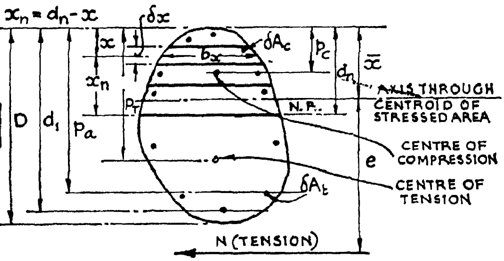
$$\left[= \frac{\Sigma p_a (p_a - d_n)}{\Sigma (p_a - d_n)} \text{ IF ALL BARS ARE OF SAME SIZE} \right]$$

EQUIVALENT AREA OF STRIP:

$$a = b_x \delta x + (m-1)\,\delta A_c$$

CENTRE OF COMPRESSION:

$$p_c = \frac{\Sigma x_n x a}{\Sigma x_n a}$$



POSITION OF CENTROID OF STRESSED AREA: $\bar{x} = \frac{m \Sigma \delta A_c p_a + \Sigma a x}{m \Sigma \delta A_t + \Sigma a}$

MAXIMUM STRESSES: $f_{cb} = \frac{N d_n (e - p_t + \bar{x})}{(p_t - p_c)\Sigma x_n a}$ $\qquad f_{st} = \frac{d_1 - d_n}{\Sigma R}\left[\left(\frac{f_{cb}}{d_n}\Sigma x_n a\right) + N\right]$

CHECK TRIAL VALUE OF d_n.

RECTANGULAR SECTION

$e > l_{as} - \bar{x}$

ASSUME d_n. EVALUATE $n_1 = d_n/d_1$ AND $J = \frac{1}{2} b d_n$.

$$\bar{x} = \frac{[J n_1 + m A_{st} + (m-1) A_{sc} f_2]\, d_1}{2J + m A_{st} + (m-1) A_{sc}} \left(= \frac{d}{2} \text{ APPROX.}\right)$$

EVALUATE $L = \frac{e + \bar{x}}{d_1} - 1$

$$G = \frac{n_1}{2}\left(1 - \frac{n_1}{3}\right)$$

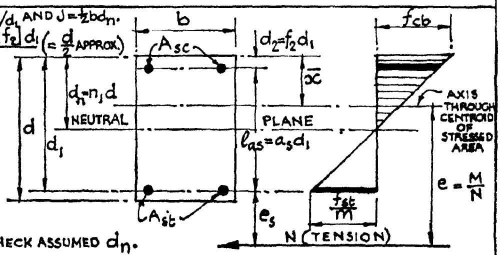
AND $H = \frac{1}{n_1}(m-1)(n_1 - f_2)$

DETERMINE STRESSES BY SUBSTITUTING IN

$$f_{cb} = \frac{NL}{G b d_1 + H A_{sc}(1 - f_2)}$$

$$f_{st} = \frac{f_{cb}(J + H A_{sc}) + N}{A_{st}}$$

; CHECK ASSUMED d_n.



$e > \frac{1}{2} d_1$

CALCULATE f_{cb} AND f_{st} DUE TO B.M. ONLY, AND DETERMINE $d_n = n_1 d_1$ FOR THESE STRESSES.

EVALUATE $f_c = \frac{N}{b d_n + m A_{st} + (m-1) A_{sc}}$. THEN $f_{cb(\mathrm{MAX})} = f_{cb} - f_c$ AND $f_{st(\mathrm{MAX})} = f_{st} + f_c m$.

APPROX. METHOD FOR SLAB

EVALUATE $e = M/N$ AND $e_s = e + \frac{1}{2} d - d_1$; A_{st} REQUIRED $= \frac{N}{p_{st}}\left(1 + \frac{e_s}{l_a}\right)$; l_a = LEVER ARM.

OR (GIVEN A_{st}) $f_{st} = \frac{N}{A_{st}}(1 + e_s/l_a)$

NOTE.—For an example of the use of this table, see page 302.

RECTANGULAR MEMBER SUBJECTED TO BENDING ABOUT TWO AXES

COMPRESSIVE STRESSES ONLY

SECTION MODULI.

Z_{xo}: ABOUT xx, AT EDGE A-B = $I_x/\bar{x}$

Z_{xd}: " " " " C-D = $I_x/_{d-\bar{x}}$

Z_{yo}: " yy, " " B-C = $I_y/\bar{y}$

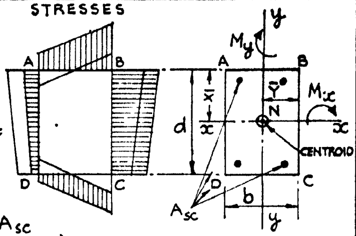
Z_{yb}: " " " " A-D = $I_y/_{b-\bar{y}}$

M_y = B.M. IN PLANE yy.

M_x = B.M. IN PLANE xx.

A_e = EQUIVALENT AREA = $bd + (m-1)A_{sc}$

STRESSES



STRESSES. $f_{cc} = \dfrac{N\,(=\text{CONCENTRIC THRUST})}{A_e}$; IF NO THRUST, $f_{cc} = 0$

AT A: $f_{cb} = \left(f_{cc} + \dfrac{M_y}{Z_{xo}}\right) - \dfrac{M_x}{Z_{yb}}$

AT B: (MAX) $f_{cb} = \left(f_{cc} + \dfrac{M_y}{Z_{xo}}\right) + \dfrac{M_x}{Z_{yo}}$

AT D: (MIN.) $f_{cb} = \left(f_{cc} - \dfrac{M_y}{Z_{xd}}\right) - \dfrac{M_x}{Z_{yb}}$

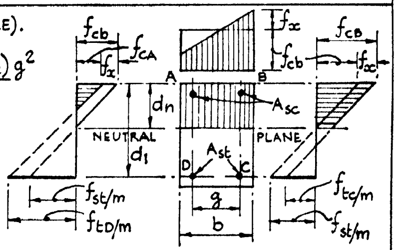
AT C: $f_{cb} = \left(f_{cc} - \dfrac{M_y}{Z_{xd}}\right) + \dfrac{M_x}{Z_{yo}}$

TENSILE & COMPRESSIVE STRESSES

NOTATION AS ABOVE (WHERE APPLICABLE).
ADDITIONAL NOTATION.

$$Z_n = \frac{d_n b^2}{6} + \frac{(m-1)(A_{st}+A_{sc})\,g^2}{2b}$$

STRESSES (APPROX.) WHEN $M_y > M_x$.



① CALCULATE f_{st}, f_{cb} AND d_n FOR M_y COMBINED WITH N OR FOR M_y ALONE IF NO THRUST.

② CALCULATE $\dfrac{M_x}{Z_n} = f_x$

③ RESULTANT STRESSES.

COMPRESSION IN CONCRETE AT A: $f_{CA} = f_{cb} - f_x$

COMPRESSION (MAX) IN CONCRETE AT B: $f_{CB} = f_{cb} + f_x$

TENSION (MAX.) IN BAR AT D: $f_{tD} = f_{st} + mf_x$

TENSION IN BAR AT C: $f_{tc} = f_{st} - mf_x$

PRINCIPAL STRESSES

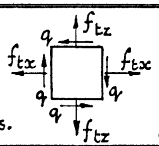
f_{pt} = PRINCIPAL TENSILE STRESS.

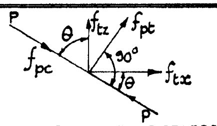
f_{pc} = PRINCIPAL COMPRESSIVE STRESS.

θ = INCLINATION OF PRINCIPAL PLANE PP.

f_{xt}, f_{zt} = APPLIED TENSILE (POSITIVE) STRESSES.

f_{xc}, f_{zc} = APPLIED COMPRESSIVE (NEG.) STRESSES.





q = APPLIED SHEARING STRESS

f_{xt} & f_{zt} TENSILE	$\left.\begin{matrix} f_{pt} \\ f_{pc} \end{matrix}\right\rangle -\frac{1}{2}(f_{xt}+f_{zt}) \mp \sqrt{\left[\frac{1}{2}(f_{xt}-f_{zt})\right]^2 + q^2}$	$\tan 2\theta = \dfrac{2q}{f_{xt}-f_{zt}}$
f_{xt} TENSILE $f_{zt}=0$	$f_{pt} = -\frac{1}{2}\left(\sqrt{f_{xt}^2+4q^2} + f_{xt}\right)$ $f_{pc} = \frac{1}{2}\left(\sqrt{f_{xt}^2+4q^2} - f_{xt}\right)$	$\tan 2\theta = \dfrac{2q}{f_{xt}}$
f_{xc} COMPN $f_{zt}=0$	$f_{pt} = -\frac{1}{2}\left(\sqrt{f_{xc}^2+4q^2} - f_{xc}\right)$ $f_{pc} = \frac{1}{2}\left(\sqrt{f_{xc}+4q^2} + f_{xc}\right)$	$\tan 2\theta = -\dfrac{2q}{f_{scc}}$
q ONLY	$f_{pt} = -q$ $f_{pc} = +q$	$\theta = 45$ DEG.

NON-PLANAR ROOFS.

Prismatic Structures.—To design a simple prismatic roof, or similar structure, comprising a number of planar slabs, the resultant loads P acting perpendicularly to each slab and the unbalanced thrusts N acting in the plane of each slab are determined first, taking into account the thrust of one slab on another. The slabs are then designed to resist the transverse bending moments due to loads P assuming continuity and combination with the thrusts N. The longitudinal forces F due to the slabs bending in their own plane under the loads N are, for any two adjacent slabs AB and BC, calculated from formula (2) in *Table* 91, in which M_{AB} and M_{BC} are from formula (1) if the structure is freely supported at the ends of the span L. For each pair of slabs, AB—BC, BC—CD, etc., there is an equation like (2) containing three unknown forces F. If there are n pairs there are $(n - 1)$ equations and $(n + 1)$ unknowns. The conditions at the outer edges (a) and (z) of the end slabs determine the forces F at these edges; for example, if the edges are unsupported, $F_a = F_z = 0$. The simultaneous equations are solved for the remaining unknown forces F_A, F_B, F_C, etc. The longitudinal stress at any junction B is calculated from the formula (in *Table* 91) for f_B. Variation of the longitudinal stress from one junction to the next is rectilinear. If f_B is negative, the stress is tensile and the consequent tensile forces should be resisted by reinforcement. Shearing stresses are generally small.

Domes.—A dome is designed for the total vertical load only, that is, for the weights of the slab and any covering on the slab, the weight of any ceiling or other distributed load suspended from the slab, and the live load. The intensity of total load $= w =$ the equivalent load per sq. ft. of surface of the dome. Horizontal loads due to the wind and the effect of shrinking and changes in temperature are allowed for by assuming an ample live load, or by inserting more reinforcement than that required for the vertical load alone, or by designing for stresses well below the permissible values, or by a combination of two or all of these methods.

Segmental Domes.—Referring to the diagram and formulæ in *Table* 91, the circumferential force acting in a horizontal plane in a unit strip S is T, and the corresponding force (the meridional thrust) acting tangentially to the surface of the dome is N. At the plane where θ is 51 deg. 48 min., that is, at the plane of rupture, $T = 0$. Above this plane T is compressive and reaches a maximum value of $0{\cdot}5wR$ at the crown of the dome ($\theta = 0$). Below this plane T is tensile, and equals $0{\cdot}167wR$ when $\theta = 60$ deg., and wR when $\theta = 90$ deg. The meridional thrust N is $0{\cdot}5wR$ at the crown, $0{\cdot}618wR$ at the plane of rupture, and $0{\cdot}667wR$ when $\theta = 60$ deg., and wR when $\theta = 90$ deg ; that is, N increases from the crown towards the support and has its greatest value at the support.

For a concentrated load W on the crown of the dome T is tensile; and T and the corresponding meridional compressive thrust N are given by the appropriate formulæ in *Table* 91, the basis of which is that the load is concentrated on so small an area at the crown that it is equivalent to a point load. The theoretical stress at the crown is therefore infinite, but the practical impossibility of obtaining a point load invalidates the application of the formulæ when θ is nought or very nearly nought. For domes of varying thickness, reference should be made to other publications.

Shallow Segmental Dome.—Approximate analysis only is sufficient in the case of a shallow dome; appropriate formulæ are given in *Table* 91.

Conical Dome.—In a conical dome, the circumferential forces are compressive throughout and at any horizontal plane x from the apex are given by the expression for T in *Table* 91, the corresponding force in the direction of the slope being N. The horizontal outward force per ft. of circumference at the bottom of the slope is P and resistance to this force must be provided by the supports or by a ring beam at the bottom of the slope.

Segmental Shells.—The longitudinal, tangential and shearing membrane forces in an element of a shell roof (defined in position by x and θ_x) are given in *Table* 91. Note the distribution of dead and live loads on which the formulæ are based. The principal stresses in any part of the shell (due to the membrane stresses only) are computed by the appropriate formula; the reinforcement is arranged to be approximately in line with the principal tensile force and to resist this force. Limitations on the length and width of the shell and the thickness of the slab are as given in the diagram. The effects of a support along the free longitudinal edges and of the end-stiffeners is to modify the forces in the vicinity of the edges and ends; publications dealing with the design of segmental shells should be consulted to determine these effects.

PRISMATIC STRUCTURE (HIPPED PLATE CONSTRUCTION)

N_{AB}, ETC. = THRUST IN PLANE OF SLAB AB, ETC., DUE TO COMPONENT OF LOAD AND WEIGHT AND OF THRUST IN SLAB BC, ETC.

① $M_{AB} = \frac{N_{AB}L^2}{8}$; $M_{BC} = \frac{N_{BC}L^2}{8}$, ETC.

LONGITUDINAL STRESSES; (TENSILE IF NEGATIVE) AT JUNCTION B:

$$f_B = \frac{2}{d_{AB}l_{AB}}\left(\frac{3M_{AB}}{l_{AB}} - F_A - 2F_B\right)$$

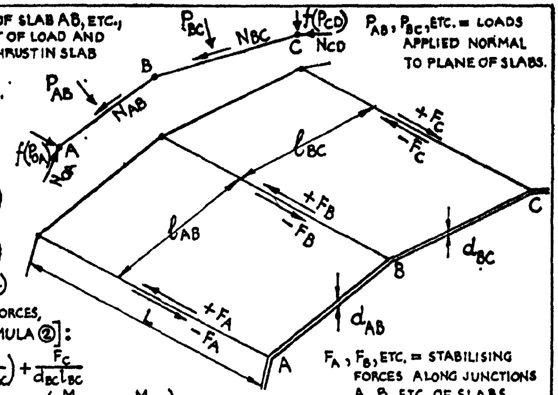
$$= \frac{2}{d_{BC}l_{BC}}\left(2F_B + F_C - \frac{3M_{BC}}{l_{BC}}\right)$$

(SIMILARLY FOR JUNCTION C, ETC.)

FORMULÆ FOR STABILISING FORCES, SLABS AB AND BC [FORMULA ②]:

$$\frac{F_A}{d_{AB}l_{AB}} + 2F_B\left(\frac{1}{d_{AB}l_{AB}} + \frac{1}{d_{BC}l_{BC}}\right) + \frac{F_C}{d_{BC}l_{BC}} = 3\left(\frac{M_{AB}}{d_{AB}l_{AB}^2} + \frac{M_{BC}}{d_{BC}l_{BC}^2}\right)$$

P_{AB}, P_{BC}, ETC. = LOADS APPLIED NORMAL TO PLANE OF SLABS.



F_A, F_B, ETC. = STABILISING FORCES ALONG JUNCTIONS A, B, ETC. OF SLABS.

DOMES

SEGMENTAL DOME.

T = CIRCUMFERENTIAL FORCE (IN HORIZONTAL PLANE) IN UNIT STRIP AT S.

N = MERIDIONAL THRUST (ACTING TANGENTIALLY) IN UNIT STRIP AT S.

UNIFORMLY-DISTRIBUTED LOAD ω PER UNIT AREA OF SURFACE OF DOME

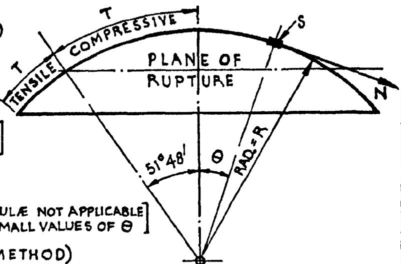
$$T = \omega R\left[\frac{1-\cos\theta-\cos^2\theta}{1+\cos\theta}\right]; \quad N = \omega R\left[\frac{1-\cos\theta}{\sin^2\theta}\right]$$

AT CROWN: $T = N = \frac{1}{2}\omega R$ (COMPRESSION)

LOAD W CONCENTRATED AT CROWN

$$T = \frac{W}{2\pi R}\operatorname{cosec}^2\theta; \quad N = \frac{W}{2\pi R\sin^2\theta}.$$

[FORMULÆ NOT APPLICABLE TO SMALL VALUES OF θ]

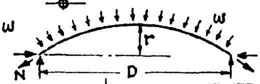


SHALLOW SEGMENTAL DOME (APPROX. METHOD)

$R = \frac{D^2}{8r} + \frac{1}{2}r$. TOTAL LOAD ON SUPPORTS $= W = 2\pi R r\omega$

AT SPRINGING: $N = \frac{2WR}{\pi D^2}$

TENSILE FORCE IN RING BEAM AT SPRINGING: $T_R = \frac{W(R-r)}{\pi D}$

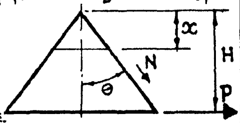


CONICAL DOME.

IN HORIZONTAL PLANE AT x: $T = \omega x\tan^2\theta$ (COMPRESSION)

$$N = \frac{\omega x}{2\cos^2\theta}$$

AT BOTTOM: $P = \omega H\frac{\tan\theta}{2\cos\theta}$ PER UNIT LENGTH OF CIRCUMFERENCE.



SEGMENTAL SHELLS

MEMBRANE FORCES. (POSITIVE = TENSILE)

TANGENTIAL:

$$F_y = -(\omega_d + \omega_\ell\cos\theta_x)R\cos\theta_x.$$

LONGITUDINAL:

$$F_x = -(\ell - x)\frac{x}{R}\left[\omega_d\cos\theta_x + 1{\cdot}5\,\omega_\ell(\cos^2\theta_x - \sin^2\theta_x)\right]$$

SHEARING:

$$Q_{xy} = (\omega_d + 1{\cdot}5\,\omega_\ell\cos\theta_x)(2x - \ell)\sin\theta_x.$$

AT MID-POINT B OF CROWN ($\theta_{xy} = 0$)

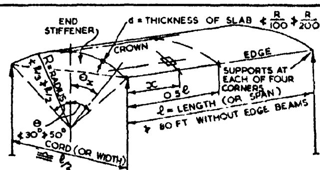
$$F_{yB} = -(\omega_D + \omega_\ell)R$$

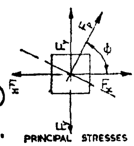
$$F_{xB} = -\frac{\ell^2}{4R}(\omega_d + 1{\cdot}5\,\omega_\ell)$$

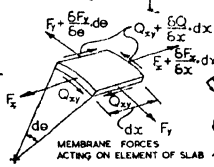
PRINCIPAL FORCES (MEMBRANE FORCES ONLY)

$$F_p = \tfrac{1}{2}(F_x + F_y) \pm \sqrt{\tfrac{1}{4}(F_x - F_y)^2 + Q_{xy}^2}.$$

$$\tan 2\phi = \frac{2Q_{xy}}{F_x - F_y}.$$







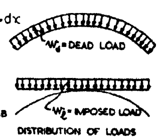
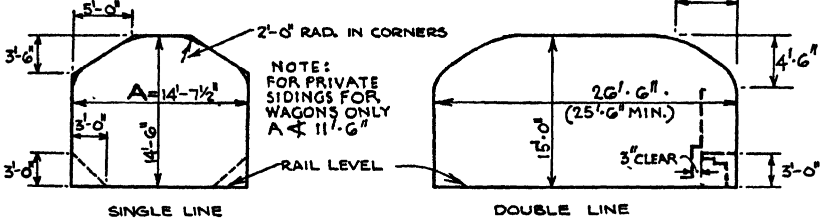
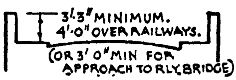


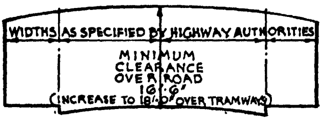
TABLE 92.—BRIDGES: GENERAL DATA.

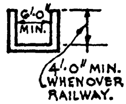
STRUCTURE GAUGE FOR RAILWAY OVERLINE BRIDGES. (APPROX.) (BRITISH RAILWAYS : MAIN LINES)



ROAD BRIDGES. BRIDGES OVER ROADS. FOOTBRIDGE.

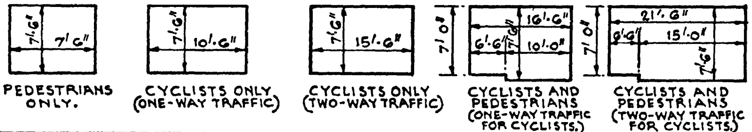




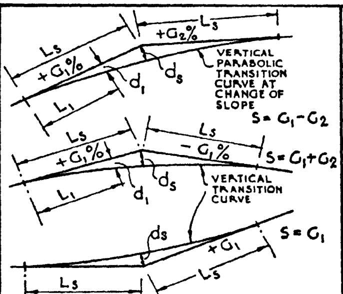


PARAPETS.

MINIMUM CLEARANCES FOR SUBWAYS.



ROAD PROFILE (MAIN ROADS)



$L_s = 12{\cdot}5\,(S + 10)$ FEET

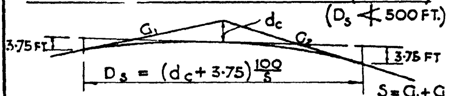
WHERE S = ALGEBRAIC DIFFERENCE OF SLOPES EXPRESSED AS PERCENTAGE. E.G. SLOPE 1 : 20 $= \frac{100}{20} = 5\% = G$

$$d_s = \frac{S L_s}{400}$$

AT ANY POINT : $d_1 = \left(\frac{L_1}{L_s}\right)^2 d$

MAX. GRADIENT FOR MAIN ROADS = 1 : 30 ; $G = 3\frac{1}{3}\%$

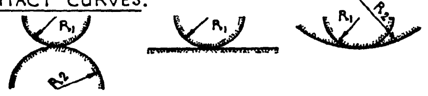
SIGHTING DISTANCE OVER CROWN OF BRIDGE (ROADWAY):– (D_s ≮ 500 FT.)



$$D_s = (d_c + 3{\cdot}75)\frac{100}{S}$$

BEARINGS

CONTACT CURVES.



$$\frac{L}{W} = \frac{1}{K}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad \frac{L}{W} = \frac{1}{KR_1} \qquad \frac{L}{W} = \frac{1}{K}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

L = LENGTH OF BEARING (IN.)
K = 342 FOR 1:1½:3 CONCRETE
= 2,840 FOR CAST STEEL (PER W.L SCOTT)
R = RAD. OF BEARING (IN.)
W = TOTAL LOAD ON BEARING (LB)

PERMISSIBLE STRESSES ON STEEL BEARINGS.

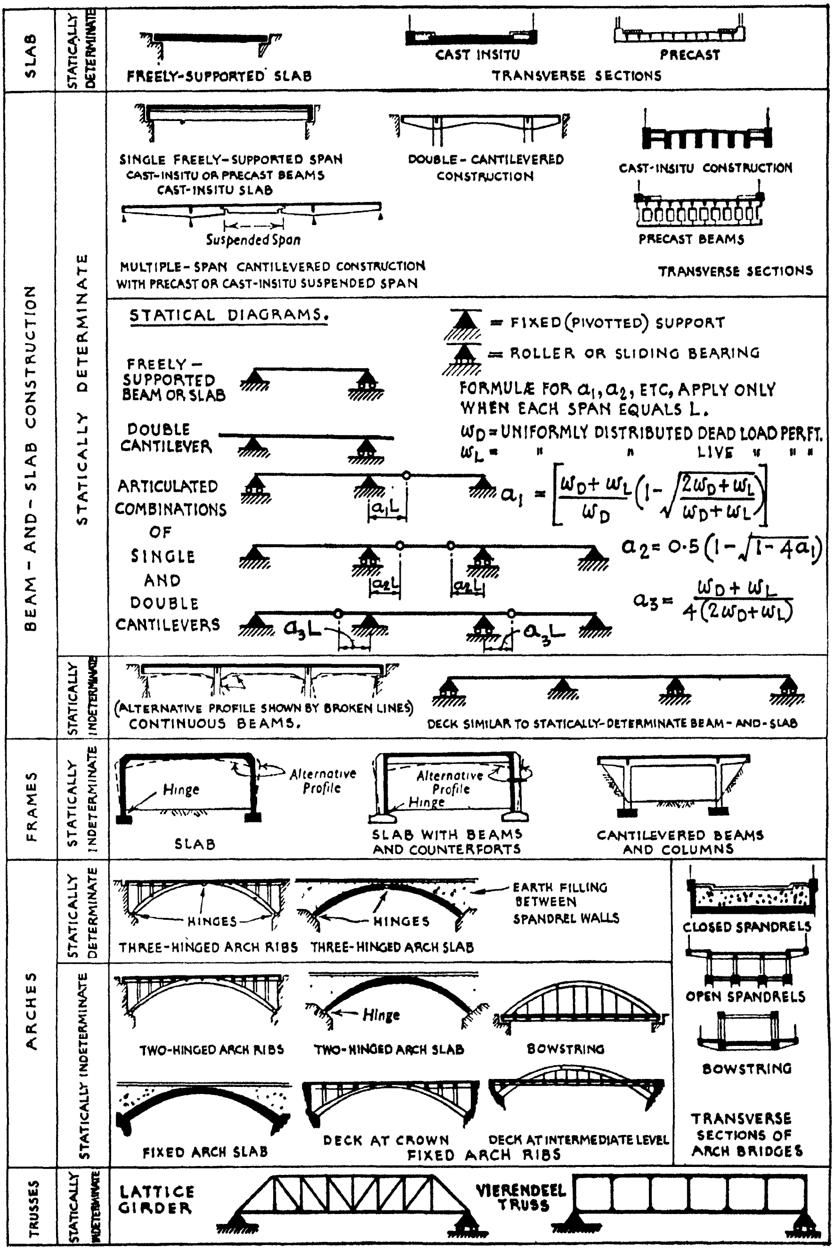
PINS.	BENDING	= 13½ TONS PER SQ IN
	BEARING	= 15 " " "
	SHEARING	= 6½ " " "

ROLLERS : D = DIA OF ROLLER (IN) ≮ 3¾
PRESSURE ON STEEL ROLLERS ≯ 0·28D TON PER LIN. IN.

LONGITUDINAL FORCES DUE TO BEARINGS.

FRICTIONAL RESISTANCE OF ROLLERS 1 PER CENT. ; 5 PER CENT. ≮ 3 ROLLERS
" " SLIDING PLATES 25 " " (STEEL ON STEEL OR CAST IRON)

EXPANSION OR CONTRACTION DUE TO TEMPERATURE : ALLOWED FOR BY CONSIDERING FRICTION ON EXPANSION BEARING UNDER DEAD LOAD AS ADDITIONAL ECCENTRIC THRUST OR PULL IN GIRDER


SLAB
STATICALLY DETERMINATE
FREELY-SUPPORTED SLAB
CAST INSITU
PRECAST
TRANSVERSE SECTIONS
BEAM - AND - SLAB CONSTRUCTION
STATICALLY DETERMINATE
SINGLE FREELY-SUPPORTED SPAN
CAST-INSITU OR PRECAST BEAMS
CAST-INSITU SLAB
DOUBLE - CANTILEVERED CONSTRUCTION
CAST-INSITU CONSTRUCTION
Suspended Span
PRECAST BEAMS
MULTIPLE-SPAN CANTILEVERED CONSTRUCTION WITH PRECAST OR CAST-INSITU SUSPENDED SPAN
TRANSVERSE SECTIONS
STATICAL DIAGRAMS.
= FIXED (PIVOTTED) SUPPORT
= ROLLER OR SLIDING BEARING
FREELY-SUPPORTED BEAM OR SLAB
FORMULÆ FOR a_1, a_2, ETC, APPLY ONLY WHEN EACH SPAN EQUALS L.
DOUBLE CANTILEVER
w_D = UNIFORMLY DISTRIBUTED DEAD LOAD PER FT.
w_L = " " LIVE " " "
ARTICULATED COMBINATIONS OF SINGLE AND DOUBLE CANTILEVERS
a_1L
$a_1 = \left[\frac{w_D + w_L}{w_D}\left(1 - \sqrt{\frac{2w_D + w_L}{w_D + w_L}}\right)\right]$
a_2L
$a_2 = 0{\cdot}5\left(1 - \sqrt{1 - 4a_1}\right)$
$a_3 = \frac{w_D + w_L}{4(2w_D + w_L)}$
a_3L
STATICALLY INDETERMINATE
(ALTERNATIVE PROFILE SHOWN BY BROKEN LINES)
CONTINUOUS BEAMS.
DECK SIMILAR TO STATICALLY-DETERMINATE BEAM-AND-SLAB
FRAMES
STATICALLY INDETERMINATE
Hinge
Alternative Profile
SLAB
Alternative Profile
Hinge
SLAB WITH BEAMS AND COUNTERFORTS
CANTILEVERED BEAMS AND COLUMNS
ARCHES
STATICALLY DETERMINATE
HINGES
THREE-HINGED ARCH RIBS
HINGES
EARTH FILLING BETWEEN SPANDREL WALLS
THREE-HINGED ARCH SLAB
CLOSED SPANDRELS
STATICALLY INDETERMINATE
Hinge
TWO-HINGED ARCH RIBS
TWO-HINGED ARCH SLAB
BOWSTRING
OPEN SPANDRELS
BOWSTRING
FIXED ARCH SLAB
DECK AT CROWN
DECK AT INTERMEDIATE LEVEL
FIXED ARCH RIBS
TRANSVERSE SECTIONS OF ARCH BRIDGES
TRUSSES
STATICALLY INDETERMINATE
LATTICE GIRDER
VIERENDEEL TRUSS

BEARINGS, HINGES, AND JOINTS.

Hinges and Bearings.—A hinge is an element that can transmit a thrust and a shearing force, but permits rotation without restraint. If it be essential that hinge action be fully realised, it can be effected by providing a steel hinge, or by forming a hinge monolithic with the member as shown at (*a*) in *Table* 94; this hinge is sometimes provided in a frame of a large bunker to isolate the container from the sub-structure or to provide a hinge at the base of the columns of a hinged frame bridge. The hinge-bars (a) resist the entire horizontal shearing force; the area of concrete at D must be sufficient to transfer all the compressive force from the upper to the lower part of the member. The hinge bars should be bound together by binders (d); the main vertical bars (e) should terminate on each side of the slots B and C. It may be advantageous during construction to provide bars extending across the slots, and to cut these bars on completion of the frame. The slots should be filled with bituminous material, lead, or a similar separating layer.

In *Table* 94 there are illustrated other types of hinges and bearings. (*b*) A hinge formed by the convex end of a concrete member bearing in a concave recess in the foundation. (*c*) A hinge suitable for the bearing of a girder where rotation, but not sliding, is required. (*d*) A bearing for a girder where sliding is required (*e*) A mechanical hinge suitable for the base of a large portal frame or the abutment of a large hinged arch rib. (*f*) A transverse expansion joint in the deck of a bridge, which is suitable if the joint is in the slab or is formed between two transverse beams. (*g*) A rocker-bearing suitable for girder bridges of spans of over 50 ft.; for spans less than 30 ft. an expansion is not always necessary, and up to 50 ft. a sliding bearing as at (*d*) is sufficient. (*h*) A hinge suitable for the crown of a three-hinge arch when a mechanical hinge is not justified. (*j*) A bearing suitable for the support of a freely-suspended span on a cantilever in an articulated bridge.

The permissible bearing stresses on bearings and the longitudinal forces due to bearings are given in *Table* 92; the load on bearings formed by two curved surfaces in contact is also given in this table Permissible bearing pressures on plain and reinforced concrete under bearing plates are given in *Table* 104.

Expansion and Contraction Joints.—The joints shown in *Table* 95 are expansion and contraction joints suitable for various structures. At (*a*) is a vertical joint in the stem of a cantilever retaining wall; the spacing depends on the height of the wall but should not exceed 60 ft. For low walls with thin stems a simple butt-joint is generally sufficient, but unequal deflection or tilting of one part of the wall relative to the next shows at the joint; a keyed joint as at (*a*) is therefore preferable for walls more than 4 ft. high. The key also reduces the risk of percolation of moisture through the joint. The double chamfer improves the appearance. Where it is necessary to provide for an amount of expansion exceeding the probable amount of shrinkage a space, say, $\frac{1}{2}$ in. wide, should be left between the faces of the concrete and filled with resilient material.

At (*b*) a method of bearing the base of the wall of a cylindrical tank on the bottom is illustrated, which ensures that bending moments are not induced in the wall due to restraint at the bottom. Because of the difficulty of maintaining such joints, monolithic construction and designing the wall to resist bending moment may be preferable.

In the walls of a reservoir a watertight joint must be provided to allow for movements due to shrinking and change of temperature; a suitable joint for a cantilevered wall or a wall that spans vertically is shown at (*c*).

In slabs laid on the ground (and not forming part of a foundation raft), construction joints should be permanent joints in predetermined positions, such as at the end of a day's work, at a restricted section, at a change of thickness, or at other positions where cracks are likely to occur. At (*d*) is shown such a joint that makes a definite break in the slab; this type of joint should be provided at intervals of about 15 ft. in the bottoms of reservoirs or in floors laid directly on the ground. The pad under the joint prevents one panel settling relatively to the other when laid on soft ground, but on firm ground the pad may not be necessary. When the floor is subject to abrasion, as in factories where wheeled containers travel across the joint, the edges should be protected by steel angles as indicated. If the ground is waterlogged, the joint at (*f*) can be adopted; this joint is also suitable for the floors of reservoirs.

In the case of cantilevered retaining walls and reservoir walls it is not always necessary to extend the joint from the walls into the base slab, but a longitudinal joint parallel to the wall should be provided in the base slab to separate the base of the wall from the remainder of the floor when the area of the floor is considerable.

If permanent joints are provided in buildings it is necessary to carry the joint through the floors, walls [as at (*h*)], and roof slab [as at (*g*)]. The joints in the walls should be made at the columns, in which case a double column as at (*e*) is provided; the space is filled with a joint filler; the copper strip or other type of water-bar must be notched where the binders occur, the ends of the notched pieces being bent horizontally or cut off. At joints through suspended floors and flat roofs it is common for a double beam to be provided. A joint filler and water-bar are required in a joint in a roof, but a joint in a floor should be sealed to prevent rubbish accumulating therein.

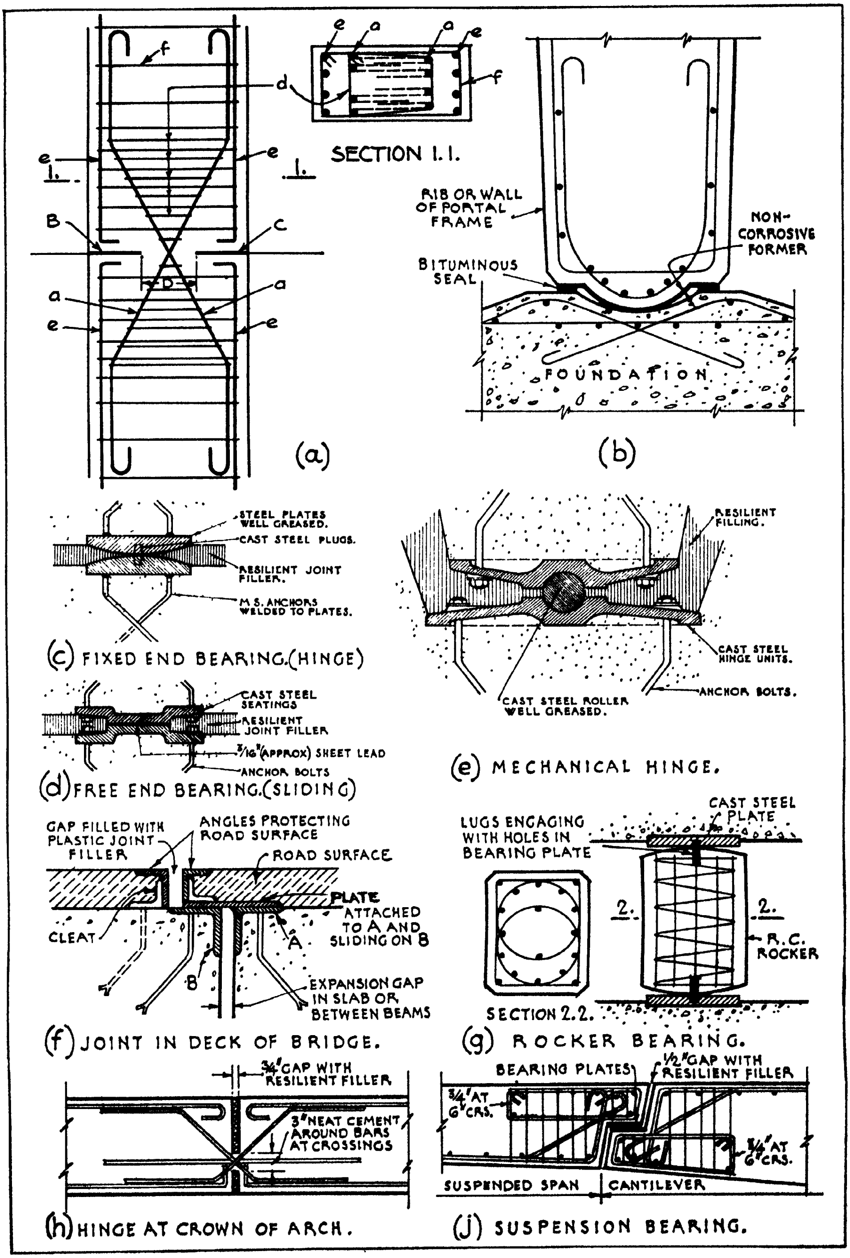
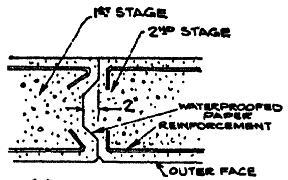
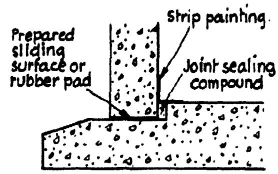

SECTION 1.1.
RIB OR WALL OF PORTAL FRAME
NON-CORROSIVE FORMER
BITUMINOUS SEAL
FOUNDATION
(a)
(b)
STEEL PLATES WELL GREASED.
CAST STEEL PLUGS.
RESILIENT JOINT FILLER.
M.S. ANCHORS WELDED TO PLATES.
(c) FIXED END BEARING.(HINGE)
RESILIENT FILLING.
CAST STEEL HINGE UNITS.
ANCHOR BOLTS.
CAST STEEL ROLLER WELL GREASED.
CAST STEEL SEATINGS
RESILIENT JOINT FILLER
3/16"(APPROX) SHEET LEAD
ANCHOR BOLTS
(d) FREE END BEARING.(SLIDING)
(e) MECHANICAL HINGE.
GAP FILLED WITH PLASTIC JOINT FILLER
ANGLES PROTECTING ROAD SURFACE
ROAD SURFACE
PLATE ATTACHED TO A AND SLIDING ON B
CLEAT
EXPANSION GAP IN SLAB OR BETWEEN BEAMS
(f) JOINT IN DECK OF BRIDGE.
CAST STEEL PLATE
LUGS ENGAGING WITH HOLES IN BEARING PLATE
R.C. ROCKER
SECTION 2.2.
(g) ROCKER BEARING.
3/4" GAP WITH RESILIENT FILLER
3" NEAT CEMENT AROUND BARS AT CROSSINGS
(h) HINGE AT CROWN OF ARCH.
BEARING PLATES
1/2" GAP WITH RESILIENT FILLER
3/4" AT 6" CRS.
SUSPENDED SPAN
CANTILEVER
(j) SUSPENSION BEARING.

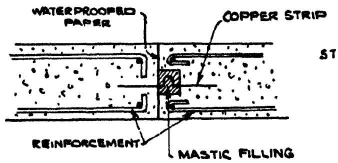
TABLE 95.—EXPANSION AND CONTRACTION JOINTS.



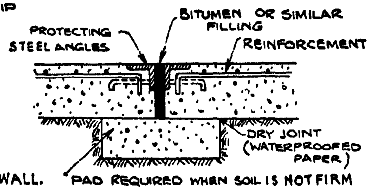
(a) VERTICAL JOINT IN CANTILEVERED RETAINING WALL.



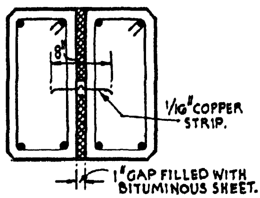
(b).- BASE OF WALL OF CYLINDRICAL TANK.



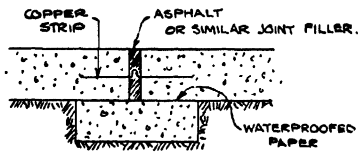
(c) VERTICAL JOINT IN RESERVOIR WALL.



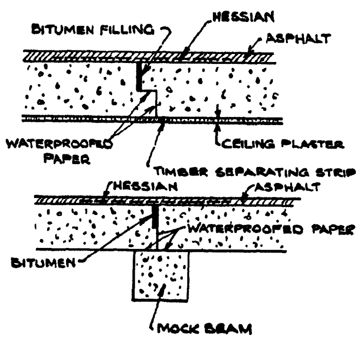
(d) JOINT IN FLOOR LAID ON THE GROUND.



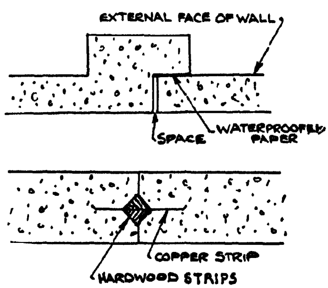
(e) EXPANSION JOINT AT COLUMN.



(f) JOINT IN FLOOR LAID ON THE GROUND.



(g) ALTERNATIVE DESIGNS FOR JOINTS IN ROOF SLABS.



(h) ALTERNATIVE DESIGNS FOR JOINTS IN EXTERNAL WALLS OF BUILDINGS.

NOTES.—(i) FOR OTHER TYPES OF JOINTS SUITABLE FOR RESERVOIRS, SEE B. S. CODE No. 2007.
(ii) COPPER-STRIP CAN BE REPLACED BY RUBBER OR SIMILAR WATER-BARS.

STRESSES IN ROAD SLABS

W = WHEEL LOAD (LB.).
d = THICKNESS OF CONCRETE SLAB (IN.).
f = MAXIMUM COMPRESSIVE AND TENSILE STRESS (LB. PER SQ. IN.).
R = RADIUS OF CONTACT AREA OF WHEEL ON SLAB.
R_1 = EQUIVALENT RADIUS OF CONTACT AREA (IF $R > 1{\cdot}724d$, $R_1 = R$).
E_c = ELASTIC MODULUS OF CONCRETE. μ = POISSON'S RATIO FOR CONCRETE (= 0 TO 0·3)
r = RADIUS OF RELATIVE STIFFNESS = $\sqrt[4]{\dfrac{E_c d^3}{12(1-\mu^2)k}}$
r_1 = COEFFICIENT (= $5r$ APPROX.). C = COEFFICIENT ($\ngtr 0{\cdot}4$).
k = MODULUS OF REACTION OF GROUND (= RECIPROCAL OF DEFORMATION IN INCHES WHEN SOIL IS LOADED WITH 1 LB. PER SQ. IN.)

SOFT AND PLASTIC SOILS: $k = 50$
FIRM SOIL: $= 200$
COMPACT GRAVEL AND ROCK: $= 500$

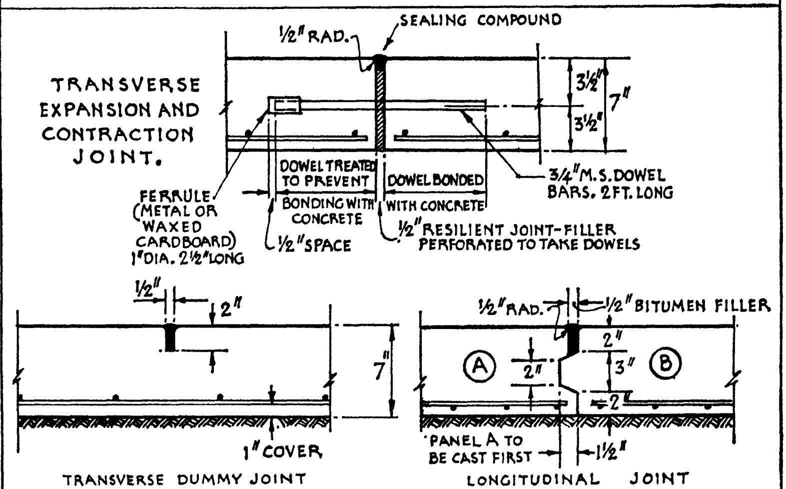
MAXIMUM STRESSES.

AT CORNER: $$f = \frac{3W}{d^2}\left[1 - \left(\frac{R\sqrt{2}}{r}\right)^{0{\cdot}6}\right]$$

AT AN EDGE: $$f = \frac{0{\cdot}53(1+0{\cdot}54\mu)W}{d^2}\left[\log\left(\frac{E_c d^3}{kR^4}\right) - 0{\cdot}71\right].$$

REMOTE FROM EDGE OR CORNER: $$f = \frac{W(1+\mu)}{d^2}\left[0{\cdot}275\log\left(\frac{E_c d^3}{kR_1}\right) - 15\left(\frac{r}{r_1}\right)^2\right]$$

TYPICAL JOINTS



TYPICAL DETAILS

TYPE OF SOIL	DENSITY OF TRAFFIC**	THICKNESS OF SLAB	REINFORCEMENT	NOTES
BAD GROUND (LOAM, PEAT, SILT, SANDY CLAY, ETC.)	LIGHT MEDIUM HEAVY VERY HEAVY	7 IN. 8 " 9 " 10 " *	10 LB PER SQ. YD. ONE LAYER. DITTO BUT IN EACH OF TWO LAYERS.	* FOR ADVERSE CONDITIONS (e.g. HEAVY DOCKS' TRAFFIC ON POOR SOIL) ≮ 12" SLAB. ** TRAFFIC:— LIGHT = ≯ 500 TONS PER DAY MEDIUM = 500 TO 5000 " " HEAVY = 5000 TO 10000 " " VERY HEAVY = ≮ 10000 " "
MEDIUM GROUND (VIRGIN CLAY, COMPACTED FILLING EXCEPT CLAY FILL)	LIGHT MEDIUM HEAVY VERY HEAVY	6 IN 7 " 8 " 9 "	8 LB. PER SQ. YD. ONE LAYER. DITTO BUT IN EACH OF TWO LAYERS.	
GOOD GROUND (GRAVEL, WELL-GRADED SAND, CHALK, ROCK, ETC.)	LIGHT MEDIUM HEAVY VERY HEAVY	5 IN. 7 " 8 " 9 "	7 LB. PER SQ. YD. ONE LAYER. DITTO BUT IN EACH OF TWO LAYERS.	

IF HIGH-YIELD-STRESS STEEL IS USED, WT. OF REINFORCEMENT = ⅔ OF WEIGHT TABULATED.

CYLINDRICAL TANKS AND WATER TOWERS.

Walls of Cylindrical Tanks.

Example.—To determine the bending moments and tensions in the wall of a cylindrical water tank 20 ft. in diameter, 30 ft. deep, with walls 10 in. thick at bottom. Thus $D = 20$ ft.; $H = 30$ ft.; $d = 0{\cdot}83$ ft.; $w = 62{\cdot}4$ lb. per cu. ft. $= \frac{H}{D} = \frac{30}{20} \cdot 1{\cdot}5$; $\frac{H}{d_A'} = \frac{30}{0{\cdot}83} = 36$; therefore from *Table* 97, $F = 0{\cdot}003$; $K_1 = 0{\cdot}20$; $K_2 = 0{\cdot}85$.

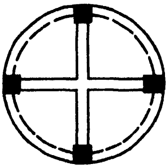
The bending moment (vertical) at A is $0{\cdot}003 \times 62{\cdot}4 \times 30^3 = 5050$ ft.-lb.

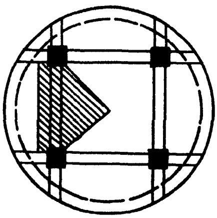
Position of point of maximum circumferential tension: $L = 0{\cdot}20 \times 30 = 6$ ft. from bottom.

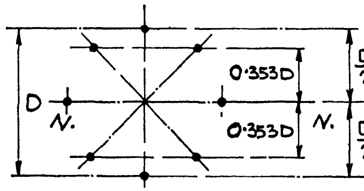
Maximum circumferential tension $= 0{\cdot}5 \times 62{\cdot}4 \times 30 \times 20 \times 0{\cdot}85 = 15{,}900$ lb. per ft. height.

Note.—If the bottom is suspended, and monolithic with the wall, deformation of the bottom slab may produce in the wall bending moments (vertical) which must be combined with those due to restraint at the base of the wall. Similarly, the bending moments and shearing forces due to this restraint affect the resultant moments and tensions in the bottom.

Bottoms and Roofs of Cylindrical Elevated Tanks.







(*a*) (*b*) (*c*)

Designs for the bottoms of elevated cylindrical tanks are given in the diagrams above and in *Table* 97.

With Beams.—At (*a*) each beam spans between opposite columns and carries one-quarter of the load of the bottom of the tank. The remaining half of the load and the weight of the wall and the load from the roof are transferred to the columns through the wall. In the arrangement shown at (*b*), each length of beam between columns carries the load on the shaded area, and the remainder of the load on the floor of the tank and the weight of the walls and the load from the roof are equally divided between the eight cantilevered lengths of the beams. An alternative to this design is for the columns to be placed almost under the wall, in which case the cantilevers are unnecessary but secondary beams may be required.

Domed Bottoms.—For a tank of large diameter a domed bottom and roof of either of the types shown in *Table* 97 are more economical, and although the shuttering is much more costly the saving in concrete and reinforcement compared with beam-and-slab construction may be considerable. The ring-beams marked R in the case of a simple domed bottom or roof resist the horizontal component of the thrust from the domes, and the thicknesses of the domes are determined by the magnitude of this thrust. The working compressive stress in the roof dome should be low, say 20 per cent. of the ordinary safe stress, in order to allow ample margin for local increases due to incidental concentrated loads and for irregular distribution of the imposed load. For the bottom dome, where the uniform distribution of the load is more assured, a higher stress can be used, and about ½ per cent. to 1 per cent. of reinforcement should be provided in each direction. The shearing stress around the periphery of the dome should also be calculated and sufficient thickness of concrete provided to resist the shearing forces. Expressions for the thrust and vertical shearing force around the edge of the dome and the resultant circumferential tension in the ring beams are given in *Table* 97. Domes can also be analysed by the method described in *Table* 91.

The bottom comprising a central dome and an outer conical part (called an Intze tank) illustrated at the bottom of *Table* 97 is economical for the largest tanks. The outward thrust from the top of the conical part is resisted by the ring beams S, and the difference between the inward thrust from the bottom of the conical part and the outward thrust from the domed part is resisted by the ring beam A_1. Expressions for the forces are given in *Table* 97. The proportions of the conical and domed parts can be arranged so that the resultant horizontal thrust on A_1 is zero. Suitable proportions for bottoms of this type are given in *Table* 97, and the volume for a tank of diameter D_0 with these proportions is $0{\cdot}585 D_0^3$. The wall of

(*Continued on page* 318.)

FORCES AND BENDING MOMENTS ON WALLS

DIRECT TENSION IN WALL DUE TO INTERNAL PRESSURE. p LB. PER SQ. FT.

$$T \text{ (LB PER FT.)} = \tfrac{1}{2} p D = \tfrac{1}{2} w h D$$

$$A_{st} = \frac{T}{p_{st}} = \frac{pD}{2p_{st}} = \frac{whD}{2p_{st}} \text{ (SQ. IN./FT.)}$$

*MINIMUM THICKNESS OF WALL = d IN. = $\dfrac{whD}{24}\left[\dfrac{1}{p_{ct}} - \dfrac{m-1}{p_{st}}\right]$

DIRECT COMPRESSION IN WALL OF UNDERGROUND OR SUBMERGED TANK. (EMPTY)

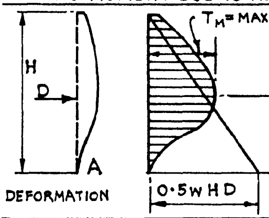
$$f_{cc} = \frac{p_e D}{2\left[12d + (m-1)A_{st}\right]} \text{ LB. PER SQ. IN.}$$

D = DIAMETER OF TANK (FT.)
w = WT. OF LIQUID CONTENTS (LB. PER CU. FT.)
h = DEPTH OF LIQUID (FT.)
$p_{st}(p_{ct})$ = PERMISSIBLE TENSILE STRESS IN STEEL (CONCRETE) LB. PER SQ. IN.
p = INTERNAL PRESSURE AT h. (LB. PER SQ. FT.)
p_e = EXTERNAL PRESSURE (LB. PER SQ. FT.)
H = DEPTH OF TANK (FT.) = DEPTH OF WATER
d'_A = THICKNESS OF WALL AT A (FT.)

IF CONTENTS ARE DRY GRANULAR MATERIALS SUBSTITUTE wk_2 FOR w WHERE $K_2 = \dfrac{1 - \sin\theta}{1 + \sin\theta}$

θ = ANGLE OF INTERNAL FRICTION **

BENDING MOMENT DUE TO RESTRAINT AT BASE OF WALL.



T_M = MAX. CIRCUMFERENTIAL TENSION $= \tfrac{1}{2} wHDK_2$ LB.

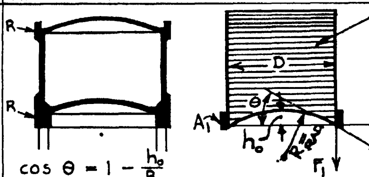
MAX. B.M. AT A $= FwH^3$ FT.-LB. PER FT.

L = POSITION OF MAX. CIRCUMFERENTIAL TENSION = $K_1 H$ FT.

VALUES OF FACTORS F, K_1 & K_2 TABULATED BELOW.

FACTORS		F				K_1				K_2			
$H \div d'_A$		10	20	30	40	10	20	30	40	10	20	30	40
VALUES OF $\frac{H}{D}$	0·2	0·046	0·028	0·022	0·015	—	0·50	0·45	0·40	0·32	0·46	0·53	0·59
	0·3	0·032	0·019	0·014	0·010	0·55	0·43	0·38	0·33	0·35	0·53	0·60	0·66
	0·4	0·024	0·014	0·010	0·007	0·50	0·39	0·35	0·30	0·44	0·58	0·65	0·70
	0·5	0·020	0·012	0·009	0·006	0·45	0·37	0·32	0·27	0·48	0·63	0·69	0·73
	1·0	0·012	0·006	0·005	0·003	0·37	0·28	0·24	0·21	0·62	0·73	0·74	0·83
	2·0	0·006	0·003	0·002	0·002	0·30	0·22	0·19	0·16	0·73	0·81	0·85	0·88
	4·0	0·004	0·002	0·002	0·001	0·27	0·20	0·17	0·14	0·80	0·85	0·87	0·90

DOMED BOTTOMS



$\cos\theta = 1 - \dfrac{h_o}{R}$

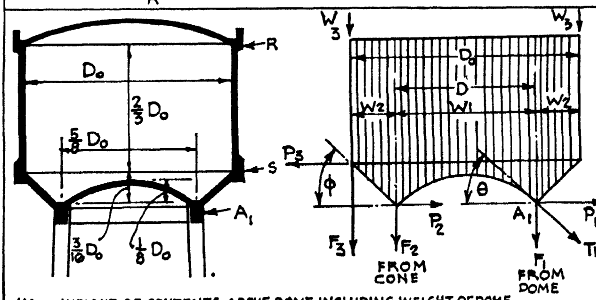
W_1 = WT. OF CONTENTS ABOVE DOME INCLUDING WEIGHT OF DOME (LB.)

SHEARING FORCE = $F_1 = \dfrac{W_1}{\pi D}$ (LB PER FT.)

CIRCUMFERENTIAL TENSION IN BEAM AT $A_1 = \tfrac{1}{2} P_1 D$ $= 0{\cdot}16 W_1 \cot\theta$

$P_1 = F_1 \cot\theta$

T_1 = THRUST AT PERIMETER OF DOME = $F_1 \operatorname{cosec}\theta$ (LB PER FT.)



VALUES OF F_1, T_1 & P_1 AS FOR SIMPLE DOME ABOVE.

$F_2 = \dfrac{W_2 + W_3}{\pi D}$ (LB. PER FT.)

$F_3 = \dfrac{W_3}{\pi D_o}$ (LB. PER FT.)

$P_2 = F_2 \cot\phi$ (LB. PER FT.)

$P_3 = F_3 \cot\phi$ (LB. PER FT.)

(IDEAL CASE: $P_1 = P_2$)

CIRCUMFERENTIAL TENSION IN BEAM AT A_1 $= \tfrac{1}{2} D (P_1 - P_2)$

W_1 = WEIGHT OF CONTENTS ABOVE DOME INCLUDING WEIGHT OF DOME.
W_2 = " " " " CONE " " " CONE.
W_3 = " " WALL, ETC., AND ALL LOADS ON ROOF INCLUDING WEIGHT OF ROOF, ETC.

NOTES.—* For design of slabs forming walls of containers in direct tension, see also *Table* 74.
** For angles of friction, see *Table* 11.

CYLINDRICAL TANKS AND WATER TOWERS

(*continued from page* 316).

Domed Bottoms (*cont.*)

the cylindrical part of the tank should be designed as described previously, account being taken of the vertical bending at the base of the wall and the effect of this bending on the conical part. The floor of a tank must be designed to resist, in addition to the forces and bending moments already described, any direct tension due to the vertical bending of the wall.

Example.—Determine the principal forces in the bottom of a cylindrical tank of the Intze type, if $D_0 = 40$ ft.; $D = 25$ ft.; $\phi = 48$ deg.; $\theta = 40$ deg.; $W_1 = 582{,}000$ lb.; $W_2 = 639{,}000$ lb.; and $W_3 = 296{,}000$ lb.

$$\text{From } \textit{Table } 10\text{: } \operatorname{cosec}\theta = \frac{1}{\sin\theta} = 1{\cdot}55;\ \cot\theta = 1{\cdot}192;\ \cot\phi = 0{\cdot}900.$$

From *Table* 97.

Vertical shearing force along periphery of dome: $F_1 = \dfrac{582{,}000}{3{\cdot}14 \times 25} = 7400$ lb. per ft.

Thrust at periphery of dome: $T_1 = 7400 \times 1{\cdot}55 = 11{,}450$ lb. per ft.

The values of F_1 and T_1 determine the thickness of the dome at the springing.

Outward horizontal thrust from dome on ring beam B: $P_1 = 7400 \times 1{\cdot}192 = 8820$ lb. per ft.

Shearing force along inner periphery of conical portion:

$$F_2 = \frac{639{,}000 + 296{,}000}{3{\cdot}14 \times 25} = 11{,}900 \text{ lb. per ft.}$$

Inward thrust on ring beam B from conical part: $P_2 = 11{,}900 \times 0{\cdot}900 = 10{,}700$ lb. per ft.

Resultant circumferential force in ring beam B $= 0{\cdot}5 \times 25(8820 - 10{,}700) = 23{,}500$ lb. (compression).

(If P_1 exceeds P_2, the circumferential force is tensile; the ideal case is for P_1 and P_2 to be equal, thereby producing zero force in B—see note below.)

Shearing force along outer periphery of conical part: $F_3 = \dfrac{296{,}000}{3{\cdot}14 \times 40} = 2350$ lb. per ft.

Outward thrust on ring beam at top of conical part: $P_3 = 2350 \times 0{\cdot}900 = 2120$ lb. per ft.

Circumferential tension in beam at top of conical section $= 0{\cdot}5 \times 40 \times 2120 = 42{,}400$ lb.

The vertical wall must be reinforced for circumferential tension due to the horizontal pressure of water: tension $= 0{\cdot}5 \times 62{\cdot}4 D_0 h$ lb. The conical part must be reinforced to resist a circumferential tension, and the reinforcement can be either distributed throughout the height of the conical part or concentrated in the ring beams at the top and bottom.

Note.—In Intze tanks of large diameter the width of the ring beam B may be considerable, in which case the weight of water immediately above the beam should not be considered as contributing to the forces on the dome and conical part. With a wide beam, W_1 is the weight of the contents over the net area of the dome and d is the internal diameter of ring beam; W_2 is the weight of the contents over the net area of the conical part and d for use with W_2 is the external diameter of ring beam. If this adjustment is made for a ring beam of reasonable width in the above example P_1 would more nearly balance P_2.

Columns Supporting Elevated Tanks.—The thrust and tension produced in columns due to wind forces on a tank supported by the column can be calculated for a group of four columns as follows. If the total wind moment is M_W ft.-lb. and the distance apart of columns is D ft., when the wind blows normal to the dimension D, the thrust on each column on the leeward side and the tension in each column on the windward side $= \dfrac{M_W}{2D}$ lb. When the wind blows normal to a diagonal at the group, the thrust on the leeward corner column and the tension in the windward corner column $= \dfrac{M_W}{D\sqrt{2}}$ lb.

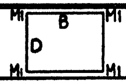
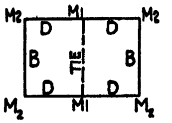
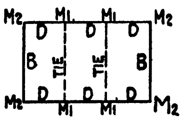
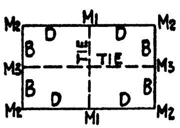
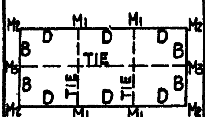
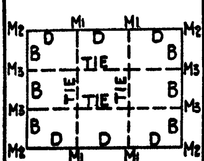
For any other number of columns, the force in any column can be calculated from the equivalent moment of inertia of the group. For example, consider a wind moment of M_W due to the wind blowing normal to the axis NN of the group of eight columns shown at (*c*) in the diagram on page 316.

Moment of inertia of the group about NN $= [2 \times (0{\cdot}5D)^2] + [4 \times (0{\cdot}353D)^2] = 1{\cdot}0D^2$.

Thrust on the extreme leeward column $= \dfrac{M_W \times 0{\cdot}5D}{1{\cdot}0D^2} = \dfrac{M_W}{2D}$.

The forces on each of the other columns can be determined similarly, substituting the appropriate "arm" for $0{\cdot}5D$.

BENDING MOMENTS AND REACTIONS IN WALLS SPANNING HORIZONTALLY AND SUBJECTED TO HORIZONTAL PRESSURE OF INTENSITY P.

FORM OF CONTAINER	BENDING MOMENTS. $M = \frac{PD^2}{k}$								REACTION FORMULÆ
	FORMULÆ	BENDING MOMENT COEFFICIENTS k							
		VALUES OF $\frac{B}{D}$							
		0·5	0·8	1·0	1·2	1·5	1·8	2·0	
	$M_1 = \frac{P(D^3+B^3)}{12(D+B)}$	16	14·3	12	9·68	6·86	4·92	4	$R_B = \frac{PB}{2}$; $R_D = \frac{PD}{2}$
	$M_1 = \frac{P(D^3+3D^2B-B^3)}{12(D+2B)}$	10·1	10·8	12	14·2	22·6	97	MINUS 60	$R_B = \frac{PB}{2}$
	$M_2 = \frac{P(D^3+2B^3)}{12(D+2B)}$	19·2	15·4	12	9·15	6·2	4·36	3·53	$R_D = \frac{PD}{2} + \frac{M_2-M_1}{D}$
	$M_1 = \frac{P(3D^3+6D^2B-B^3)}{12(3D+5B)}$	11·2	11·5	12	12·75	14·7	18	22·3	$R_B = \frac{PB}{2}$; R_D (END SPAN) $= \frac{PD}{2} + \frac{M_2-M_1}{D}$
	$M_2 = \frac{P(3D^3+5B^3)}{12(3D+5B)}$.	18·2	15·1	12	9·3	6·35	4·5	3·6	R_D (CENTRE SPAN) $= \frac{PD}{2}$
	$M_1 = \frac{P(D+B)(2D-B)}{24}$	10·68	11·1	12	13·6	19·2	42·9	–	$R_B = \frac{PB}{2} + \frac{M_2-M_3}{B}$
	$M_2 = \frac{P(D^3+B^3)}{12(D+B)}$	16	14·3	12	9·68	6·86	4·92	4	$R_D = \frac{PD}{2} + \frac{M_2-M_1}{D}$
	$M_3 = \frac{P(D+B)(2B-D)}{24}$	–	22·2	12	7·8	4·8	3·3	2·67	
	$M_1 = \frac{P(6D^3+6D^2B-B^3)}{12(6D+5B)}$	11·5	11·66	12	12·55	13·95	16·4	19·2	$R_B = \frac{PB}{2} + \frac{M_2-M_3}{B}$
	$M_2 = \frac{P(6D^3+5B^3)}{12(6D+5B)}$	15·4	14·10	12	9·85	7·09	5·12	4·17	R_D (END SPAN) $= \frac{PD}{2} + \frac{M_2-M_1}{D}$
	$M_3 = \frac{P(5B^3+9DB^2-3D^3)}{12(6D+5B)}$.	–	22·6	12	7·75	4·75	3·25	2·65	R_D (CENTRE SPAN) $= \frac{PD}{2}$
	$M_1 = \frac{P(5D^3+6D^2B-B^3)}{60(D+B)}$	11·4	11·61	12	12·6	14·11	16·71	20	R_B (END SPAN) $= \frac{PB}{2} + \frac{M_2-M_3}{B}$; R_B (CENTRE SPAN) $= \frac{PB}{2}$
	$M_2 = \frac{P(D^3+B^3)}{12(D+B)}$	16	14·3	12	9·68	6·86	4·92	4	R_D (END SPAN) $= \frac{PD}{2} + \frac{M_2-M_1}{D}$
	$M_3 = \frac{P(6B^3+6DB^2-D^3)}{60(D+B)}$	80	20	12	8·1.	5·12	3·53	2·86	R_D (CENTRE SPAN) $= \frac{PD}{2}$

For example of use of this table, see page 322.

DESIGN OF BUNKER WITH PYRAMIDAL HOPPER BOTTOM.

DIMENSIONS OF BUNKER. 15 FT. SQUARE. 12 FT. DEEP TO TOP OF HOPPER SLOPES.

DESIGN DATA

CONTENTS:— DRY COAL: $w = 50$ LB./CU.FT. $\theta = 35°$. $K_2 = 0{\cdot}27$ [PER TABLE (11)]

CONCRETE 1:2:4 ORDINARY GRADE. $p_{cb} = 1000$ LB./SQ.IN. [PER TABLE (56)]

MILD STEEL REINFORCEMENT. $p_{st} = 18{,}000$ " " "

COVER: $\frac{3}{4}$ IN. GENERALLY. ⊄ 1 IN. ON WEARING SURFACES.

CONSIDER $\frac{1}{2}$-IN. NON-STRUCTURAL THICKNESS TO ALLOW FOR WEAR.

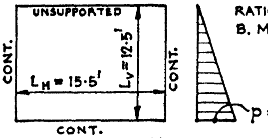
DESIGN FACTORS $(m = 15)$. $p_{st}/p_{cb} = 18$; $n_1 = 0{\cdot}455$; $a_1 = 0{\cdot}848$; $Q_c = 193$ [PER TABLE (68)]

WALLS

VERT. SPAN

$d_1 = 3\frac{3}{4}$"

$l_a = 0{\cdot}85 \times 3\frac{3}{4} = 3{\cdot}2$"



RATIO OF EFFECTIVE SPANS $= 15{\cdot}5/12{\cdot}5 = 1{\cdot}24$

B. M. COEFFICIENTS [PER TABLE (40)]

VERTICAL SPAN.—AT BOTTOM: 0·050

HORIZONTAL SPAN.—AT CORNERS: 0·035

AT MID-SPAN: 0·022

$p = 0{\cdot}27 \times 50 \times 12{\cdot}5 \doteqdot 170$ LB./SQ.FT.

5 IN. WALLS (MIN.) $4\frac{1}{2}$ IN. EFFECTIVE THICKNESS

VERTICAL SPAN.— $M = 0{\cdot}05 \times 170 \times 12{\cdot}5^2 \times 12 = 15{,}950$ IN.-LB./FT.

[PER TABLE (72) $4\frac{1}{2}$ IN. SLAB AMPLE]

$$A_{st} = \frac{15{,}950}{18{,}000 \times 3{\cdot}2} = 0{\cdot}277 \text{ SQ IN/FT.}$$ PER TABLE (60)

$\frac{1}{2}$" BARS AT 6" C/C VERTICALLY AT INNER FACE

HORIZ. SPAN AT MID-SPAN $d_1 = 3\frac{1}{2}$" $l_a = 3$"

AT CORNERS $d_1 = 3\frac{1}{4}$" $l_a = 0{\cdot}86 \times 3\frac{1}{4} = 2{\cdot}8$"

HORIZONTAL SPAN.— AT CORNERS: $M = 0{\cdot}035 \times 170 \times 15{\cdot}5^2 \times 12 = 17{,}150$ IN.-LB./FT.

AT $x = 0{\cdot}5$ [TABLE (40)]: $p_x = 0{\cdot}27 \times 50 \times (0{\cdot}5 \times 12{\cdot}5) = 85$ LB./SQ.FT.

$N \doteqdot 85 \times \frac{15}{2} \doteqdot 637$ LB./FT.

[PER TABLE (89)] $e = \frac{17{,}150}{637} = 27$ IN.; $e_s = 27 - \frac{4\frac{1}{2}}{2} + \frac{3}{4} = 25\frac{1}{2}$ IN.

$$A_{st} = \frac{637}{18{,}000}\left[1 + \frac{25\frac{1}{2}}{2{\cdot}8}\right] = 0{\cdot}36 \text{ SQ.IN.}$$

$\frac{1}{2}$" BARS AT 6" C/C HORIZONTALLY AT MID-HEIGHT AT INNER FACE AT CORNERS

AT MID-SPAN: $M = 0{\cdot}022 \times 170 \times 15{\cdot}5^2 \times 12 = 10{,}750$ IN.-LB./FT.

$e = \frac{10{,}750}{637} \doteqdot 17$ IN.; $e_s = 17 - \frac{4\frac{1}{2}}{2} + 1 = 15\frac{3}{4}$ IN.

$$A_{st} = \frac{637}{18{,}000}\left[1 + \frac{15\frac{3}{4}}{3}\right] = 0{\cdot}221 \text{ SQ IN.}$$

$\frac{1}{2}$" BARS AT 6" C/C HORIZONTALLY AT MID-HEIGHT AT OUTER FACE AT MID-SPAN

HOPPER BOTTOM

[FORMULÆ PER TABLE (89)] $L = B = 15$ FT. $a = b = 1{\cdot}5$ FT. $H = 12$ FT. $E = 7{\cdot}5$ FT.

BY SCALE (OR CALCULATION): $d = 10$ FT.

$\sin\theta = \frac{7{\cdot}5}{10} = 0{\cdot}75$ $\cos\theta = \frac{6{\cdot}75}{10} = 0{\cdot}675$

DIAMETER OF INSCRIBED CIRCLE (BY SCALE): $D = 8$ FT. $D_1 = 9{\cdot}5$ FT.

$h = 15$ FT. $S = 4{\cdot}5$ FT. $l = 9{\cdot}5$ FT. $w_s = 63$ LB./SQ.FT.

5" SLAB (MIN) $4\frac{1}{2}$" EFFECTIVE THICKNESS

w_1 = WT. OF BOTTOM BELOW C = 8,500 LB. APPROX.

w_2 = WT. OF ENTIRE BOTTOM = 22,500 " "

$$W = 50\left[\frac{7{\cdot}5}{3}(1{\cdot}5^2 + 15^2 + \sqrt{15^2 + 1{\cdot}5^2}) + (15^2 \times 12)\right] + 22{,}500 = 188{,}750 \text{ LB.}$$

$$W_1 = 50\left[\frac{4{\cdot}5}{3}(1{\cdot}5^2 + 9{\cdot}5^2 + \sqrt{9{\cdot}5^2 + 1{\cdot}5^2}) + (9{\cdot}5^2 \times 15)\right] + 8{,}500 = 93{,}650 \text{ LB.}$$

$$p_n = 50 \times 15\left[(0{\cdot}27 \times 0{\cdot}75^2) + 0{\cdot}675^2\right] + (63 \times 0{\cdot}675) \doteqdot 500 \text{ LB./SQ.FT.}$$

APPROX FOR ALL SECTIONS $d_1 = 3\frac{1}{2}$" $l_a = 3$"

M (AT ALL CRITICAL SECTIONS) $= 0{\cdot}375 \times 500 \times 8^2 = 12{,}000$ IN.-LB./FT.

NEGLECTING DIRECT TENSION, $4\frac{1}{2}$ IN. SLAB IS AMPLE [TABLE (72)]

HORIZONTAL REINFORCEMENT. $N = 0{\cdot}5 \times 500 \times 0{\cdot}75 \times 9{\cdot}5 = 1775$ LB/FT.

$e = \frac{12{,}000}{1775} = 6\frac{3}{4}$ IN. $e_s = 6\frac{3}{4} - \frac{4\frac{1}{2}}{2} + 1 = 5\frac{1}{2}$ IN.

$$A_{st} = \frac{1775}{18{,}000}\left[1 + \frac{5\frac{1}{2}}{3}\right] = 0{\cdot}28 \text{ SQ.IN./FT.}$$

$\frac{1}{2}$" BARS AT 8" C/C HORIZONTALLY

LONGITUDINAL (INCLINED) REINFORCEMENT.

AT CENTRE OF SIDE: $N = \frac{93{,}650}{2 \times 0{\cdot}75 \times 2 \times 9{\cdot}5} = 3300$ LB./FT.

$e = \frac{12{,}000}{3{,}300} = 3\frac{5}{8}$ IN. $e_s = 3\frac{5}{8} - \frac{4\frac{1}{2}}{2} + 1 = 2\frac{3}{8}$ IN.

$$A_{st} = \frac{3{,}300}{18{,}000}\left[1 + \frac{2\frac{3}{8}}{3}\right] = 0{\cdot}33 \text{ SQ.IN./FT.}$$

$\frac{1}{2}$" BARS AT 6" C/C ALONG SLOPE AT OUTER FACE

AT TOP OF SLOPE: $N = \frac{188{,}750}{2 \times 0{\cdot}75 \times 2 \times 15} = 4150$ LB./FT.

$e = \frac{12{,}000}{4150} \doteqdot 3$ IN. $e_s = 3 - \frac{4\frac{1}{2}}{2} + 1 = 1\frac{3}{4}$ IN.

$$A_{st} = \frac{4150}{18{,}000}\left[1 + \frac{1\frac{3}{4}}{3}\right] = 0{\cdot}365 \text{ SQ.IN./FT.}$$

$\frac{1}{2}$" BARS AT 6" C/C AT TOP OF SLOPE

BEAM AT TOP OF SLOPE

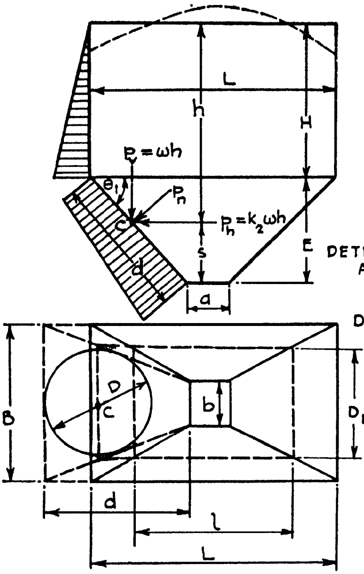
W + WEIGHT OF WALLS = 240,000 LB. APPROX. (TOTAL)

LOAD PER BEAM $= \frac{1}{4} \times 240{,}000 = 60{,}000$ LB.

$$A_{st} = \frac{60{,}000}{4 \times 18{,}000} = 0{\cdot}833 \text{ SQ.IN.}$$

2-$\frac{3}{4}$" BARS

PYRAMIDAL HOPPER BOTTOMS



ω = WT. PER CU. FT. OF FILLING.

ω_s = WT. OF SLAB PER SQ. FT.

INTENSITY OF PRESSURE NORMAL TO SLAB:

$$p_n = \omega h(k_2 \sin^2\theta_1 + \cos^2\theta_1) + \omega_s \cos\theta_1$$

$$W_1 = \omega\left[\frac{S}{3}\left(ab + lD_1 + \sqrt{ab.lD_1}\right) + lD_1h\right] + \omega_1$$

$$W = \omega\left[\frac{E}{3}\left(ab + LB + \sqrt{ab.LB}\right) + LBH\right] + \omega_2$$

ω_1 = WT. OF BOTTOM BELOW LEVEL OF C.

ω_2 = WT. OF COMPLETE BOTTOM.

DETERMINATION OF HORIZONTAL BARS AT MIDSPAN AND CORNERS:

$$\text{B.M.} = 0{\cdot}375\, p_n D^2 \quad \text{IN.-LB. PER FT.}$$

$$\text{DIRECT TENSION} = 0{\cdot}5\, p_n l \sin\theta_1 \quad \text{LB. PER FT.}$$

DETERMINATION OF LONGITUDINAL BARS

AT CENTRE OF SLOPE:

$$\text{B.M.} = 0{\cdot}375\, p_n D^2 \quad \text{IN.-LB. PER FT.}$$

$$\text{DIRECT TENSION} = \frac{W_1}{2\sin\theta(l+D)} \quad \text{LB. PER FT.}$$

AT TOP OF SLOPE:

$$\text{B.M.} = 0{\cdot}375\, p_n D^2 \quad \text{IN.-LB. PER FT.}$$

$$\text{DIRECT TENSION} = \frac{W}{2\sin\theta_1(L+B)} \quad \text{LB. PER FOOT}$$

VERTICAL HANGING-UP FORCE AT BASE OF WALLS

$$= \frac{W}{2(L+B)} \quad \text{LB. PER FT.}$$

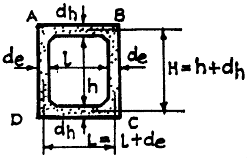
BOX CULVERTS

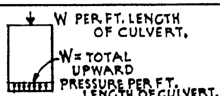
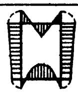
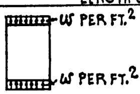
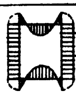
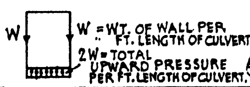
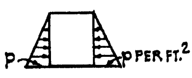
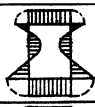
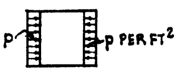
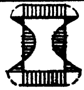
BENDING MOMENTS AT CORNERS.

$$M_A = M_B;\quad M_D = M_C$$

$$k = \frac{H}{L}\cdot\left(\frac{d_e}{d_h}\right)^3$$

LOADS AND BENDING MOMENTS ARE PER FOOT LENGTH OF CULVERT.



		B.M. diagrams	
CONCENTRATED LOAD ON ROOF		 B.M. DIAGRAMS	$M_A = -\frac{WL}{12}\left[\frac{2k+4{\cdot}5}{(k+3)(k+1)}\right]$ $M_D = -\frac{WL}{24}\left[\frac{k+6}{(k+3)(k+1)}\right]$
UNIFORMLY-DISTRIBUTED LOAD ON ROOF			$M_A = M_D = -\frac{wL^2}{12(k+1)}$
WEIGHT OF WALLS			$M_A = +\frac{WL}{6}\left[\frac{k}{(k+3)(k+1)}\right]$ $M_D = -\frac{WL}{6}\left[\frac{3+2k}{(k+3)(k+1)}\right]$
EARTH PRESSURE ON WALLS			$M_A = -\frac{pH^2}{60}\left[\frac{(2k+7)k}{(k+3)(k+1)}\right]$ $M_D = -\frac{pH^2}{60}\left[\frac{(3k+8)k}{(k+3)(k+1)}\right]$
PRESSURE ON WALLS DUE TO SURCHARGE			$M_A = M_D = -\frac{pH^2k}{24(k+1)}$

RECTANGULAR CONTAINER.

Example.—A single rectangular container, the inside dimensions of which are 12 ft. 9 in. by 10 ft. 6 in., is subject to a uniform horizontal pressure of 200 lb. per sq. ft. at a certain depth. Find the maximum bending moments and direct tensions at this depth, assuming that the walls span horizontally.

Assume the walls are 6 in. thick; effective spans = 13 ft. 3 in. and 11 ft.; hence $\frac{B}{D} = \frac{13{\cdot}25}{11{\cdot}0} = 1{\cdot}2$; from *Table* 98, $k = 9{\cdot}7$ (approx.).

$$\text{Bending moment at corners} = M_1 = \frac{PD^2}{k} = \frac{200 \times 11^2}{9{\cdot}7} = 2500 \text{ ft.-lb.}$$

$$\text{Free bending moment on } D = \frac{200 \times 11^2}{8} = 3025 \quad ,,$$

Deduct bending moment at corner = 2500 ,,

Positive bending moment at mid-span of D = 525 ,,

$$\text{Free bending moment on } B = \frac{200 \times 13{\cdot}25^2}{8} = 4390 \quad ,,$$

Deduct bending moment at corner = 2500 ,,

Positive bending moment at mid-span of B = 1890 ,,

$$\text{Direct tension in short side} = \frac{PB}{2} = 0{\cdot}5 \times 200 \times 12{\cdot}75 = 1275 \text{ lb. per ft.}$$

$$\text{Direct tension in long side} = \frac{PD}{2} = 0{\cdot}5 \times 200 \times 10{\cdot}5 = 1050 \text{ lb. per ft.}$$

The bending moments and direct tensions are combined as described in *Table* 89.

RETAINING WALLS.

Examples.—See pages 356 to 358 (Appendix II) for examples of designs of retaining walls.

Sheet-pile Wall.

Prepare a preliminary design for a simple cantilevered sheet-pile wall 12 ft. high; angle of internal friction of earth = 35 deg. No surcharge.

H = 12 ft.; θ = 35 deg.; w = 100 lb. per cu. ft. Hence $k_1' = 1{\cdot}4$; $k_2' = 1{\cdot}0$.

Embedded length of pile = 1·0 × 12 = 12 ft. Total length of pile = 12 + 12 = 24 ft.

Span of sheet-pile for calculating bending moment = L = 1·4 × 12 = 16·8 ft.

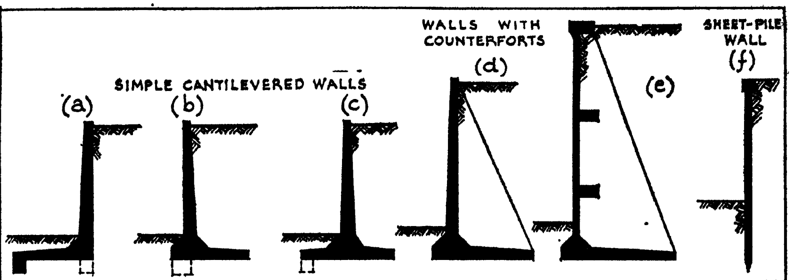
Total active pressure on back of wall on height L(= 16·8 ft.) = $k_2 \times 100 \times 16{\cdot}8^2 \times 0{\cdot}5$; $k_2 = 0{\cdot}27$ from *Table* 11; therefore the total pressure = 3810 lb. The bending moment = 3810 × 0·33 × 16·8 × 12 = 256,000 in.-lb. per ft. width, that is, 21,333 in.-lb. per inch, which from *Table* 70A with equal areas of steel in tension and compression and with maximum stresses of 18,000 lb. per sq. in. and 1000 lb. per sq. in. $d_1 = \sqrt{\frac{21{,}333}{507}} = 6{\cdot}3$ in.; with not less than $1\frac{1}{2}$-in. cover, the thickness of pile required is about 9 in., l_a is about 6 in., and the reinforcement required in a pile 15 in. wide is approximately $\frac{21{,}333 \times 15}{18{,}000 \times 6} = 2{\cdot}97$ sq. in., say a total of six $1\frac{1}{8}$-in. bars (three bars on each face).

CHIMNEYS.

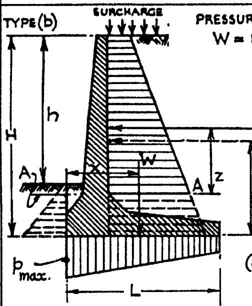
The factors and formulæ given in the table below enable the approximate thickness of the shaft and the amount of reinforcement therein to be determined to a degree of accuracy sufficient for preliminary designs; the stresses in a final design should be checked by more exact methods.

DIMENSIONS OF CHIMNEY SHAFTS (FOR PRELIMINARY DESIGNS).						
STRESSES LB. PER SQ. IN.	IN CONCRETE	600	600	600	400	R = MEAN RADIUS OF SHAFT (IN.)
	IN STEEL	12 000	14000	16 000	16 000	TOTAL AREA OF VERTICAL STEEL $A_{st} = \frac{M}{AR} - \frac{W}{B}$ SQ. IN.
FACTORS	A	6 408	7551	8685	8 600	
	B	7776	8990	10230	9698	THICKNESS OF CONCRETE
	C	4582	3409	2867	1376	$d = \frac{M}{CR^2} + \frac{W}{DR}$ IN.
	D	1372	1410	1419	911	M = BENDING MOMENT (IN.-LB.) W = TOTAL WEIGHT (LB.)

TYPES OF RETAINING WALLS



CANTILEVERED RETAINING WALLS



PRESSURES ** AND LOADS IN LB. PER FT. LENGTH OF WALL.

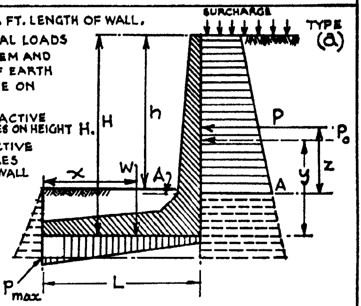
W = RESULTANT OF ALL VERTICAL LOADS INCLUDING WEIGHTS OF STEM AND BASE OF WALL, WEIGHT OF EARTH ON BASE; AND SURCHARGE ON GROUND BEHIND WALL.

P = RESULTANT OF ALL ACTIVE HORIZONTAL PRESSURES ON HEIGHT H.

P_o = RESULTANT OF ALL ACTIVE HORIZONTAL PRESSURES ACTING ON STEM OF WALL ABOVE A-A.

B.M. AT A-A $= P_o z$ FT.-LB./FT.

(ALSO CHECK M_r WITH B.M. AT BOTTOM OF SPLAY)



FOR STABILITY: $Wx \nless F_s Py$.

RESISTANCE TO SLIDING: FRICTIONAL RESISTANCE ONLY — $\mu W \nless F_\mu P$ (GRANULAR SOILS ONLY); PASSIVE RESISTANCE ONLY — PASSIVE RESISTANCE OF GROUND IN FRONT OF WALL $\nless F_p P$

MINIMUM FACTORS OF SAFETY: $F_s \nless 1\tfrac{1}{2}$. $F_\mu \nless 1\tfrac{1}{2}$. $F_p \nless 2$. μ FOR GRANULAR SOILS $\ngtr 0.4$.

GROUND PRESSURE. e = ECCENTRICITY OF W AND P COMBINED $= \dfrac{Py}{W} + \dfrac{L}{2} - x \ngtr \dfrac{L}{6}$.

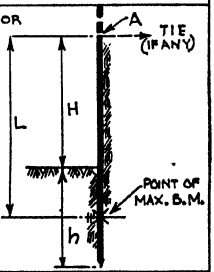
MAX. GROUND PRESSURE $= p_{max.} = \dfrac{W}{L}\left[1 + \dfrac{6e}{L}\right] \ngtr$ SAFE BEARING CAPACITY.

SHEET-PILE RETAINING WALLS

ANGLE OF INTERNAL FRICTION OF GROUND	CONDITION AT A: FREE k_1'	FREE k_2'	HINGED k_1'	HINGED k_2'	FIXED k_1'	FIXED k_2'
20°	2·0	2·0	1·23	1·23	1·11	1·11
30°	1·5	1·3	1·08	0·88	1·07	0·87
35°	1·4	1·0	1·07	0·67	1·06	0·66
45°	1·3	0·8	1·06	0·56	1·05	0·55

SPAN OF SHEET PILES FOR CALCULATION OF B.M.* $= L = k_1' H$.

MINIMUM EMBEDDED LENGTH OF SHEETING $= h = k_2' H$



FREE = SIMPLE CANTILEVERED WALL.
FIXED = TIE BELOW TOP OF WALL.
HINGED = TIE AT TOP OF WALL.
TABULATED VALUES ASSUME PRESSURES TRIANGULARLY DISTRIBUTED AND ARE APPLICABLE TO PRELIMINARY DESIGNS. FOR FINAL DESIGNS ACTUAL CONDITIONS SHOULD BE CONSIDERED.

B.M. REDUCTION FACTORS (DO NOT APPLY TO SIMPLE CANTILEVERED WALLS) FACTORS ARE DUE TO PRESSURE REDISTRIBUTION AND NOT TO REDUCTION OF PRESSURE

THICKNESS OF WALL / SPAN \ ANGLE OF INTERNAL FRICTION OF GROUND	5 DEG. OR LESS	15 DEG.	20 DEG.	30 DEG.	35 DEG.	45 DEG. OR OVER
0·02	1·00	0·80	0·70	0·56	0·48	0·36
0·10	1·00	0·88	0·80	0·69	0·62	0·50
0·20	1·00	0·92	0·86	0·78	0·72	0·60
0·30	1·00	0·93	0·90	0·83	0·78	0·69
0·40	1·00	0·96	0·91	0·87	0·83	0·76

* See *Table* 101 for calculation of bending moments.
** See *Tables* 11, 12 and 13 for calculation of earth pressures on walls.

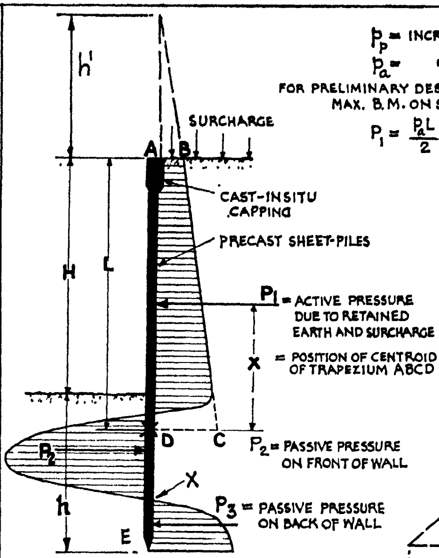
TABLE 101.—SIMPLE SHEET-PILE RETAINING WALLS.

SIMPLE CANTILEVERED SHEET-PILE WALLS WITHOUT TIES

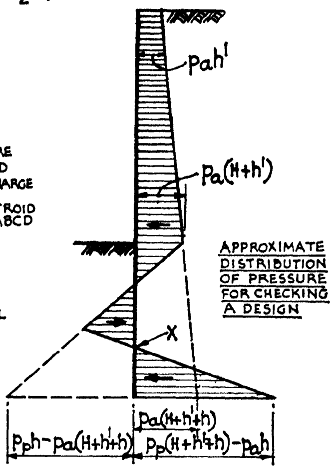
p_p = INCREASE OF PASSIVE RESISTANCE PER FOOT DEPTH.
p_a = " " ACTIVE PRESSURE " " "

FOR PRELIMINARY DESIGN:–
MAX. B.M. ON SHEET-PILE = $P_1 x$

$$P_1 = \frac{p_a L}{2}(2h' + L)$$



PROBABLE VARIATION OF PRESSURE



APPROXIMATE DISTRIBUTION OF PRESSURE FOR CHECKING A DESIGN

FORMULÆ FOR CHECKING A DESIGN

SUBSTITUTE KNOWN VALUES OF p_a, H, h' AND h IN

$$p_a(H+h'+h)^3 - p_p h^3 + \frac{[p_p h^2 - p_a(H+h'+h)^2]^2}{(p_p - p_a)(H+h'+2h)} = 0$$

TO GIVE AN EXPRESSION IN THE FORM

$$N_1 - N_2 p_p + \frac{N_3(p_p - N_4)^2}{p_p - N_5} = 0$$

IN WHICH N_1, N_2, N_3, N_4 AND N_5 ARE NUMERICAL FACTORS; CALCULATE

$$a = (N_3 - N_2);\quad b = (N_1 + N_2 N_5 - 2N_3 N_4);\quad c = (N_4^2 N_3 - N_1 N_5)$$

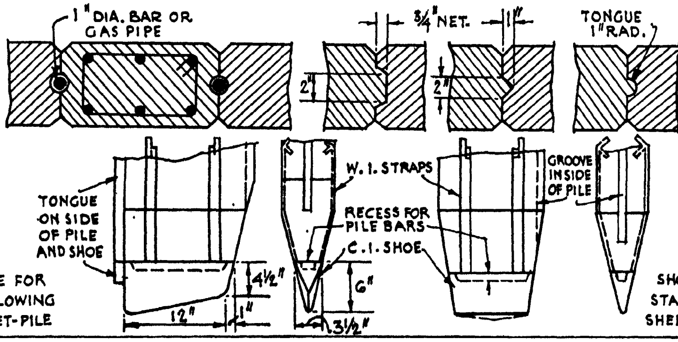
SUBSTITUTE IN $p_a = \frac{1}{2a}\left[\sqrt{b^2 - 4ac} - b\right]$

COMPARE CALCULATED VALUE OF p_a WITH PROBABLE VALUE WITH A FACTOR OF SAFETY.

NOTE

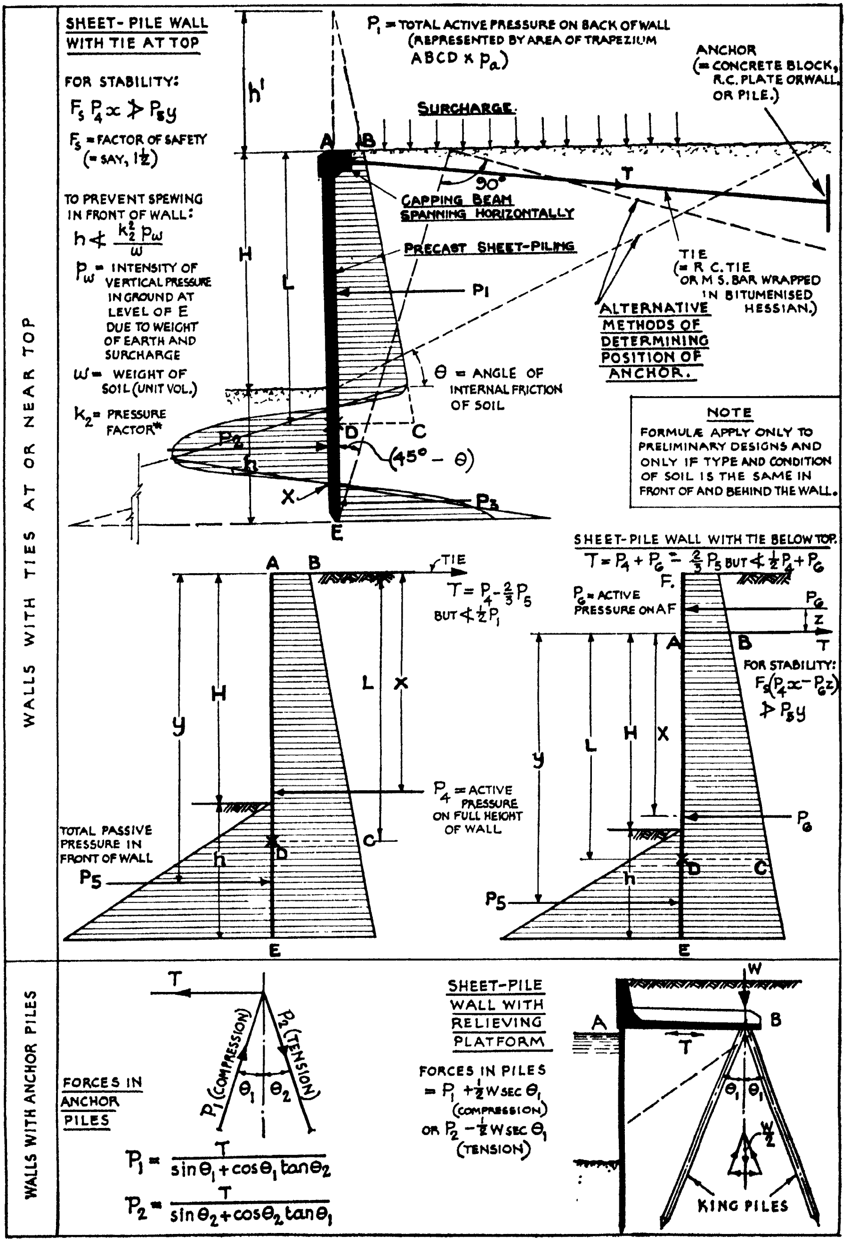
FORMULÆ APPLICABLE ONLY IF TYPE AND CONDITION OF SOIL IS THE SAME IN FRONT OF AND BEHIND WALL.

SHOES AND INTERLOCKS



SHOE FOR FOLLOWING SHEET-PILE

SHOE FOR STARTING SHEET-PILE



* See *Table* 11 for values of k_2.

COMBINED FOUNDATIONS.

Combined Bases (Strip Bases).—When more than one column or load is carried on a single base the centre of gravity of the several loads should, if possible, coincide with the centre of the area of the base, in which case the pressure under the base is uniformly distributed. The base should be symmetrically disposed about the line of the loads and may be rectangular in plan as in *Table* 105, trapezoidal as in *Table* 105 or at (i) in the upper part of *Table* 103, or made up of a series of rectangles as at (ii) in *Table* 103. In the last case each rectangle should be proportioned so that the load upon it acts at the centre of its area, and the area of each rectangle should be equal to the corresponding applied load divided by a safe bearing pressure, the value adopted for this pressure being the same for all the rectangles.

If it is not practicable to proportion the bases as described the load will be eccentric, and thus the centre of pressure of the upward ground pressure will have the same eccentricity relative to the centre of the area of the base. If the base is so thick that it may be considered to act as a single rigid member, the ground pressure will vary according to the formulæ for eccentric loads given in *Table* 104, and the pressure-distribution diagram is as at (iii) in *Table* 103. If the base is comparatively thin this distribution may not be realised, and owing to the flexibility of the base the ground pressure may be greater immediately under the loads, giving a pressure-distribution as at (iv) in *Table* 103.

In the case of uniform distribution, or uniform variation of distribution, of pressure the longitudinal bending moment on the base at any section is the sum of anti-clockwise moments of each load to the left of the section minus the clockwise moment of the upward pressure between the section and the left-hand end of the base. This method of analysis gives larger values for longitudinal bending moments on the base than if a non-linear variation is assumed, and therefore the assumption of linear distribution on which the formulæ in *Table* 105 (concentric load) and on page 330 and in *Table* 105 (eccentric load) are based, is safe.

Balanced Bases.—Referring to the diagrams in the upper right-hand corner of *Table* 103, a column is supported on the overhanging end C of the beam BC at (i) which is supported on a base at A and subjected to a counterbalance at B. The reaction at A, which depends on the relative values of BC and BA, can be provided by an ordinary reinforced concrete or plain concrete base designed for a concentric load. The counterbalance can mostly be provided by the load from another column as at (ii), in which case the dead load on this column at B should be sufficient to counterbalance the dead and live loads on the column at C, and vice versa, with a sufficient margin of safety. It is often possible to arrange the column B immediately over A_1. Formulæ giving the values of the reactions at A and A_1 are given in *Table* 105, but the notes on the page facing the table should be observed. From the reactions the shearing forces and bending moments on the beam can be calculated.

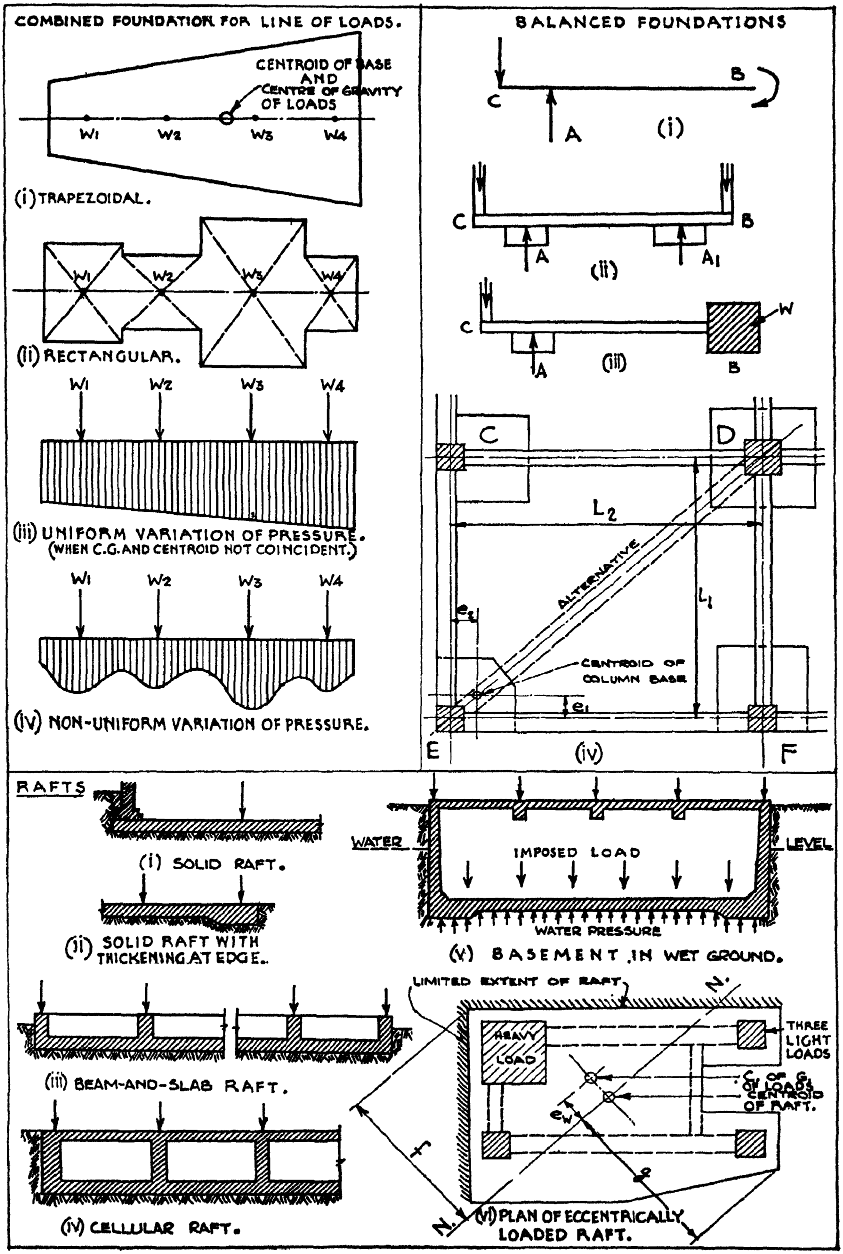
If no column loads can be conveniently brought into service to counterbalance the column at C an anchorage must be provided at B by other means, such as the construction of a plain concrete counterweight as at (iii) or the provision of tension piles.

If the column to be supported is a corner column loading the foundation eccentrically in two directions, one parallel to each building line, it is sometimes possible to introduce a diagonal balancing beam which is anchored by the adjacent internal column D as at (iv) In other cases, however, the two wall beams meeting at the column can be designed as balancing beams to overcome the double eccentricity. The bending moment due to the cantilever action in the beam EC is equal to $W_E e_1$, where W_E is the column load, and the upward force on column C is $\frac{W_E e_1}{L_1 - e_1}$. Similarly the bending moment in beam EF is $W_E e_2$ and the upward force on column F is $\frac{W_E e_2}{L_2 - e_2}$.

Rafts.—The load on and the spacing of the columns determine the shearing forces and bending moments which in turn determine the thickness of the raft. If this thickness does not exceed 12 in. a solid slab as at (i) in the lower part of *Table* 103 is generally the most convenient form. If a slab at ground level is required, it is nearly always necessary to thicken the slab at the edge, as at (ii), to ensure that the edge of the raft is deep enough below the ground to avoid weathering of the ground under the raft. If a greater thickness is required beam-and-slab construction, designed as an inverted floor, as at (iii), is more economical. In cases where the total depth required exceeds 3 ft., or where a level top surface is required, cellular construction as at (iv), consisting of a top and bottom slab with intermediate ribs, is used.

When the columns on a raft are not equally loaded or are not symmetrically arranged, the raft should be designed so that the centroid coincides with the centre of gravity of the loads. As this gives uniformly-distributed pressure on the ground, the area of the raft is equal to the total load (including the weight of the raft) divided by the safe bearing pressure. If this coincidence of centres of gravity is impracticable owing to the extent of the raft being limited on one or more sides, the plan of the raft should be made so that the eccentricity e_w

(*Continued on page* 328.)


COMBINED FOUNDATION FOR LINE OF LOADS.
CENTROID OF BASE AND CENTRE OF GRAVITY OF LOADS
W1
W2
W3
W4
(i) TRAPEZOIDAL.
(ii) RECTANGULAR.
(iii) UNIFORM VARIATION OF PRESSURE.
(WHEN C.G. AND CENTROID NOT COINCIDENT.)
(iv) NON-UNIFORM VARIATION OF PRESSURE.
BALANCED FOUNDATIONS
C
A
B
(i)
A1
(ii)
W
(iii)
D
L2
L1
ALTERNATIVE
e2
e1
CENTROID OF COLUMN BASE
E
F
(iv)
RAFTS
(i) SOLID RAFT.
(ii) SOLID RAFT WITH THICKENING AT EDGE.
(iii) BEAM-AND-SLAB RAFT.
(iv) CELLULAR RAFT.
WATER
LEVEL
IMPOSED LOAD
WATER PRESSURE
(v) BASEMENT IN WET GROUND.
LIMITED EXTENT OF RAFT
N.
HEAVY LOAD
THREE LIGHT LOADS
C. OF G. OF LOADS
CENTROID OF RAFT.
e_w
r
(vi) PLAN OF ECCENTRICALLY LOADED RAFT.

COMBINED FOUNDATIONS (*continued from page* 326).

of the total loading W_T is a minimum, and this may produce a raft which is not rectangular in plan, as in the example illustrated at vi.

Maximum pressure on the ground (which should not exceed the safe bearing resistance and occurs at f) $= \dfrac{W_T}{A_R} + \dfrac{W_T e_{wf}}{I_R}$, where A_R = total area of raft, and I_R = moment of inertia about the axis NN which passes through the centroid of the raft and is normal to the line joining the centroid and the centre of gravity of the loads.

Minimum ground pressure (at g) $= \dfrac{W_T}{A_R} - \dfrac{W_T e_{wg}}{I_R}$.

Intensity of ground pressure along the line NN $= \dfrac{W_T}{A_R}$.

When the three pressures have been determined, the pressure at any other point or the mean pressure over any area can be assessed. Having arranged a rational system of beams or ribs dividing the slab into suitable panels, as suggested by the dotted lines at (vi), the panels of slabs and the beams can be designed for the bending moments and shearing forces due to the net upward pressures to which they are subjected.

SEPARATE BASES (*Table* 104).

Examples.—(*a*) Determine the variation in pressure under a plain concrete foundation 10 ft. long by 2 ft. thick by 8 ft. wide carrying a load of 100 tons placed 1 ft. eccentrically.

Weight of base $= \dfrac{8 \times 10 \times 2 \times 140}{2240} = 10$ tons. Total load = 110 tons.

Moments about the short side of the base $= (10 \times 5) + (100 \times 4) = 450$ ft.-tons.

Eccentricity $= \dfrac{10}{2} - \dfrac{450}{110} = 0{\cdot}91$ ft. Since $\dfrac{L}{6} = \dfrac{10}{6} = 1{\cdot}67$ ft. and $e = 0{\cdot}91$ ft., $e < \dfrac{L}{6}$. Thus $k = \left(1 \pm \dfrac{6 \times 0{\cdot}91}{10}\right) = 1{\cdot}545$ or $0{\cdot}455$.

Maximum ground pressure $= \dfrac{1{\cdot}545 \times 110}{10 \times 8} = 2{\cdot}13$ tons per sq. ft.

Minimum ground pressure $= \dfrac{0{\cdot}455 \times 110}{10 \times 8} = 0{\cdot}625$ ton ,, ,,

(*b*) Determine the ground pressure in (*a*) if the load is 2 ft. eccentric.

Moments about the short side of the base $= (10 \times 5) + (100 \times 3) = 350$ ft.-tons.

Eccentricity $= 5 - \dfrac{350}{110} = 1{\cdot}82$ ft.; $e > \dfrac{L}{6}$ Thus $k = \dfrac{4}{3\left(1 - \dfrac{2 \times 1{\cdot}82}{10}\right)} = 2{\cdot}09$.

(This condition is not advisable.)

Maximum ground pressure $= \dfrac{2{\cdot}09 \times 110}{10 \times 8} = 2{\cdot}89$ tons per sq ft.

(*c*) Design a base for a reinforced concrete column 15-in. square carrying a concentric load of 120,000 lb.; ground pressure not to exceed 3000 lb. per sq. ft. Permissible stresses 18,000 lb. and 1000 lb. per sq. in. $Q_c = 193$ (*Table* 68).

Assume $W_B = 4000$ lb. $A \nless \sqrt{\dfrac{124{,}000}{3000}} = 6{\cdot}4$ ft., say, 6 ft. 6 in. square.

$p_{net} = \dfrac{120{,}000}{6{\cdot}5^2} = 2840$ lb. per sq. ft. $M_{XX} = \dfrac{2840 \times 6{\cdot}5}{8}(6{\cdot}5 - 1{\cdot}25)^2 = 63{,}500$ ft.-lb.

Make $B = 18$ in.; $d \nless \sqrt{\dfrac{63{,}500}{193 \times 1{\cdot}5}} = 14{\cdot}8$ in., say, $D_B = 18$ in.

If $D_C = 0{\cdot}25$ ft., $d = 18 - 3 = 15$ in.; $A_{st} = \dfrac{12 \times 63{,}500}{\frac{7}{8} \times 15 \times 18{,}000} = 3{\cdot}21$ sq. in.

Provide twelve $\frac{5}{8}$-in. bars (*Table* 60) in each layer.

If the base is tapered from 18 in. at the column to $D_V = 6$ in. at the edge, and

$$F = 1{\cdot}25 + 2(1{\cdot}5 - 0{\cdot}25) = 3{\cdot}75 \text{ ft.}$$

$$d_1 = 12\left[\frac{(D_B - D_V)(A - F)}{A - B} + D_V - D_C\right] = 12\left[\frac{1{\cdot}0 \times 2{\cdot}75}{5} + 0{\cdot}5 - 0{\cdot}25\right] = 10 \text{ in. approx.}$$

Check for shearing resistance: $Q = \dfrac{2840}{4 \times 3{\cdot}75}(6{\cdot}5^2 - 3{\cdot}75^2) = 5330$ lb. per ft. approx.

$d_1 \nless \dfrac{5330}{10{\cdot}5 \times 100} = 5{\cdot}1$ in., which is less than d_1 provided (10 in.).

SAFE BEARING PRESSURES (TONS PER SQ. FT.) ON GROUND AND CONSTRUCTIONAL MATERIALS

TYPE OF GROUND			TOTAL PER SQ. FT.
SILT, ALLUVIAL EARTH, ETC.			0 TO 3/4
CLAY:	SOFT OR VERY SOFT		< 3/4
	SANDY	FIRM	3/4 TO 1½
		STIFF	1½ TO 3
	FIRM		3/4 TO 1½
	STIFF		1½ TO 3
	HARD SHALEY CLAY		3 TO 6
	VERY STIFF (BOULDER)		3 TO 6
	SOUND YELLOW		3 TO 5
	BLUE		4 TO 6
SAND:	UNIFORM	LOOSE	1 TO 2
		COMPACT	2 TO 4
	WELL GRADED	LOOSE	2 TO 4
		COMPACT	4 TO 6
GRAVEL:	SANDY	LOOSE	2 TO 4
		COMPACT	4 TO 6
	CLEAN	LOOSE	3
		COMPACT	4 TO 7
ROCK:	CHALK	SOFT	1½
		HARD	3 TO 6
	SOFT		2
	MODERATELY HARD		5 TO 10
	HARD		12

CONSTRUCTIONAL MATERIAL: DESCRIPTION		CRUSHING STRENGTH LB. PER SQ. IN.	BEARING PRESSURES TONS PER SQ. FT.: UNIFORM PRESSURE	ECCENTRIC LOAD	CONCENTRATED LOAD
PLAIN CONCRETE	1:8	1200	15	18¾	22½
	1:4:8	1250	16	20	24
	1:6	1600	20	25	30
	1:3:6	1650	22½	28	33¼
	1:2:4	2250	38¾	47¼	58
	1:2:4	3000	64¼	80	96
MASONRY OR BRICKWORK (NOT REINFORCED) IN 1:3 CEMENT MORTAR		3000	13	16¼	19½
		5000	23¼	29	34¾
		7500	32¾	40¾	49
		10000	42½	53	63¾

REDUCTION FOR SLENDER PIERS, ETC.
S = SLENDERNESS
R = REDUCTION FACTOR

FOR $S \nless 1 \ngtr 24$
$R = 1{\cdot}03 - 0{\cdot}03S$

S =	R =
1	1·0
2	0·96
4	0·88
6	0·80
(INTERPOLATE)	
14	0·40
16	0·35
18	0·30

BEARING PRESSURES

ROCK FILLING: DEPENDS ON CONSOLIDATION; DETERMINE BY TEST.
ROCK: NOT MORE THAN 12½% OF CRUSHING STRENGTH.
SAND, GRAVEL, ETC. TABULATED PRESSURES ARE FOR DRY MATERIALS. ASSUME 50% IF SATURATED.
EARTH FILLING, PEAT, ASHES: MAY BE NEGLIGIBLE

MAXIMUM GROUND PRESSURE TO PREVENT SPEWING

AT DEPTH h FT.

GRANULAR (COHESIONLESS) SOIL:— $p_{max.} = \dfrac{wh}{k_2^2}$

COHESIVE SOIL:— $p_{max} = \dfrac{wh}{k_2^2} + \dfrac{2c}{\sqrt{k_2}}\left(\dfrac{1}{k_2}+1\right)$

BUT NOT GREATER THAN SAFE PRESSURE AS TABULATED

VALUES OF k_2 AND C: SEE FOOTNOTE

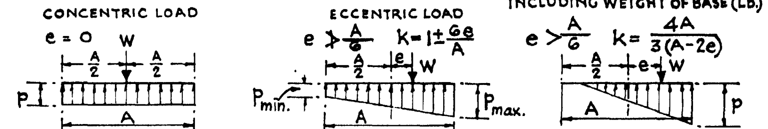
DISTRIBUTION OF PRESSURE

INTENSITY OF PRESSURE (LB. PER SQ. FT.) $p = \dfrac{kW}{AB}$

A = LENGTH OF FOUNDATION (FT.)
B = WIDTH OF FOUNDATION (FT.)
W = TOTAL LOAD ON FOUNDATION INCLUDING WEIGHT OF BASE (LB.)

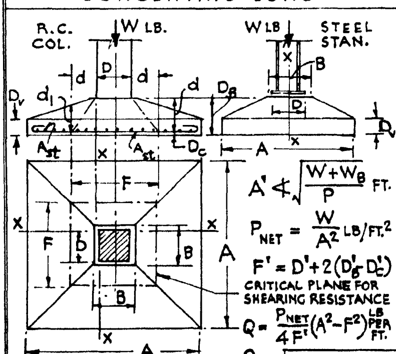
CONCENTRIC LOAD: $e = 0$

ECCENTRIC LOAD: $e \ngtr \dfrac{A}{6}$ $\quad k = 1 \pm \dfrac{6e}{A}$ $\qquad e > \dfrac{A}{6}$ $\quad k = \dfrac{4A}{3(A-2e)}$



INDEPENDENT BASES

CONCENTRIC LOAD



$A' \nless \sqrt{\dfrac{W+W_B}{p}}$ FT.

$P_{NET} = \dfrac{W}{A^2}$ LB/FT.²

$F' = D' + 2(D'_B - D'_C)$

CRITICAL PLANE FOR SHEARING RESISTANCE

$Q = \dfrac{P_{NET}}{4F'}(A^2 - F^2)$ LB PER FT.

$d_1 \nless \dfrac{Q}{10{\cdot}5a}$

R.C. COLUMN.
$M'_{xx} = \dfrac{P_{NET}A'}{8}(A'-D')^2$ FT. LB.

STEEL STAN.
$M'_{xx} = \dfrac{P_{NET}(A')^2}{8}(A'-D')$ FT. LB.

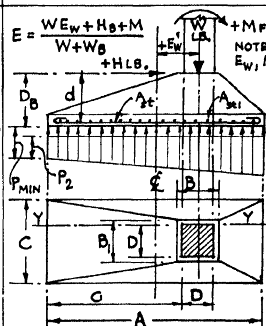
$d \nless \sqrt{\dfrac{M'}{QB\ (\text{OR } B_1)}}$ IN.

IF BASE IS OF UNIFORM THICKNESS B = A AND B = C

TOTAL A_{st} IN EACH LAYER $= \dfrac{12M'}{7/8\, d\, p_{st}}$ SQ. IN.

ECCENTRIC LOAD

$E = \dfrac{WE_W + H_B + M}{W + W_B}$



$P_B = \dfrac{W_B}{A'C'}$

$P_1 = P_{MAX.} - P_B$

$P_2 = P_{MIN.} - P_B$

$G' = \dfrac{A'-D'}{2} + E'_W$

$\begin{matrix}P_{MAX.}\\P_{MIN.}\end{matrix} = \dfrac{W+W_B}{A'C'}\left[1 \pm \dfrac{6E'}{A'}\right] \begin{matrix}\ngtr p\\ \text{NOT NEG.}\end{matrix}$

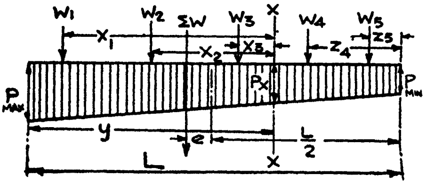
$M'_{xx} = \dfrac{(G')^2C'}{2}\left[P_2 + \dfrac{G'}{3A'}(P_1+P_2)\right]$ FT. LB.

$M'_{YY} = \dfrac{W}{8C'}(C'-D')$ FT. LB.

NOTES.—See *Tables* 11 and 12 for values of k_2 and C.
For notation and units, see *Table* 105.

COMBINED BASES.

Strip Base Supporting Several Loads Arranged Eccentrically.—



$$e = \frac{\Sigma Wz}{\Sigma W} - \frac{L}{2} \not> \frac{L}{6}.$$

Calculate $p_{min.}$ and $p_{max.}$ from formula in *Table* 104.

$$p_x = \left(\frac{L-y}{L}\right)(p_{max.} - p_{min.}) + p_{min.}$$

$$M_{XX} = \Sigma Wx - \frac{y^2}{6}(2p_{max.} + p_x)C.$$

Example.—Find the bending moment at the position of load W_2 on a base 50 ft. long and 5 ft. wide, carrying five unequal point loads as shown in the diagram. The loads (in tons) are $W_1 = 50$, $W_2 = 45$, $W_3 = 40$, $W_4 = 35$, and $W_5 = 30$; the distances (in feet) are $z_1 = 45$, $z_2 = 37$, $z_3 = 28$, $z_4 = 18$, and $z_5 = 5$. Thus $\Sigma W = 200$ and
$\Sigma Wz = (50 \times 45) + (45 \times 37) + (40 \times 28) + (35 \times 18) + (30 \times 5) = 5815.$

Therefore $e = \frac{5815}{200} - \frac{50}{2} = 4{\cdot}075$ ft., which is less than $\frac{L}{6}(= 8\frac{1}{3}$ ft.). Thus

$$k = 1 \pm \frac{6 \times 4{\cdot}075}{50} = 1{\cdot}489 \text{ or } 0{\cdot}511.$$

Maximum pressure: $p_{max.} = \frac{1{\cdot}489 \times 200}{50 \times 5} = 1{\cdot}19$ tons per sq. ft.

Minimum pressure: $p_{min.} = \frac{0{\cdot}511 \times 200}{50 \times 5} = 0{\cdot}41$ ton ,, ,,

At the load W_2, $y = 50 - 37 = 13$ ft.

Therefore $p_x = \left(\frac{50-13}{50}\right)(1{\cdot}19 - 0{\cdot}41) + 0{\cdot}41 = 0{\cdot}987$ ton per sq ft.

Also $x_1 = 8$, $x_2 = 0$. Therefore $\Sigma Wx = W_1x_1 + W_2x_2 = 400 + 0 = 400$ ft.-tons.

$$M_{XX} = 400 - \frac{13^2}{6}(2 \times 1{\cdot}19 + 0{\cdot}987)5 = 400 - 470 = -70 \text{ ft.-tons.}$$

Balanced Bases.—The probable variation in loads W_1 and W_2 (see diagram in *Table* 105) must be considered for bases of this type. If W_1 can vary from $W_{1(max.)}$ to $W_{1(min.)}$, and W_2 from $W_{2(max.)}$ to $W_{2(min.)}$, the reaction R will vary from

$$R_{max.} = \frac{EW_{1(max.)}}{L} \text{ to } R_{min.} = \frac{EW_{1(min)}}{L}.$$

Therefore R_1 and R_2 may have the following values:

$$R_{1(max.)} = W_{1(max.)} + W_{B1} + R_{max.} + 0{\cdot}5W_L$$
$$R_{1(min.)} = W_{1(min.)} + W_{B1} + R_{min.} + 0{\cdot}5W_L$$
$$R_{2(max.)} = W_{2(max.)} + W_{B2} - R_{min.} + 0{\cdot}5W_L$$
$$R_{2(min.)} = W_{2(min.)} + W_{B2} - R_{max.} + 0{\cdot}5W_L$$

Base (1) must therefore be designed for a maximum load of $R_{1(max.)}$ and base (2) for $R_{2(max.)}$; but $R_{2(min.)}$ must always be positive, and should be sufficiently large to ensure a margin of safety in the counterbalance provided by base (2).

Shearing Forces.—The shearing forces on combined bases, rectangular or trapezoidal in plan and carrying two loads, are calculated as for a double-cantilevered beam. For a strip base the shearing force can be calculated by using the basic principle that the shearing force at any section is the algebraic sum of the vertical forces on one side of the section.

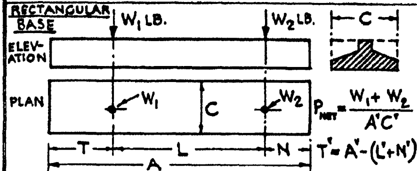
Longitudinal Bending Moments.—The longitudinal positive bending moment on a base carrying two loads can be determined graphically from the two negative bending moments under the loads W_1 and W_2 and the "free" positive bending moment M_X.

Minimum Depth of Foundation.—The theoretical minimum depth at which a foundation should be placed can be determined from the formulæ in *Table* 104 transposed to give

$$h = \frac{p_{max.}\,k_2^2}{w} \text{ or } \left[p_{max.} - \frac{2C}{\sqrt{k_2}}\left(\frac{1}{k_2} + 1\right)\right]\frac{k_2^2}{w}.$$

CONCENTRIC LOAD

RECTANGULAR BASE



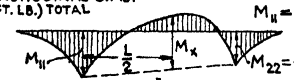
$P_{NET} = \dfrac{W_1 + W_2}{A'C'}$

$T' = A' - (L' + N')$

$A'C' \not< \dfrac{W_1 + W_2 + W_B}{P}$; $N' = \dfrac{A'}{2} - \dfrac{W_1 L'}{W_1 + W_2}$

LONGITUDINAL B.MS. (FT. LB.) TOTAL

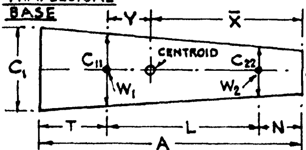
$M_{11} = -\dfrac{P_{NET}}{2} C'(T')^2$



$M_{22} = -\dfrac{P_{NET}}{2} C'(N')^2$

TRANSVERSE B.MS. AT EACH LOAD (FT.LB.) $-\dfrac{C'}{8} W_1 (\text{OR } W_2)$ $M_x = +\dfrac{P_{NET}}{8} C'(L')^2$

TRAPEZOIDAL BASE



$\bar{X} = \dfrac{A'}{3}\left[\dfrac{2C_1' + C_2'}{C_1' + C_2'}\right]$

$Y' = \dfrac{W_2 L'}{W_1 + W_2}$

$T' = A' - (Y' + \bar{X})$

$N' = A' - (L' + T')$

$\dfrac{A'}{2}(C_1' + C_2') \not< \dfrac{W_1 + W_2 + W_B}{P}$ $P_{NET} = \dfrac{2(W_1 + W_2)}{A'(C_1' + C_2')}$

$C_{11}' = C_2' + (C_1' - C_2')\dfrac{A' - T'}{A'}$; $C_{22}' = C_2' + (C_1' - C_2')\dfrac{N'}{A'}$

LONGITUDINAL B.MS. (FT.LB.) TOTAL:—

$M_{11}' = -\dfrac{(T')^2}{6}(2C_1' + C_{11}') P_{NET}$

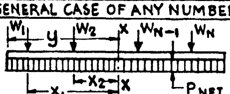
$M_{22}' = -\dfrac{(N')^2}{6}(2C_2' + C_{22}') P_{NET}$

DIAGRAM SIMILAR TO CONCENTRICALLY LOADED RECTANGULAR BASE

$M_x' = +\dfrac{(L')^2}{16}(C_1' + C_{22}') P_{NET}$ (APPROX)

TRANSVERSE B.MS. (FT.LB) $-\dfrac{C_{11}'}{8} W_1$ AT W_1; $-\dfrac{C_{22}'}{8} W_2$ AT W_2

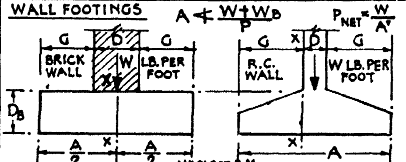
GENERAL CASE OF ANY NUMBER OF LOADS.



B.M. AT X $= \sum Wx - \dfrac{Cy^2}{2} P_{NET}$

WALL FOOTINGS

$A \not< \dfrac{W + W_B}{P}$ $P_{NET} = \dfrac{W}{A'}$



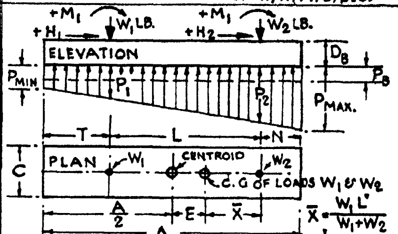
NEGLECT B.M IF $G \ngtr D_B$

$M_x' = \dfrac{A'G'}{4} P_{NET}$ FT. LB./FT. $M_x' = \dfrac{(G')^2}{2} P_{NET}$ FT. LB./FT.

MAX. SHEARING FORCE = G P_{NET} LB. PER FT.

ECCENTRIC LOAD

NOTE SIGNS & DIRECTION OF W, H, M, E, ETC.



$\bar{X} = \dfrac{W_1 L'}{W_1 + W_2}$

$E = \dfrac{A'}{2} - (N' + \bar{X})$

$M_o' = (W_1 + W_2) E' + M_1' + M_2' + (H_1' + H_2') D_B'$ FT. LB.

$\left.\begin{matrix} P_{MAX} \\ P_{MIN} \end{matrix}\right\} \dfrac{1}{A'C'}\left[(W_1 + W_2 + W_B) \pm \dfrac{6M_o'}{A'}\right] \left\{\begin{matrix} \ngtr P \\ \not< 0 \end{matrix}\right.$

$P_1 = P_{MIN} + \dfrac{T'}{A'}(P_{MAX} - P_{MIN})$; $P_2 = P_{MIN} + \dfrac{T' + L'}{A'}(P_{MAX} - P_{MIN})$

LONGITUDINAL B.MS. (FT. LB.) TOTAL:—

$M_{11} = -\dfrac{C'}{6}(T')^2(2P_{MIN} + P_1 - 3P_B)$

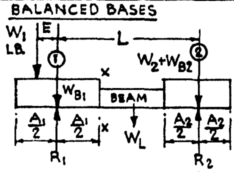
$M_{22} = -\dfrac{C'}{6}(N')^2(2P_{MAX} + P_2 - 3P_B)$

DIAGRAM SIMILAR TO CONCENTRICALLY LOADED RECTANGULAR BASE

$M_x = +(P_1 + P_2)\dfrac{C'}{16}(L')^2$ APPROX.

TRANSVERSE B.MS. (AT EACH LOAD) $= -\dfrac{C'}{8} W_1 (\text{OR } W_2)$ FT. LB.

BALANCED BASES



$R = \dfrac{E'}{L'} W_1$

$R_1 = W_1 + R + W_{B1} + \dfrac{W_L}{2}$

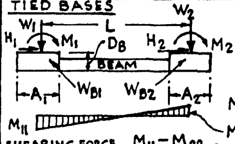
$R_2 = W_2 - R + W_{B2} + \dfrac{W_L}{2}$

$A_1'C_1' \not< \dfrac{R_1}{P}$

$A_2'C_2' \not< \dfrac{R_2}{P}$

LONG^L B. M. VARIES FROM $W_1 E'$ AT ℄ ① TO ZERO AT ℄ ②

TIED BASES



$A_1'C_1' \not< \dfrac{W_1 + W_{B1}}{P}$

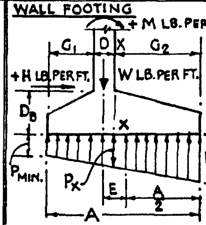
$A_2'C_2' \not< \dfrac{W_2 + W_{B2}}{P}$

$M_{11}' = H_1 D_B' + M_1$ FT. LB.

$M_{22}' = -[H_2 D_B' + M_2]$ FT. LB.

SHEARING FORCE ON BEAM $\dfrac{M_{11} - M_{22}}{L'}$ LB.

WALL FOOTING



$M_o' = M' + H'D_B' - WE'$

$\left.\begin{matrix} P_{MAX} \\ P_{MIN} \end{matrix}\right\} \dfrac{1}{A'}\left[W \pm \dfrac{6M_o}{A'}\right] \left\{\begin{matrix} \ngtr P \\ \not< 0 \end{matrix}\right.$

$P_x = P_{MIN} + \dfrac{G_1 + D'}{A'}(P_{MAX} - P_{MIN})$

P_{MAX}. $M_x' = \dfrac{(G_2')^2}{6}(P_x + 2P_{MAX} - 3P_B)$

SHEARING FORCE AT X:— $= \dfrac{G_2}{2}(P_x + P_{MAX} - 2P_B)$ LB.

UNITS AND NOTATION

(ADDITIONAL TO DIMENSIONS, ETC. ON DIAGRAMS)

LOADS, FORCES, ETC.... POUNDS (LB); PRESSURES, MOMENTS, ETC..... FT. & LB.

DIMENSIONS, ETC. IN "CAPITALS"....FEET (INDICATED THUS A', ETC.)

DIMENSIONS ETC. IN "LOWER CASE" LETTERS....INCHES (INDICATED THUS d, ETC.)

W = IMPOSED LOAD (LB. OR LB./FT.) P = SAFE BEARING PRESSURE ON GROUND. P_{NET} = ACTUAL NET UPWARD PRESSURE

W_B = TOTAL WEIGHT OF BASE (DITTO.) P_B = BEARING PRESSURE DUE TO BASE ONLY.

IMPACT-DRIVEN PILES.

In calculations for the loads on piles a formula can only give comparative values that must be combined with the results of tests and experience when assessing the safe load on a pile.

Example.—Estimate the safe load on a pile 40 ft. long, 14 in. square, driven to gravel by a 2-ton single-acting steam hammer dropping 42 in. with 12 blows per inch of final penetration; the weight of the helmet, dolly, and stationary part of hammer is $\frac{1}{2}$ ton. The dolly is in good condition at the end of the driving. The weight of the pile is

$$\frac{40 \times 196}{2240} = 3\tfrac{1}{2} \text{ tons}; \; P = 3\tfrac{1}{2} + \tfrac{1}{2} = 4 \text{ tons}; \; \frac{P}{w} = \frac{4}{2} = 2.$$

From *Table* 106: Effective drop $= H_1 = 0{\cdot}90 \times 42 = 38$ in. For $\frac{P}{w} = 2$, $e = 0{\cdot}37$. For medium driving, $c = 0{\cdot}29$ for $L = 40$ ft.

The settlement load $= \dfrac{2 \times 38 \times 0{\cdot}37 \times 12}{1 + (0{\cdot}29 \times 12)} + 2 + 4 = 75{\cdot}3 + 6 = 81{\cdot}3$ tons.

For this load the driving pressure $= \dfrac{81{\cdot}3 \times 2240}{14^2} = 930$ lb. per sq. in., which is sufficiently close to the assumed value of 1000 lb. per sq. in. Thus the working load is, say, $\dfrac{81{\cdot}3}{2} - 3\tfrac{1}{2} = 37\tfrac{1}{2}$ tons.

DESIGNS FOR PILED JETTIES.

Vertical Piles Only [Diagram (*a*) on page facing *Table* 107].—Consider a single row of piles; therefore $N = 1$ for each line and $\Sigma N = 4$. Since the group is symmetrical, $X = \frac{1}{2} \times 28 = 14$ ft. From *Table* 107: $M = (80 \times 2{\cdot}5) - (10 \times 15) = +50$ ft.-tons.

The calculation of the load on the piles can be made by tabulation as follows:

Pile No.	x (ft.)	x^2 (ft.^{2})	k_W $\left(\frac{1}{\Sigma N}\right)$	k_M $\left(\frac{x}{NI}\right)$	Axial load ($k_W W + k_M M$)
P_1	-14	196	$+0{\cdot}25$	$\frac{-14}{435{\cdot}6} = -0{\cdot}0321$	$(0{\cdot}25 \times 80) - (0{\cdot}0321 \times 50) = 18{\cdot}4$ tons
P_2	$-4{\cdot}67$	21·8	$+0{\cdot}25$	$\frac{-4{\cdot}67}{435{\cdot}6} = -0{\cdot}0107$	$20 - (0{\cdot}0107 \times 50) = 19{\cdot}5$ tons
P_3	$+4{\cdot}67$	21·8	$+0{\cdot}25$	$+0{\cdot}0107$	$20 + 0{\cdot}5 = 20{\cdot}5$ tons
P_4	$+14$	196	$+0{\cdot}25$	$+0{\cdot}0321$	$20 + 1{\cdot}6 = 21{\cdot}6$ tons
$I = \Sigma N x^2 =$		435·6 (ft.^{4})			

The shearing force on each pile is $\frac{10}{4} = 2{\cdot}5$ tons $= 5600$ lb., and the bending moment on each pile is $0{\cdot}5 \times 2{\cdot}5 \times 15 = 18{\cdot}75$ ft.-tons $= 506{,}000$ in.-lb. The maximum load on any pile is 21·6 tons $= 48{,}300$ lb., or 56,000 lb. including the pile.

Vertical and Inclined Piles [Diagram (*b*) on page facing *Table* 107] ($\tan\theta = 0{\cdot}25$). For each pile, A is assumed to be the same. All piles are driven to the same depth, therefore if $\frac{A}{L}$ for piles P_1 and P_4 is unity, $\frac{A}{L}$ for P_2 and $P_3 = \dfrac{4}{\sqrt{1 + 4^2}} = 0{\cdot}97$. Since the group is symmetrical, $\Sigma_2 = 0$, Σ_3 is not required, $\Sigma_4 = 0$, $G = \Sigma_1 \Sigma_5$, $x_0 = \frac{28}{2} = 14$ ft., and $y_0 = 0$. $M = 80(16{\cdot}5 - 14) + 0 = 200$ ft.-tons.

(*Continued on page* 334.)

HILEY FORMULA (MODIFIED)

SETTLEMENT LOAD (TONS) $= W_M = \dfrac{wH_1en}{1+cn} + w + P.$

WORKING LOAD (INCLUDING WEIGHT OF PILE) $= \dfrac{W_M}{3}$ TO $\dfrac{W_M}{1\cdot5}$ (TONS) (LOWER DENOMINATOR FOR HARD DRIVING)

w = WEIGHT OF HAMMER (TONS)

H_1 = EFFECTIVE DROP OF HAMMER (IN.)

VALUES OF $\dfrac{\text{EFFECTIVE DROP}}{\text{ACTUAL DROP}}$ = 1·0 FOR FREELY-FALLING DROP HAMMER; 0·9 " SINGLE-ACTING STEAM "; 0·8 " WINCH-OPERATED DROP "

L = LENGTH OF PILE (FT.)

P = WEIGHT OF HELMET, DOLLY, AND STATIONARY PARTS OF HAMMER, AND PILE (TONS)

n = NUMBER OF BLOWS PER FINAL INCH OF PENETRATION.

c = TEMPORARY ELASTIC COMPRESSION FACTOR $= 0\cdot5\,(c_1 + c_2 + c_3)$ IN (SEE TABLE BELOW)

c_1 (PILE) $= 0\cdot0134\,pL$.

c_2 (HELMET AND DOLLY) $= 0\cdot56p$ FOR HELMET AND PACKING; $0\cdot11p$ WITHOUT HELMET.

c_3 (GROUND) $= 0\cdot22p$ FOR GRAVEL; $0\cdot44p$ FOR CLAY; $1\cdot1p$ FOR PEAT.

DRIVING PRESSURE p (TONS PER SQ. IN.) $= W_M \div$ AREA OF PILE; p' (LB. PER SQ. IN.) $= 2240p$.

EFFICIENCY OF BLOW e	P/w	0·5	1·0	1·5	2·0	2·5	3	4	5	6	7
	WITH HELMET AND WOODEN DOLLY.	0·69	0·53	0·44	0·37	0·33	0·30	0·25	0·21	0·19	0·17
	PILE-HEAD & CAP IN POOR CONDITION	0·67	0·50	0·40	0·33	0·28	0·25	0·20	0·16	0·14	0·12

COMPRESSION FACTOR c	LENGTH OF PILE (FT)	20	30	40	50	60	
	EASY DRIVING $p' = 500$	0·13	0·16	0·17	0·19	0·20	TABULATED VALUES OF C ARE MINIMA AND ASSUME DOLLY AND PACKING IN GOOD CONDITION AND COMPACT GROUND.
	MEDIUM DRIVING $p' = 1\,000$	0·24	0·27	0·29	0·33	0·36	
	HARD DRIVING $p' = 1\,500$	0·33	0·37	0·42	0·46	0·51	
	VERY HARD DRIVING $p' = 2000$	0·40	0·46	0·52	0·58	0·64	

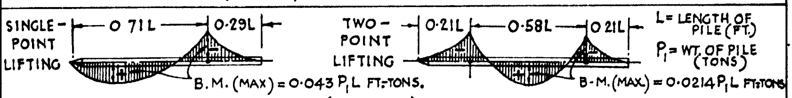
FABER FORMULA FOR PILES DRIVEN INTO CLAY

ADDITIONAL NOTATION: H = FREE FALL OF HAMMER (IN.); d = DIAMETER OR SIZE OF PILE (IN.); A_s = AREA (SQ. FT.) OF SIDES OF PILE EMBEDDED IN GROUND.

$$\text{SAFE LOAD} = \frac{w\left(H - \frac{d}{7}\right)}{F\left(\frac{1}{n} + 0\cdot8H\right)} + \frac{A_s}{20} \text{ TONS}$$

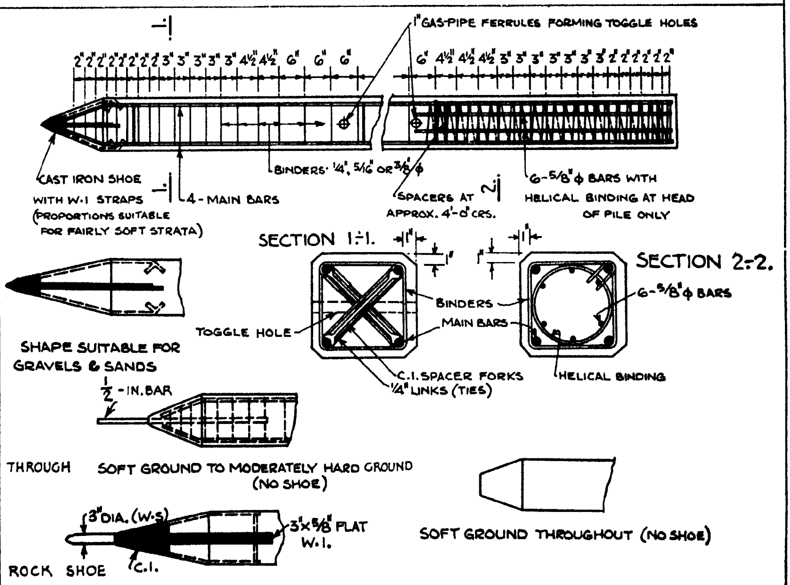
F = FACTOR OF SAFETY ($1\tfrac{1}{2}$ TO $2\tfrac{1}{2}$)

POINTS OF LIFTING TO PRODUCE MIN. B. Ms.

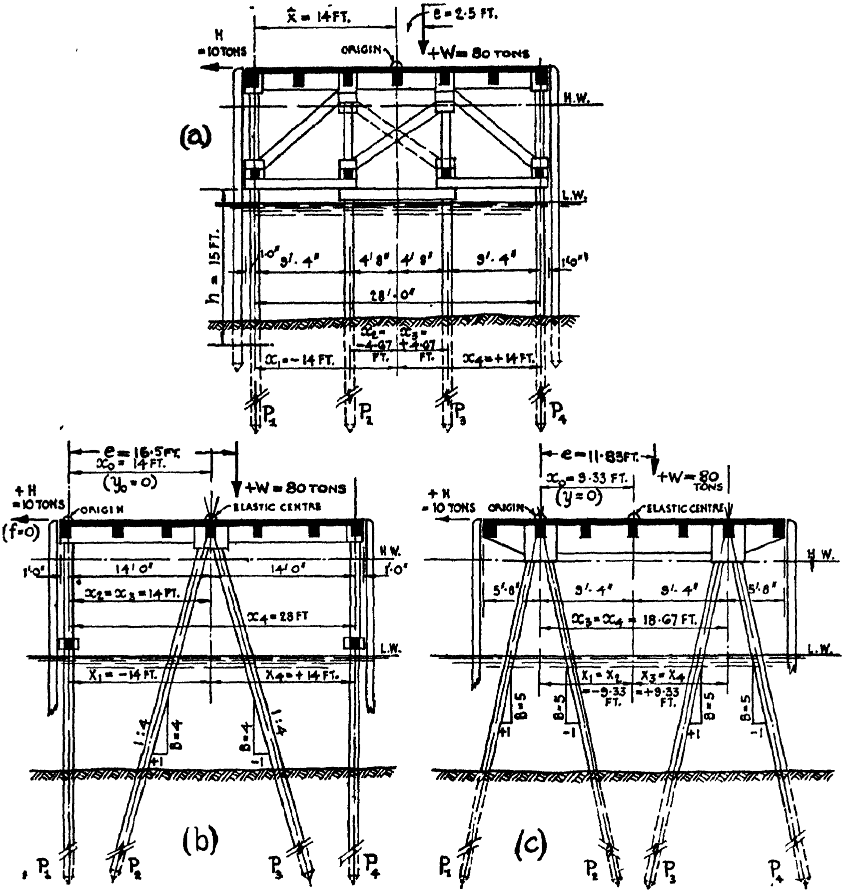


SHORT PILES.— TOGGLE HOLES NEAR HEAD (FOR GUIDING) CAN GENERALLY BE USED FOR LIFTING WITHOUT PRODUCING EXCESSIVE B.Ms.

TYPICAL DETAILS OF PILES AND SHAPES OF SHOES

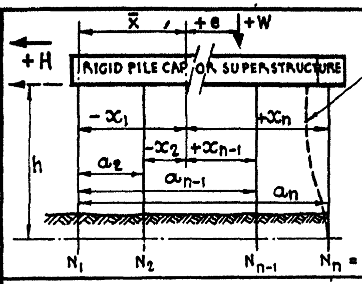


DESIGNS FOR PILED JETTIES (*continued from page* 332).



Pile No.	Σ_1	Σ_5	x (ft.)	X (ft.)	Σ_6 $(= I)$	k_P	k_W	k_H	k_M
P_1	$+1$	0	0	-14	$+196$	$+1$	$+\frac{0\cdot114}{0\cdot436} = +0\cdot261$	0	$\frac{-14}{392} = -0\cdot036$
P_2	$+0\cdot97\frac{4^2}{1+4^2} = +0\cdot912$	$+0\cdot97\frac{1}{1+4^2} = +0\cdot057$	$+14$	0	0	$+0\cdot97\frac{4}{\sqrt{1+4^2}} = +0\cdot941$	$+0\cdot261$	$+\frac{3\cdot824}{4\times0\cdot436} = +2\cdot19$	0
P_3	$+0\cdot912$	$+0\cdot057$	$+14$	0	0	$+0\cdot941$	$+0\cdot261$	$-2\cdot19$	0
P_4	$+1$	0	$+28$	$+14$	$+196$	$+1$	$+0\cdot261$	0	$+\frac{14}{392} = +0\cdot036$
Totals	$+3\cdot824$	$+0\cdot114$	—	—	392 (ft.^{4})	—	—	—	—

GROUPS CONTAINING VERTICAL PILES ONLY



$\bar{x} = \frac{\Sigma aN}{\Sigma N}$ $\quad I = \Sigma N x^2$

ASSUMED DEFORMATION.

AXIAL LOAD ON ANY PILE:

$P_x = K_W W + K_m M$ PLUS WEIGHT OF PILE

$M = We - Hh$

$K_W = \frac{1}{\Sigma N}$ $\quad K_m = \frac{x}{NI}$

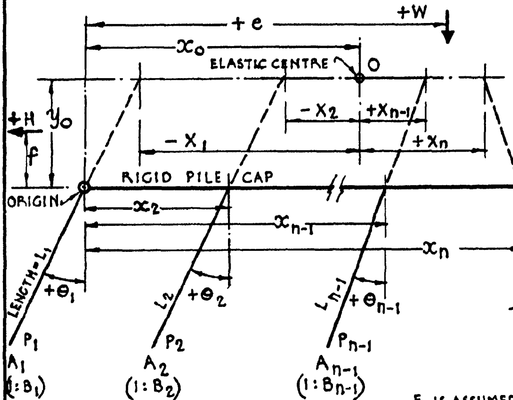
SHEARING FORCE ON ANY PILE $= S = \frac{H}{\Sigma N}$

BENDING MOMENT ON ANY PILE $= 0{\cdot}5\,S h$

(FOR SYMMETRICAL GROUP: $\bar{x} = \frac{a_n}{2}$)

N_n = NUMBER OF PILES IN EACH ROW.

GROUPS CONTAINING INCLINED PILES WITH OR WITHOUT VERTICAL PILES



AXIAL LOAD ON ANY PILE:

$P_x = K_p(K_W W + K_H H + K_M M)$

$M = W(e - x_o) + (y_o - f)H$

CO-ORDINATES OF ELASTIC CENTRE:

$$x_o = \frac{\Sigma_3\Sigma_5 - \Sigma_2\Sigma_4}{G}$$

$$y_o = \frac{\Sigma_2\Sigma_3 - \Sigma_1\Sigma_4}{G}$$

$$G = \Sigma_1\Sigma_5 - \Sigma_2^2$$

$$V = Q\cos^2\theta$$

$$Q = \frac{A}{L}$$

P_n = PILE REFERENCE & LOAD

A_n = CROSS-SECTIONAL AREA

$(1:-B_n)$ = SLOPE OF PILE = $\tan\theta$

E IS ASSUMED TO BE CONSTANT.

SUMMATIONS	PILES INCLINED TOWARDS RIGHT	VERTICAL PILES	PILES INCLINED TOWARDS LEFT
$\Sigma_1 = \Sigma V = \Sigma Q\cos^2\theta$	$+\Sigma_R\frac{A}{L}\cdot\frac{B^2}{1+B^2}$	$+\Sigma_V\frac{A}{L}$	$+\Sigma_L\frac{A}{L}\cdot\frac{B^2}{1+B^2}$
$\Sigma_2 = \Sigma V\tan\theta = \Sigma Q\cos\theta\sin\theta$	$+\Sigma_R\frac{A}{L}\cdot\frac{B}{1+B^2}$	NIL	$-\Sigma_L\frac{A}{L}\cdot\frac{B}{1+B^2}$
$\Sigma_3 = \Sigma Vx = \Sigma x Q\cos^2\theta$	$+\Sigma_R\frac{A}{L}\cdot\frac{B^2}{1+B^2}x$	$+\Sigma_V\frac{A}{L}x$	$+\Sigma_L\frac{A}{L}\cdot\frac{B^2}{1+B^2}x$
$\Sigma_4 = \Sigma Vx\tan\theta = \Sigma xQ\cos\theta\sin\theta$	$+\Sigma_R\frac{A}{L}\cdot\frac{B}{1+B^2}x$	NIL	$-\Sigma_L\frac{A}{L}\cdot\frac{B}{1+B^2}x$
$\Sigma_5 = \Sigma V\tan^2\theta = \Sigma Q\sin^2\theta$	$+\Sigma_R\frac{A}{L}\cdot\frac{1}{1+B^2}$	NIL	$+\Sigma_L\frac{A}{L}\cdot\frac{1}{1+B^2}$
$\Sigma_6 = I = \Sigma VX^2 = \Sigma X^2 Q\cos^2\theta$	$+\Sigma_R\frac{A}{L}\cdot\frac{B^2}{1+B^2}X^2$	$+\Sigma_V\frac{A}{L}X^2$	$+\Sigma_L\frac{A}{L}\cdot\frac{B^2}{1+B^2}X^2$
$X = x - x_o + y_o\tan\theta$	$x - x_o + \frac{y_o}{B}$	$x - x_o$	$x - x_o - \frac{y_o}{B}$
COEFFICIENTS IN FORMULA FOR P_x			
$K_p = Q_x\cos\theta_x = \frac{A_x}{L_x}\cdot\frac{B}{\sqrt{1+B^2}}$	$+\frac{A}{L}\cdot\frac{B}{\sqrt{1+B^2}}$	$+\frac{A}{L}$	$+\frac{A}{L}\cdot\frac{B}{\sqrt{1+B^2}}$
$K_W = \frac{\Sigma_5 - \tan\theta\,\Sigma_2}{G}$	$+\frac{\Sigma_5 - \frac{\Sigma_2}{B}}{G}$	$+\frac{\Sigma_5}{G}$	$+\frac{\Sigma_5 + \frac{\Sigma_2}{B}}{G}$
$K_H = \frac{\tan\theta\,\Sigma_1 - \Sigma_2}{G}$	$+\frac{\frac{\Sigma_1}{B} - \Sigma_2}{G}$	$-\frac{\Sigma_2}{G}$	$-\frac{\frac{\Sigma_1}{B} + \Sigma_2}{G}$
$K_M = \frac{X}{I} = \frac{X}{\Sigma_6}$	$+\frac{X}{\Sigma_6}$	$+\frac{X}{\Sigma_6}$	$+\frac{X}{\Sigma_6}$

NOTE ON SYMMETRICAL GROUPS $\Sigma_2 = 0$; $G = \Sigma_1\Sigma_5$; $x_o = \frac{x_n}{2}$; Σ_3 IS NOT REQUIRED.

DESIGN FOR PILED JETTIES

Vertical and Inclined Piles (*continuation of example from pages* 332 *and* 334).

Therefore $G = 3·824 \times 0·114 = 0·436$. The axial loads on the piles are

$P_1 = 1[(0·261 \times 80) + 0 - (0·036 \times 200)] = 20·9 - 7·2 = 13·7$ tons.

$P_2 = 0·941[20·9 + (2·19 \times 10) + 0] = 40·3$ tons.

$P_3 = 0·941(20·9 - 21·9 + 0) = -0·94$ tons (tension) or $+2$ tons including the pile.

$P_4 = 20·9 + 0 + 7·2 = 28·1$ tons.

The maximum load on any pile is 40·3 tons, say, 43 tons including the pile.

Inclined Piles Only [Diagram (*c*) on page facing *Table* 107] ($\tan \theta = 0·2$). For each pile L and A are the same; so unity can be substituted for $\frac{A}{L}$. For Piles P_1 and P_3, $B = +5$ and for P_2 and P_4, $B = -5$. Since the group is symmetrical, $\Sigma_2 = 0$, Σ_3 is not required. $\Sigma_4 = 0$, $G = \Sigma_1\Sigma_5$, $x_0 = 9·33$ ft., $y_0 = 0$. $M = 80(11·83 - 9·33) + 0 = 200$ ft.-tons.

Pile No.	Σ_1	Σ_5	x (ft.)	X (ft.)	$\Sigma_6(=I)$	k_P	k_W	k_H	$\frac{X}{I}$
P_1	$\frac{5^2}{1+5^2} = +0·96$	$\frac{1}{1+5^2} = +0·0384$	0	$-9·33$	$0·96(-9·33)^2$	$\frac{5}{\sqrt{1+5^2}}$	$\frac{0·1536}{0·59}$	$+\frac{3·84}{5 \times 0·59}$	$\frac{-9·33}{334}$
					$= +83·5$	$= +0·98$	$= +0·26$	$= +1·305$	$= -0·028$
P_2	$+0·96$	$+0·0384$	0	$-9·33$	$+83·5$	$+0·98$	$+0·26$	$\frac{3·84}{-5 \times 0·59}$	$-0·028$
P_3	$+0·96$	$+0·0384$	$+1·67$	$+9·33$	$0·96(+9·33)^2$	$+0·98$	$+0·26$	$= -1·305$	$\frac{+9·33}{334}$
					$= +83·5$			$+1·305$	$= +0·028$
P_4	$+0·96$	$+0·0384$	$+18·67$	$+9·33$	$+83·5$	$+0·98$	$+0·26$	$-1·305$	$-0·028$
Totals	$+3·84$	$+0·1536$	—	—	$+334$	—	—	—	—

Therefore $G = 3·84 \times 0·1536 = 0·59$. The axial loads on the piles are:

$P_1 = 0·98[(0·26 \times 80) + (1·305 \times 10) - (0·028 \times 200)]$
$= 0·98(20·8 + 13·1 - 5·6) = 28·3$ tons.

$P_2 = 0·98(20·8 - 13·1 - 5·6) = 2·1$ tons. $P_3 = 0·98(20·8 + 13·1 + 5·6) = 38·6$ tons.

$P_4 = 0·98(20·8 - 13·1 + 5·6) = 13·1$ tons.

The maximum load on any pile is 38·6 tons, say, $41\frac{1}{2}$ tons including the pile.

Notes on Designs (a), (b) and (c).—Design (*a*) comprises vertical piles only, design (*b*) vertical and inclined piles, and design (*c*) inclined piles only. In each case the group is symmetrical and is subjected to the same imposed loads. Designs (*b*) and (*c*) are special cases of symmetrical groups for which $\Sigma_4 = 0$ and therefore $y_0 = 0$; this condition applies only if the inclined piles are in symmetrical pairs, both piles in a pair meeting at the same pile-cap.

The design in (*c*) requires the smallest pile, but there is little difference between the designs in (*b*) and (*c*). Although the maximum load on any pile is least in the design in (*a*), the bending moment on the pile requires a pile of greater cross-sectional area to provide the necessary resistance to combined thrust and bending. The superiority of design (*c*) is greater if the horizontal force H is greater, and if H were 20 tons (instead of 10 tons) the maximum loads (excluding the weight of the pile) are $23\frac{1}{4}$ tons (and a large bending moment of $37\frac{1}{2}$ ft.-tons) in design (*a*), $60\frac{3}{4}$ tons in design (*b*), and $51\frac{1}{2}$ tons in design (*c*). Design (*b*) is the most suitable when H is small; if H were 1 ton, the maximum loads are 26 tons (and a small bending moment of $1\frac{7}{8}$ ft.-tons) in design (*a*), 22 tons in design (*b*), and 27 tons in design (*c*). Design (*a*) is most suitable when there is no horizontal load.

DECIMAL EQUIVALENTS: TO CONVERT INCHES TO FEET.—TABLE 108.

INS.	0	1	2	3	4	5	6	7	8	9	10	11
	-	·08333	·16666	·25	·33333	·41666	·5	·58333	·66666	·75	·83333	·91666
1/32	·002604	·08594	·16927	·2526	·33594	·41927	·5026	·58594	·66927	·7526	·83594	·91927
1/16	·00521	·08854	·17187	·25521	·33854	·42187	·50521	·58854	·67187	·75521	·83854	·92187
3/32	·00781	·09114	·17448	·25781	·34114	·42448	·50781	·59114	·67448	·75781	·84114	·92448
1/8	·01041	·09374	·17707	·26041	·34374	·42707	·51041	·59374	·67707	·76041	·84374	·92707
5/32	·01302	·09635	·17969	·26302	·34635	·42969	·51302	·59635	·67969	·76302	·84835	·92969
3/16	·01562	·09895	·18228	·26562	·34895	·43228	·51562	·59895	·68228	·76562	·84895	·93228
7/32	·01823	·10156	·18489	·26823	·35156	·43489	·51823	·60156	·68489	·76823	·85156	·93489
1/4	·02083	·10416	·1875	·27083	·35416	·4375	·52083	·60416	·6875	·77083	·85416	·9375
9/32	·02344	·10677	·1901	·27344	·35677	·4401	·52344	·60677	·6901	·77344	·85677	·9401
5/16	·02604	·10937	·1927	·27604	·35937	·4427	·52604	·60937	·6927	·77604	·85937	·9427
11/32	·02864	·11198	·19531	·27864	·36198	·44531	·52864	·61198	·6951	·77864	·86198	·94531
3/8	·03125	·11458	·19791	·28125	·36458	·44791	·53125	·61458	·69791	·78125	·86458	·94791
13/32	·03385	·11718	·20052	·28385	·36719	·45052	·53385	·61719	·70052	·78385	·86719	·95052
7/16	·03646	·11979	·20312	·28646	·36979	·45312	·53646	·61979	·70312	·78646	·86979	·95312
15/32	·03906	·12239	·20573	·28906	·37239	·45573	·53906	·62239	·70573	·78906	·87239	·95573
1/2	·04166	·125	·20832	·29166	·375	·45833	·54166	·625	·70832	·79166	·875	·95833
17/32	·04427	·1276	·21094	·29427	·3776	·46094	·54427	·6276	·71094	·79427	·8776	·96094
9/16	·04687	·1302	·21353	·29687	·3802	·46353	·54687	·6302	·71353	·79687	·8802	·96353
19/32	·04948	·13281	·21614	·29948	·38281	·46614	·54948	·63281	·71614	·79948	·88281	·96614
5/8	·05208	·13541	·21874	·30208	·38541	·46875	·55208	·63541	·71874	·80208	·88541	·96875
21/32	·05469	·13809	·22135	·30469	·38802	·47135	·55469	·63802	·72135	·80469	·88802	·97135
11/16	·05729	·14062	·22395	·30729	·39062	·47395	·55729	·64062	·72395	·80729	·89062	·97395
23/32	·05989	·14323	·22656	·30989	·39323	·47656	·55989	·64323	·72656	·80989	·89323	·97656
3/4	·0625	·14583	·22916	·3125	·39583	·47916	·5625	·64583	·72916	·8125	·89583	·97916
25/32	·0651	·14844	·23177	·3151	·39844	·48177	·5651	·64844	·73177	·8151	·89844	·98177
13/16	·06771	·15104	·23437	·31771	·40104	·48437	·56771	·65104	·73437	·81771	·90104	·98437
27/32	·07031	·15364	·23698	·32031	·40364	·48698	·57031	·65394	·73698	·82031	·90364	·98698
7/8	·07292	·15625	·23958	·32292	·40625	·48958	·57292	·65625	·73958	·82292	·90625	·98958
29/32	·07552	·15885	·24219	·32552	·40885	·49219	·57552	·65885	·74219	·82552	·90885	·99219
15/16	·07813	·16146	·24479	·32813	·41146	·49479	·57813	·66146	·74479	·82813	·91146	·99479
31/32	·08073	·16406	·24739	·33075	·41406	·49739	·58073	·66406	·74739	·83073	·91406	·99739
	0	1	2	3	4	5	6	7	8	9	10	11

NOTE.—For example of the use of this table, see page facing *Table* 109.

NUMERICAL CONVERSIONS.

Converting Feet and Inches (*Table* 108).

(*a*) To express 10 ft. $5\frac{5}{32}$ in. as a decimal of a foot:

10 ft. = 10·00000
$5\frac{5}{32}$ in. = 0·42969

10·43 ft.

(*b*) To convert 0·6732 ft. to inches (nearest $\frac{1}{32}$ in.):

0·67187 ft. = $8\frac{1}{16}$ in.

Difference = 0·00133 ft. which is slightly more than $\frac{1}{64}$ in.
Therefore 0·6732 ft. = $8\frac{3}{32}$ in.

Metric Conversion Factors.—1 in. = 2·54 cm. 1 metre = 39·3701 in.
1 pound (av.) = 0·4536 kg. 1 kg. = 2·2406 lb.
1 metric tonne = 0·98421 ton (= 19·684 cwt.)

To convert the moment of resistance factor Q_c in inch-pounds to Q_M in kg.-cm. ($M_r = Q_c b d_1^2$):

$$Q_M = 0{\cdot}0703 Q_c$$

M_r in kilogramme-centimetres = $Q_M b d_1^2$ (where b and d_1 are in centimetres).

E_c =	2,000,000 lb. per sq. in.	=	140,614	kg. per sq. cm.
E_s =	30,000,000 ,, ,, ,,	=	2,109,210	,, ,, ,,
p_{st} =	18,000 ,, ,, ,,	=	1266	,, ,, ,,
p_{sc} =	1000 ,, ,, ,,	=	70·31	,, ,, ,,
w =	100 lb. per sq. ft.	=	488·24	kg. per sq. cm.
I =	1000 in.^{4}	=	41,623	cm.^{4}.

Metric Conversions (*Table* 109).

(*a*) To convert 17 ft. $8\frac{5}{16}$ in. to metric units:

17 ft. 8 in. = 5·385 metres.
$\frac{5}{16}$ in. = 0·0079 metre.

17 ft. $8\frac{5}{16}$ in. = 5·393 metres.

(*b*) To convert 4·067 metres to English units:

4·064 metres = 13 ft. 4 in.
0·003 metre = $\frac{1}{8}$ in. (At foot of *Table* 109: $\frac{1}{8}$ in. = 3·18 mm.)

13 ft. $4\frac{1}{8}$ in.

(*c*) To express $\frac{1}{2}$-in. bars at 8-in. centres in metric units:

$\frac{1}{2}$ in. = 12·7 mm. 8 in. = 0·203 m.
Nearest practical values: 12-mm. bars at 20-cm. centres.

(*d*) To express 600 lb. per sq. in. in metric units:

$$\frac{600}{14{\cdot}22} = 42 \text{ kg. per sq. cm. approximately.}$$

(*e*) To convert 6 lb. per sq. yd. to metric units:

$$6 \text{ lb.} = \frac{6}{2{\cdot}21} = 2{\cdot}71 \text{ kg.}$$

$$1 \text{ sq. yd.} = \frac{1}{1{\cdot}196} = 0{\cdot}836 \text{ sq. m.}$$

$$\text{Thus 6 lb. per sq. yd.} = \frac{2{\cdot}71}{0{\cdot}836} = 3{\cdot}24 \text{ kg. per sq. m.}$$

FT.	0"	1"	2"	3"	4"	5"	6"	7"	8"	9"	10"	11"
0	–	0·025	0·051	0·076	0·102	0·127	0·152	0·178	0·203	0·229	0·254	0·279
1	0·305	0·330	0·360	0·381	0·406	0·432	0·457	0·483	0·508	0·533	0·559	0·584
2	0·610	0·635	0·660	0·686	0·711	0·737	0·762	0·787	0·813	0·838	0·864	0·889
3	0·914	0·940	0·965	0·991	1·016	1·041	1·067	1·092	1·118	1·143	1·168	1·194
4	1·219	1·245	1·270	1·295	1·321	1·346	1·372	1·397	1·422	1·448	1·473	1·499
5	1·524	1·549	1·575	1·600	1·626	1·651	1·676	1·702	1·727	1·753	1·778	1·803
6	1·829	1·854	1·880	1·905	1·930	1·956	1·981	2·007	2·032	2·057	2·083	2·108
7	2·134	2·159	2·184	2·210	2·235	2·261	2·286	2·311	2·337	2·362	2·388	2·413
8	2·438	2·464	2·489	2·515	2·539	2·565	2·591	2·616	2·642	2·667	2·692	2·718
9	2·743	2·769	2·794	2·819	2·845	2·871	2·896	2·921	2·946	2·972	2·997	3·023
10	3·048	3·073	3·099	3·124	3·150	3·175	3·200	3·226	3·251	3·277	3·302	3·327
11	3·353	3·378	3·404	3·429	3·454	3·480	3·505	3·531	3·556	3·581	3·607	3·632
12	3·658	3·683	3·708	3·734	3·759	3·785	3·810	3·835	3·861	3·886	3·912	3·937
13	3·962	3·988	4·013	4·039	4·064	4·089	4·115	4·140	4·166	4·191	4·216	4·242
14	4·267	4·293	4·318	4·343	4·369	4·395	4·420	4·445	4·470	4·496	4·521	4·547
15	4·572	4·597	4·623	4·648	4·674	4·699	4·724	4·750	4·775	4·801	4·826	4·851
16	4·877	4·902	4·928	4·954	4·978	5·004	5·029	5·055	5·081	5·105	5·131	5·156
17	5·182	5·207	5·232	5·258	5·283	5·309	5·334	5·359	5·385	5·410	5·436	5·461
18	5·486	5·512	5·537	5·562	5·588	5·613	5·639	5·664	5·690	5·715	5·741	5·766
19	5·791	5·817	5·842	5·867	5·893	5·918	5·944	5·969	5·994	6·020	6·045	6·071
20	6·096	6·121	6·147	6·172	6·198	6·223	6·248	6·274	6·299	6·325	6·350	6·375
21	6·401	6·426	6·452	6·477	6·502	6·528	6·553	6·579	6·604	6·629	6·655	6·680
22	6·706	6·731	6·756	6·782	6·807	6·833	6·858	6·883	6·909	6·934	6·960	6·985
23	7·010	7·036	7·061	7·087	7·112	7·137	7·163	7·188	7·214	7·239	7·264	7·290
24	7·315	7·341	7·366	7·391	7·417	7·442	7·468	7·493	7·518	7·544	7·569	7·595
25	7·620	7·645	7·671	7·696	7·722	7·747	7·772	7·798	7·823	7·849	7·874	7·899
26	7·925	7·950	7·975	8·001	8·026	8·052	8·077	8·102	8·128	8·153	8·179	8·204
27	8·229	8·255	8·280	8·306	8·331	8·356	8·382	8·407	8·433	8·458	8·483	8·509
28	8·534	8·560	8·585	8·610	8·636	8·661	8·687	8·712	8·737	8·763	8·788	8·814
29	8·839	8·864	8·890	8·915	8·941	8·966	8·991	9·017	9·042	9·068	9·093	9·118
30	9·144	9·169	9·195	9·220	9·246	9·271	9·297	9·322	9·348	9·373	9·399	9·424
31	9·449	9·474	9·500	9·525	9·551	9·576	9·602	9·627	9·653	9·678	9·704	9·729
39	11·887	11·912	11·938	11·963	11·989	12·014	12·040	12·065	12·091	12·116	12·142	12·167
40	12·192	12·217	12·243	12·268	12·294	12·319	12·345	12·370	12·396	12·421	12·447	12·472
50	15·240	15·265	15·291	15·316	15·342	15·367	15·393	15·418	15·444	15·469	15·495	15·520

FRACTIONS OF INCH

1/16"	1·59 M/M	7/16"	11·11 M/M	13/16"	20·64 M/M
1/8"	3·18 "	1/2"	12·7 "	7/8"	22·23 "
3/16"	4·76 "	9/16"	14·29 "	15/16"	23·81 "
1/4"	6·35 "	5/8"	15·88	0·10"	2·54 "
5/16"	7·94 "	11/16"	17·46	0·20"	5·08 "
3/8"	9·53 "	3/4"	19·05	0·30"	7·62 "

TO CONVERT	MULTIPLY BY
METRES TO YARDS:	1·094
SQ. M. TO SQ. YD.:	1·196
CU. M. TO CU. YD.:	1·308
KGM./CM.^{2} TO LB./IN.^{2}:	14·22
KGM./M.^{3} TO LB./FT.^{3}:	0·0624

TRIGONOMETRICAL COMPUTATIONS.

Definitions.—If a, b, and c are the sides of a right-angled triangle, a being the height, b the base, and c the hypotenuse, and if the angle enclosed by b and c is θ: sine $\theta = \frac{a}{c}$; cosine $\theta = \frac{b}{c}$; tangent $\theta = \frac{a}{b}$; and cotangent $\theta = \frac{b}{a}$. Values of these functions for θ from 1 degree to 89 degrees are given in *Table* 110.

For $\theta = 0$: $\sin\theta = \tan\theta = 0$; $\cos\theta = 1$; $\cot\theta =$ infinity.

For $\theta = 90$ degrees: $\sin\theta = 1$; $\tan\theta =$ infinity; $\cos\theta = \cot\theta = 0$.

Secant $\theta = \frac{c}{b} = \frac{1}{\cos\theta}$; cosecant $= \frac{c}{a} = \frac{1}{\sin\theta}$. As these functions are the inverse of cosine and sine respectively, the table can be used to obtain their values by determining the reciprocals of the corresponding cosine and sine.

Interpolation.—*Table* 110 gives the trigonometrical ratios to four places of decimals for angles of a whole number of degrees. For angles expressed in degrees and minutes, the rules for interpolation given in the following are sufficiently accurate for most calculations in reinforced concrete design.

(*a*) Sines and cosines of all angles, tangents of angles less than 88 degrees, and cotangents of angles greater than 2 degrees: Interpolate by direct linear proportion, noting whether the trigonometrical ratio increases or decreases as the angle increases.

Example (i). To determine the sine of 26 deg. 28 min.

$$\sin 27^\circ - \sin 26^\circ = 0{\cdot}4540 - 0{\cdot}4384 = 0{\cdot}0156.$$

$$\sin 26^\circ\, 28' = 0{\cdot}4384 + \left(\frac{28}{60} \times 0{\cdot}0156\right) = 0{\cdot}4457.$$

Example (ii). To determine the angle the cosine of which is 0·3457. 0·3457 is the cosine of an angle between 69 deg. and 70 deg.

$$\cos 69^\circ - \cos 70^\circ = 0{\cdot}3584 - 0{\cdot}3420 = 0{\cdot}0164.$$

$$0{\cdot}3584 - 0{\cdot}3457 = 0{\cdot}0127.$$

$\frac{0{\cdot}0127}{0{\cdot}0164} \times 60 = 46\frac{1}{2}'$. Therefore the angle required is $69^\circ\ 46\frac{1}{2}'$.

(*b*) Tangents of angle θ not less than 88 deg. From *Table* 110 and by the foregoing method of interpolation, determine $\sin\theta$ and $\cos\theta$. Tan $\theta = \frac{\sin\theta}{\cos\theta}$. (Use logarithms for evaluating this quotient if greater accuracy is required.)

Example (iii). Determine the tangent of 89 degrees 10 minutes.

$$\sin 90^\circ - \sin 89^\circ = 1{\cdot}000 - 0{\cdot}9998 = 0{\cdot}0002.$$

$$0{\cdot}0002 \times \frac{10}{60} = 0{\cdot}00003. \quad \sin 89^\circ\, 10' = 0{\cdot}00003 + 0{\cdot}9998 = 0{\cdot}99983.$$

$$\cos 89^\circ - \cos 90^\circ = 0{\cdot}0175 - 0 = 0{\cdot}0175.$$

$$0{\cdot}0175 \times \frac{10}{60} = 0{\cdot}00292. \quad \cos 89^\circ\, 10' = 0{\cdot}0175 - 0{\cdot}00292 = 0{\cdot}01458.$$

$$\tan 89^\circ\, 10' = \frac{0{\cdot}99983}{0{\cdot}01458} = 68{\cdot}5 \text{ (by slide-rule; true value is } 68{\cdot}7501\text{)}.$$

(*c*) Cotangents of angle θ not greater than 2 degrees. Determine the tangent of $(90^\circ - \theta)$ by procedure in (*b*).

Example (iv). To determine the cotangent of 0 deg. 50 min., calculate the tangent of $90^\circ - 50' = 89^\circ\ 10'$. (See example iii.)

Trigonometrical Formulæ.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi \qquad \sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi \qquad \cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} \qquad \tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}.$$

$$\sin\phi + \sin\theta = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right).$$

ANGLE DEGREES	SINE	TANGENT	COTANGENT	COSINE	-
1	0·0175	0·0175	57·2900	0·9998	89
2	0·0349	0·0349	28·6363	0·9994	88
3	0·0523	0·0524	19·0811	0·9986	87
4	0·0698	0·0699	14·3007	0·9976	86
5	0·0872	0·0875	11·4301	0·9962	85
6	0·1045	0·1051	9·5144	0·9945	84
7	0·1219	0·1228	8·1443	0·9925	83
8	0·1392	0·1405	7·1154	0·9903	82
9	0·1564	0·1584	6·3138	0·9877	81
10	0·1736	0·1763	5·6713	0·9848	80
11	0·1908	0·1944	5·1446	0·9816	79
12	0·2079	0·2126	4·7046	0·9781	78
13	0·2250	0·2309	4·3315	0·9744	77
14	0·2419	0·2493	4·0108	0·9703	76
15	0·2588	0·2679	3·7321	0·9659	75
16	0·2756	0·2867	3·4874	0·9613	74
17	0·2924	0·3057	3·2709	0·9563	73
18	0·3090	0·3249	3·0777	0·9511	72
19	0·3256	0·3443	2·9042	0·9455	71
20	0·3420	0·3640	2·7475	0·9397	70
21	0·3584	0·3839	2·6051	0·9336	69
22	0·3746	0·4040	2·4751	0·9272	68
23	0·3907	0·4245	2·3559	0·9205	67
24	0·4067	0·4452	2·2460	0·9135	66
25	0·4226	0·4663	2·1445	0·9063	65
26	0·4384	0·4877	2·0503	0·8988	64
27	0·4540	0·5095	1·9626	0·8910	63
28	0·4695	0·5317	1·8807	0·8829	62
29	0·4848	0·5543	1·8040	0·8746	61
30	0·5000	0·5774	1·7321	0·8660	60
31	0·5150	0·6009	1·6643	0·8572	59
32	0·5299	0·6249	1·6003	0·8480	58
33	0·5446	0·6494	1·5399	0·8387	57
34	0·5592	0·6745	1·4826	0·8290	56
35	0·5736	0·7002	1·4281	0·8192	55
36	0·5878	0·7265	1·3764	0·8090	54
37	0·6018	0·7536	1·3270	0·7986	53
38	0·6157	0·7813	1·2799	0·7880	52
39	0·6293	0·8098	1·2349	0·7771	51
40	0·6428	0·8391	1·1918	0·7660	50
41	0·6561	0·8693	1·1504	0·7547	49
42	0·6691	0·9004	1·1106	0·7431	48
43	0·6820	0·9325	1·0724	0·7314	47
44	0·6947	0·9657	1·0355	0·7193	46
45	0·7071	1·0000	1·0000	0·7071	45
-	COSINE	COTANGENT	TANGENT	SINE	ANGLE DEGREES

MATHEMATICAL DATA.

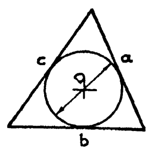
$\pi = \frac{22}{7}$ (approx.) $= 3{\cdot}14$ (approx.) $= 3{\cdot}1416$.

One radian $= \frac{180 \text{ deg.}}{\pi} = 57{\cdot}3$ deg. (approx.)

Length of arc subtended by an angle of one radian = radius of arc.
One degree Fahrenheit $= \frac{5}{9}$ degree Centigrade or Celcius.
Temperature of t deg. F. $= \frac{5}{9}(t - 32)$ deg. C.
Temperature of t deg. C. $= (1{\cdot}8t + 32)$ deg. F.
Base of Naperian logarithms, $e = 2{\cdot}7183$.
To convert common into Naperian logarithms, multiply by $2{\cdot}3026$.
Value of g (London) $= 32{\cdot}182$ ft. per second per second.

Diameter of inscribed circle of a triangle:

$$D = \frac{2b\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}}{a + b + c}$$



For isosceles triangle, $a = c$:

$$D = \frac{b\sqrt{4a^2 - b^2}}{2a + b}.$$

SOLUTION OF TRIANGLES.

Applicable to any triangle ABC in which $AB = c$; $BC = a$; $AC = b$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\text{Area} = \frac{bc \sin A}{2} = \frac{ac \sin B}{2} = \frac{ab \sin C}{2} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a + b + c)$.

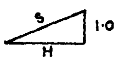
$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

ROOTS OF QUADRATICS.

$$ax^2 + bx + c = 0. \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

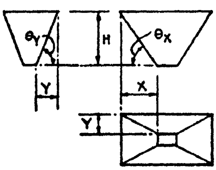
APPLICATIONS.

(*a*) Roof slopes. $s = \sqrt{1 + H^2}$.



(i) Limiting slope for inclined roof loading = 20 deg.
$H = \cot 20$ deg. $= 2{\cdot}7475$;
Therefore limiting slope = 1 : $2\frac{3}{4}$.
(ii) Limiting slope for inclined roofs = 10 deg.
$H = \cot 10$ deg. $= 5{\cdot}6713$;
Therefore limiting slope = 1 : $5\frac{2}{3}$.

(*b*) Earth pressures.



$$k_2 = \frac{1 - \sin\theta}{1 + \sin\theta} = \tan^2\left(45° - \frac{\theta}{2}\right)$$

$$\frac{1}{k_2} = \frac{1 + \sin\theta}{1 - \sin\theta} = \tan^2\left(45° + \frac{\theta}{2}\right).$$

(*c*) Hopper bottom slopes.
Specified minimum slope in valley $= \phi$.

$$\frac{Y}{X} = R;\ X = \frac{H \cot\phi}{\sqrt{1 + R^2}};\ Y = RX.$$

$$\tan\theta_X = \tan\phi\sqrt{1 + R^2}. \qquad \tan\theta_Y = \frac{R\tan\phi\sqrt{1 + R^2}}{R}.$$

APPENDIX I

THE EFFECT OF VARIOUS MATERIALS ON CONCRETE

THE following is a list of substances and liquids, some of which have been found to be injurious to concrete. Some protective treatments are described, but this information is given with reserve because the quality of the concrete, the degree of concentration of the injurious substance, the conditions of exposure, and the temperature are among many factors that determine the severity of the attack, if any, and the efficacy of the treatment. Some of the recommendations are based on experience in Britain; others are taken from foreign publications.

ACETIC ACID.—Causes slow disintegration from which concrete can be protected by spar varnish, bituminous enamels, phenol-formaldehyde varnish, paraffin wax, hard rubber, or, for concentrations up to 60 per cent., acid-resisting paints. Vinegar is a weak solution of acetic acid which slowly disintegrates concrete, and protection by paraffin wax has been recommended.

ACIDS.—All acids attack concrete to some extent, strong acids being particularly virulent, although weak acids can be stored in untreated concrete tanks if the concrete is impermeable. Greater immunity from attack by weak acids can be obtained by using high-alumina cement or by impregnation with fluosilicate solution. The latter, and similar, treatments become ineffective if the skin is broken. The calcium salts contained in set cement are soluble in nearly all mineral or organic acids, and some types of acid-resisting preparations form insoluble substances upon combination with the constituents of cement which are soluble in acid solutions; some preparations, without this chemical combination, of merely acid-proof insoluble materials are not sufficient to withstand the attacks of strong acids. For the most corrosive acids, linings of asphalt, bricks, tiles, paints, enamel, rubber, or acid-resistant coatings are generally recommended as given for specific acids in the present list. It should be noted that while most common acids, for example, nitric and sulphuric acid, are more destructive in high concentrations, hydrochloric acid is most injurious in lower concentrations Acid-resisting cements are obtainable, some of which are said to be resistant to all acids (except hydrochloric) in all strengths and at high temperatures. For weak and medium acid solutions, paraffin wax or sodium silicate is used.

ALCOHOL.—Little or no effect on concrete, but some authorities recommend protection by acid-resisting paints.

ALKALIS.—The effect of alkalis varies; therefore see under specific materials such as ammonia, caustic soda, calcium, etc.

ALUMINIUM.—Aluminium (and to a less extent zinc and lead) can be seriously corroded when in contact with freshly placed Portland cement mortar or concrete. The corrosion is said to be due to the alkaline moisture that exudes from the concrete as a result of the setting process or changes in humidity. This mixture only attains an appreciable calcium hydroxide content after prolonged leaching out, but when the calcium hydroxide exceeds 50 per cent. of the saturation value, crystalline calcium aluminate is precipitated, forming a protective coating which retards further corrosion. Corrosion can be prevented by inserting an impermeable layer of bituminous material between the concrete and the aluminium lining. Glycerine–litharge mortars have been recommended in place of Portland cement and similar mortars when setting aluminium linings, and a fairly strong material can be made by mixing 9 parts of crushed limestone (run of crusher with fines) and one part of litharge with a solution of approximately 35 per cent. of crude commercial glycerine and 65 per cent. of water.

ALUMINIUM SULPHATE.—See " Sulphates ".

AMMONIA AND AMMONIUM COMPOUNDS.—Hard rubber or special paints are recommended as protection for concrete against ammonia. The remarks for magnesium chloride apply to ammonium chloride. Ammoniacal liquor, which is encountered in gasworks and coal by-products plants, has little effect on concrete, although treatment is sometimes recommended. Ammonium nitrate disintegrates concrete, especially if

the solution is in excess of ½ to 1 per cent., and a protective coating as for sulphates is required. Ammonium sulphate is probably the most aggressive of the sulphate salts, and concrete floors of sulphate stores are protected by blue bricks laid on edge and set in pitch. In one case where sulphuretted hydrogen and ammonia, dissolved in moisture condensed on the surface of the concrete, had oxidised to ammonium sulphate causing disintegration of the concrete, a successful protection was obtained by treating the new concrete with a 30-per-cent. solution of high-silica sodium silicate. Ammonium sulphide attacks concrete, necessitating protection as for sulphates.

ANHYDRITES.—High-alumina cement concrete resists attack.

ANTHROCENE.—No effect on concrete, but in some cases tanks are rendered with cement mortar to reduce seeping.

BEER.—Fermentation tanks of concrete should be lined with enamel, glass, or stainless metal linings. Deleterious effects are most virulent when using Portland cement, high-alumina cement concrete being immune.

BENZENE.—The remarks for petrol apply.

BENZOL.—No injurious effect on concrete, but slight seeping occurs. Recommended protection is as for petrol. A skimming coat of waterproofed Portland cement well worked into the surface of the concrete, followed by two coats of cement grout, has proved successful. Asphalt should not be used as asphalt is soluble in this oil.

CALCIUM COMPOUNDS.—Calcium chloride has no injurious effect on concrete, and is sometimes used to accelerate setting and for curing. It may have corrosive effects on reinforcement. Calcium nitrate has no effect on concrete. Calcium sulphate (gypsum) is often found in clays, the ground water from which may contain solutions of sufficient strength to attack Portland cement concrete, although high-alumina cement concrete may be immune (see " Sulphates ").

CARBOLIC ACID.—Attacks poor concrete, although cases have been recorded where untreated 1 : 1½ : 3 concrete 6 in. thick with a steel-float finish has been successful.

CARBOLINEUM.—As for creosote.

CARBONATES.—At normal concentrations these have practically no effect on concrete.

CARBONIC ACID.—When dry has no effect on concrete, but causes slow disintegration when in solution. Recommended treatments include asphalt, bituminous paints, coal-tar paints, fluosilicate, spar varnish, or phenol-formaldehyde resin.

CAUSTIC SODA.—Caustic alkalis have little effect on Portland cement concrete, although they may attack high-alumina cement concrete. Some protection for all concrete is recommended, but cases are reported where 4 per cent. solutions have been stored successfully without treatment. Alkali-resisting paints will provide protection for solutions of any concentration.

CHLORIDES.—Some chlorides have little or no effect on concrete except in high concentrations, while others cause slow disintegration and require protection as for sulphates. See specific chlorides.

CHLORINE SOLUTION.—Little effect on concrete, but treatment as for acids is desirable.

CIDER.—Paraffin-wax treatment reported to be successful.

CITRIC ACID.—Acid-resisting paints provide protection against all concentrations; see also " Acids ".

COBALT SULPHATE.—See " Sulphates ".

COPPER COMPOUNDS.—Copper chloride attacks concrete slightly and requires treatment as for sulphates. For copper nitrate protection is afforded by acid-resisting paints. For copper sulphate, see " Sulphates ".

CORN SYRUP.—As for glucose.

COTTON-SEED OIL.—Opinions differ regarding the extent to which concrete is disintegrated by this oil; some suggest that no protective treatment is required as the attack is only slight; others report virulent attack, particularly on Portland cement concrete but less on high-alumina cement concrete.

CREOSOTE.—Has a tendency to disintegrate concrete slowly, requiring coatings to prevent seeping as recommended for petrol.

CRESOL.—As for creosote.

Cumol.—As for benzol.

Fish Oil.—Attacks concrete only slowly, and treatment with fluosilicate, sodium silicate, or linseed oil is recommended.

Foot Oil.—As for fish oil.

Formaldehyde.—Protection is provided by paints.

Formic Acid.—Acid-resisting paints provide protection against solutions up to 60 per cent.; see " Acids ".

Fumes.—Fumes, such as the acid vapours that occur in dye-works, pickling plants, bleaching works, and galvanising works, attack concrete. Protection can be provided by lacquers. Protection is also required for concrete exposed to smoke, the sulphur in which leads to the formation of sulphuric acid (which see).

Gallic Acid.—Mostly contained in ink. Storage vats require protection by one of the coatings given for acids or by ink-resisting linings.

Glucose.—Slowly attacks concrete. Requires protection as for sulphates

Glycerine.—Protection required as for sulphates.

Hydrochloric Acid.—Particularly destructive, especially when hot. Diluted hydrochloric acid can penetrate thick coatings of paraffin wax. Protective treatment as for sulphuric acid, including rubber linings.

Hydrofluoric Acid.—Concrete requires protection. Lead linings are recommended, and rubber is reported to be effective up to 150 deg. Fahr. and for 50 per cent. solutions.

Iron and Iron Compounds.—Iron or steel built into or attached to exposed concrete may lead to discoloration of the concrete due to rust. For the compounds of iron, the remarks for magnesium chloride apply to ferric chloride; those for copper nitrate apply to ferric nitrate; and those for sulphates apply to ferric sulphate.

Lactic Acid.—Derived from milk and occurs in whey and sour milk. Portland cement concrete is attacked and can be treated as for carbonic acid in water, or coatings of linseed oil, paraffin wax, or paints. High-alumina cement concrete is attacked to a less degree. Sodium silicate provides only temporary protection against deterioration of concrete floors subject to milk spillings, and tiles laid in asphalt are recommended for permanent protection. Channels are often lined with vitreous material.

Lard.—As for fish oil; this also applies to lard oil.

Lead and Lead Compounds.—For the effect on lead of contact with wet concrete see " Aluminium ". Lead pipes passing through concrete can be protected by a coating of pitch or tar or by embedding the pipes in clay. For lead nitrate, see the remarks for copper nitrate; and for lead sulphate, see " Sulphates ".

Lime.—See calcium compounds. Gasworks lime attacks Portland cement, high-alumina cement concrete being more resistant. Portland cement concrete treated with special paint is also effective. Piles driven through gasworks' lime should be protected by a coating of tar.

Magnesium Chloride.—Slowly attacks concrete; protection as for sulphates. No protection required if in weak solutions as in ground waters.

Magnesium Sulphate (Epsom Salts).—Remarks as for calcium sulphate.

Mercuric Chloride.—As for magnesium chloride.

Naphthalene.—As for petrol.

Nitrates.—Some nitrates have a deleterious effect, while concrete may be immune from attack by others; see separate references. At the low concentrations met in practice these salts may have no effect.

Nitric Acid.—As for sulphuric acid, but rubber lining is not recommended by some authorities except for low concentrations.

Nitro-hydrochloric Acid.—Particularly destructive, especially when hot. See " Hydrochloric Acid ".

Nut Oil.—Almond oil, coco-nut oil, peanut oil, and walnut oil slowly disintegrate concrete, which can be protected by fluosilicate, sodium silicate, or linseed oil.

Oils.—Vegetable oils attack concrete and saponify it. Mineral oils have no destructive effect, and thus light fuel oils, such as petrol, paraffin, and benzol, can be stored in concrete without injurious effect, but applications of spar varnish, linseed oil, or sodium silicate are recommended to provide protection against seeping of oils of specific gravity less than 0·875 (54½ lb. per cubic foot) equivalent to not under

30 deg. Baumé. No protective treatment is required for heavier oils, as seeping may be negligible. Resinous coatings such as an alcoholic solution of a phenol condensation product, or an oil-resisting paint, are recommended as protection against fatty oils. Concrete can be protected from the penetrating effects of lubricating oils or transformer oils by using oil-resisting lacquer. See the remarks for separate oils in this list.

OLIVE OIL.—Causes slow disintegration. Treatment is as for nut oil, or with spar varnish or plastic varnish.

PARAFFIN.—As for petrol.

PETROL.—Although it has no deleterious effect on concrete, the loss due to seeping may be considerable as petrol can find its way through pores that are impermeable to water. Applications of fluosilicate, spar varnish, sodium silicate, or phenol-formaldehyde varnish are recommended as surface treatments. Impregnation of the concrete with water reduces the permeation of petrol, and a water barrier within the thickness of the wall has proved effective. Petrol vapour can also pass through concrete; rich mixtures of concrete or a high water-cement ratio apparently slightly increases the rate of penetration; glycerine, glue, or asphalt does not seem to provide much protection.

PHENOL.—As for creosote.

PHOSPHATES.—These salts attack concrete only when in high concentrations.

PHOSPHORIC ACID.—Slowly disintegrates concrete due to the formation of soluble acid phosphates. Requires protection as for acids generally, acid-resisting paint providing protection from all concentrations. If rubber is used it should be specially compounded.

POPPYSEED OIL.—As for nut oil.

POTASSIUM COMPOUNDS.—Potassium chloride and potassium nitrate have no effect on concrete. For potassium sulphate, see remarks under " Sulphates ".

PYRITES.—Slowly attacks concrete, requiring protection as for sulphates.

RAPE-SEED OIL.—As for olive oil.

RUBBER.—Used as a protection for concrete containers holding corrosive liquids. Most inorganic acids (except nitric acid) of any concentration and at temperatures up to 150 deg. F. can be stored in soft rubber or hard rubber (vulcanite) linings, although 50 per cent. concentration is the limit for hydrofluoric and sulphuric acids and 85 per cent. for phosphoric acid. If soft rubber (and in some cases, hard rubber) is used for certain acids, it is necessary to use a special grade of rubber. Similarly most solutions of inorganic salts and alkalis up to saturation point and 150 deg. F. temperature, and most organic materials up to the same limits, can be stored in rubber-lined tanks. Hard rubber is generally more resistant to chemical attack than soft rubber. Rubber is not generally recommended for protection against powerful oxidising agents such as nitric and chromic acids, ozone, etc.

SALT (COMMON).—Common salt (sodium chloride) has no injurious effect on concrete when the salt is dry, but sodium silicate is recommended as a treatment for plain concrete. If the concrete is reinforced, a lining of asphalt or similar material is required to prevent corrosion of the steel. Untreated concrete tanks are suitable for weak solutions of brine, but strong solutions require protection by brick linings, the jointing in which should be resistant to prevent seeping of brine behind the brickwork attacking the concrete. High-alumina cement concrete offers greater immunity than Portland cement concrete.

SALTS.—Soluble inorganic salts attack concrete; the descending order of virulence has been given as follows: sulphates, sulphides, nitrates, chlorides, and carbonates; see under each of these headings and separate substances.

SEWAGE.—The acid content is generally low, otherwise sewage may have a deleterious effect on concrete. High-alumina cement concrete is more resistant than Portland cement concrete. Protection and impermeability have been obtained by ½-in. renderings to walls and ¾-in. renderings to inverts, using a Portland cement mortar with a waterproofer and applying the mortar to a concrete surface that had been well hacked immediately after the removal of the shuttering. Surface treatments with bitumen or fat are not permanent. Where disintegration has been very pronounced, concrete pipes have been lined with vitrified clay tiles and the joints pointed

with bitumen emulsion. The attack arising from sewage, which results in the presence of hydrogen sulphide, is mostly limited to the concrete at and above normal liquid level.

SODIUM COMPOUNDS.—Sodium nitrate has no effect on concrete, neither has sodium silicate (water-glass), which is widely used as a protective coating and pore filler. For treatments for sodium sulphate (Glauber salt), see "Sulphates" and "Calcium Sulphate"; steam curing has been recommended for protection of concrete against sodium sulphate. For sodium chloride, see "Salt (Common)".

SOYA BEAN OIL.—As for nut oil but is harmless in thin coats.

STRONTIUM CHLORIDE.—No effect on concrete unless alternately wet and dry.

SUGAR.—In general, sugar has a marked deleterious effect on concrete and complete protection should be provided. Sugar juices injure Portland cement concrete but have little effect on high-alumina cement concrete. Concrete requires protection from boiling jam, aluminium linings often being provided; where floors are exposed to spillings from the boiling of sweets or jam the concrete needs protection. Molasses slightly attack concrete which requires treatment as for sulphates.

SULPHATES.—Most sulphates vigorously attack concrete, the intensity of the attack increasing with the concentration of the solution up to 1 per cent., with little increased virulence beyond this limit. Recommended protective coatings include fluosilicate, sodium silicate, linseed oil, bitumen, glass linings, vitrified brick or tile laid in litharge, rubber, and special paints. See also separate references to particular sulphates.

SULPHIDES.—Most sulphides deleteriously affect concrete, necessitating protection as used for sulphates. Sulphide ores attack concrete only slowly. Soluble sulphides, with the exception of ammonium sulphide, have no effect.

SULPHITE LIQUOR.—Slowly attacks concrete; protection as for sulphates.

SULPHURIC ACID.—The disintegrating effect on concrete is guarded against by lining the container with glass, lead, or vitrified brick or tile. The effect of lead lining in contact with concrete should be considered (see "Lead" and "Aluminium"). Rubber is also effective up to 150 deg. F. and for 50 per cent. solutions, while paints may be effective for any concentration. The acid used in accumulators is sulphuric acid diluted in distilled water to a specific gravity of approximately 1·22, and the necessary protection of concrete is obtained by bituminous compositions or acid-resistant asphalt.

SULPHUROUS ACID.—Remarks as for sulphuric acid, but rubber is effective for all concentrations.

TANNIC ACID.—Derived from tea and causes slow disintegration of Portland cement concrete, but has little effect on high-alumina cement concrete. Treatment of Portland cement concrete as for carbonic acid in water.

TANNING LIQUOR.—If of the non-acid type, the concrete does not require protection, but if of the acid type it should be protected by acid-resisting bituminous paint.

TOLUOL.—As for benzol.

TURPENTINE.—As for petrol.

WINE.—The treatment of the interior of wine vats with a solution of fluosilicate or with tartaric acid has been successful and had no ill-effect on the contents. Vats treated with potassium silicate solution are reported to contaminate wine.

XYLOL.—As for benzol.

ZINC.—For the effect of zinc in contact with wet concrete, see "Aluminium". For zinc sulphate and chloride, see "Sulphates".

APPENDIX II

TYPICAL DETAILS AND CALCULATIONS

CONTENTS

* * * *

Publisher's Note

Other examples of the design of reinforced concrete structures are given in "Examples of the Design of Reinforced Concrete Buildings" and in other books in the "Concrete Series" dealing with specific types of structures.

SLAB-AND-BEAM FLOOR.—CALCULATIONS.

For Details, see page 350.

WAREHOUSE FLOOR. COLUMNS AT 20-FT. CENTRES BOTH DIRECTIONS

FINISH: 1-IN. GRANOLITHIC WEARING SURFACE (NOT INCLUDED IN STRUCTURAL THICKNESS)

DESIGN DATA

CONCRETE. 1 : 2 : 4 (ORDINARY GRADE). TABLE (57) $p_{cb} = 1000$ LB./SQ.IN. $p_{cc} = 760$ LB./SQ.IN.

$q = 100$ " " " $s_b = 120$ " " "

MILD STEEL [TABLE (58)] $p_{st} = 20{,}000$ " " " $p_{sc} = 18{,}000$ " " "

DESIGN FACTORS. (m = 15) [TABLE (68)] $p_{st}/p_{cb} = 20$; $n_1 = 0{\cdot}428$; $a_1 = 0{\cdot}857$; $Q_c = 184$

SPECIFIED IMPOSED LOAD. 2 CWT./SQ.FT. [$\nless$ 200 LB./SQ.FT. PER TABLE (3)]

SLAB

$\ell = 6'\text{-}8''$ (SPANNING IN ONE DIRECTION) FULLY CONTINUOUS

4-IN. SOLID SLAB; $\frac{3}{4}$ IN. COVER AT BOTTOM; $\frac{1}{2}$ IN. COVER AT TOP [$\nless \frac{1}{2}''$]

LOAD. DEAD LOAD.— 1-IN. GRANOLITHIC = 12 LB./SQ.FT.

[TABLE (1)] 4-IN. SLAB = 50 " " "

TOTAL = 62 " " "

IMPOSED (LIVE) LOAD = 224 " " "

TOTAL LIVE + DEAD LOAD = 286 " " "

$\dfrac{\text{LIVE LOAD}}{\text{DEAD LOAD}} = \dfrac{224}{62} = 3{\cdot}7$ (> 2; PROVIDE BARS IN TOP THROUGHOUT)

B. Ms. [USE SEPARATE COEFFICIENTS IN TABLE (20)]

MID-SPAN (INTERIOR SPAN) (POSITIVE) $M = \left[\dfrac{62}{24} + \dfrac{224}{12}\right] 6{\cdot}67^2 \times 12 = 11{,}350$ IN-LB./FT.

INTERIOR SUPPORTS $M = \left[\dfrac{62}{12} + \dfrac{224}{9}\right] 6{\cdot}67^2 \times 12 = 16{,}250$ " " "

MID-SPAN (NEGATIVE) [TABLE (20)] $M = \left[\dfrac{224}{2} - 62\right] \dfrac{6{\cdot}67^2}{24} \times 12 = 1110$ " " "

MIDSPAN: $d_1 = 3{\cdot}06''$; $\ell_a = 0{\cdot}86 \times 3{\cdot}06 = 2{\cdot}63''$

SUPPORT: $d_1 = 3{\cdot}31''$; $\ell_a = 0{\cdot}86 \times 3{\cdot}31 = 2{\cdot}85''$

THICKNESS. FROM TABLE (72), $d = 3\frac{1}{2}$ IN.— BUT 4 IN. IS PRACTICAL MINIMUM.

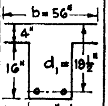
REINFORCEMENT. (SQ.IN./FT.) — TABLE (60)

			Provided
MID-SPAN (BOTTOM)	$A_{st} = \dfrac{11{,}350}{20{,}000 \times 2{\cdot}63}$	= 0·220	$\frac{3}{8}''$ AT 6" c/c
SUPPORTS (TOP)	$A_{st} = \dfrac{16{,}250}{20{,}000 \times 2{\cdot}85}$	= 0·285	$\frac{3}{8}''$ AT $4\frac{1}{2}''$ c/c
MID-SPAN (TOP)		ABOUT 0·02	$\frac{3}{8}''$ AT 9" c/c PROVIDED
DISTRIBUTION BARS:	0·15 % × 4 × 12	= 0·072	$\frac{1}{4}''$ AT 12" c/c TOP & BOTTOM
BARS OVER PRIMARY BEAMS:	0·3 % × 4 × 12	= 0·144	$\frac{3}{8}''$ AT $7\frac{1}{2}''$ c/c

SECONDARY BEAMS

$\ell = 20$ FT. (FULLY CONT.) CLEAR SPAN = 19' T.

16 IN. × 8 IN. (NET); 1 IN. COVER



$\ell_a = 18\frac{1}{2} - \frac{4}{2} = 16\frac{1}{2}''$

$d_n = 0{\cdot}43 \times 18\frac{1}{2} = 8''$

LOAD. DEAD LOAD.— SLAB, ETC. = 6·67 × 62 = 413 LB./FT.

BEAM RIB = $16 \times 8 \times \frac{150}{144}$ = 134 " "

TOTAL = 547 " "

LIVE LOAD.— IMPOSED LOAD = 6·67 × 224 = 1500 " "

TOTAL LOAD = 2047 " "

$\dfrac{\text{LIVE LOAD}}{\text{DEAD LOAD}} = \dfrac{1500}{547} = 2{\cdot}76 = 3$ APPROX.; USE COMBINED B. M. COEFFICIENTS [TABLE (20)]

MID-SPAN (INTERIOR SPAN) (POSITIVE) $M = \dfrac{2047 \times 20^2 \times 12}{13{\cdot}7} = 719{,}000$ IN-LB.

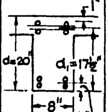
$A_{st} = \dfrac{719{,}000}{20{,}000 \times 16{\cdot}5} = 2{\cdot}18$ SQ.IN. [TABLE (60)] — 3 - 1" BARS AT BOTTOM

FLANGE [PER TABLE (69)]: $b \ngtr 6{\cdot}67$ FT (= 80 IN.) OR $\frac{1}{3} \times 20$ FT. (= 80 IN.) OR (12 × 9) + 8 = 56 IN.

M_{rc} [PER FORMULA (9) IN TABLE (66)] $= \dfrac{(16-4) \times 16{\cdot}5 \times 1000 \times 56 \times 4}{2 \times 8} = 2{,}770{,}000$ IN-LB. APPROX. (O.K. > 719,000)

SUPPORT (INTERIOR) $M = \dfrac{2047 \times 20^2 \times 12}{9{\cdot}6} = 1{,}023{,}500$ IN-LB.

$A_{st} = \dfrac{1{,}023{,}500}{20{,}000 \times 15} = 3{\cdot}41$ SQ. IN. — 5 - 1" BARS AT TOP (4 + EFFECTIVE)



$\ell_a = 0{\cdot}86 \times 17\frac{1}{2} = 15''$

$\ell_{as} = 17\frac{1}{2} - 2\frac{1}{2} = 15''$

$d_n = 0{\cdot}43 \times 17\frac{1}{2} = 7{\cdot}5''$

COMPRESSIVE RESISTANCE [PER FORMULÆ IN TABLE (66)] AT FACE OF SUPPORT

$M_s = (90\% \times 1{,}023{,}500) - (184 \times 8 \times 17\frac{1}{2}^2) = 469{,}150$ IN-LB.

$f_{sc} = 1000 \times 14 \left(\dfrac{7{\cdot}5 - 2{\cdot}5}{7{\cdot}5}\right) = 9320$ LB./SQ.IN.

$A_{sc} = \dfrac{469{,}150}{9320 \times 15} = 3{\cdot}35$ SQ. IN. — 4 - 1" BARS AT BOTTOM

MIN. OVERLAP [PER TABLE (62)] $= 12'' + 2(0{\cdot}8 \times 21 \times 1'') \cong 4$ FT. (42")

SHEARING RESISTANCE [PER TABLE (81)] Q [AT FACE OF SUPPORT] $= 2047 \times \frac{19}{2} = 19{,}500$ LB.

$q = \dfrac{19500}{15 \times 8} = 163$ LB./SQ.IN. (> 100; REINFT. REQ'D.)

1 - 1" BAR AT 45° SINGLE SYSTEM RESISTS 11,100 LB.

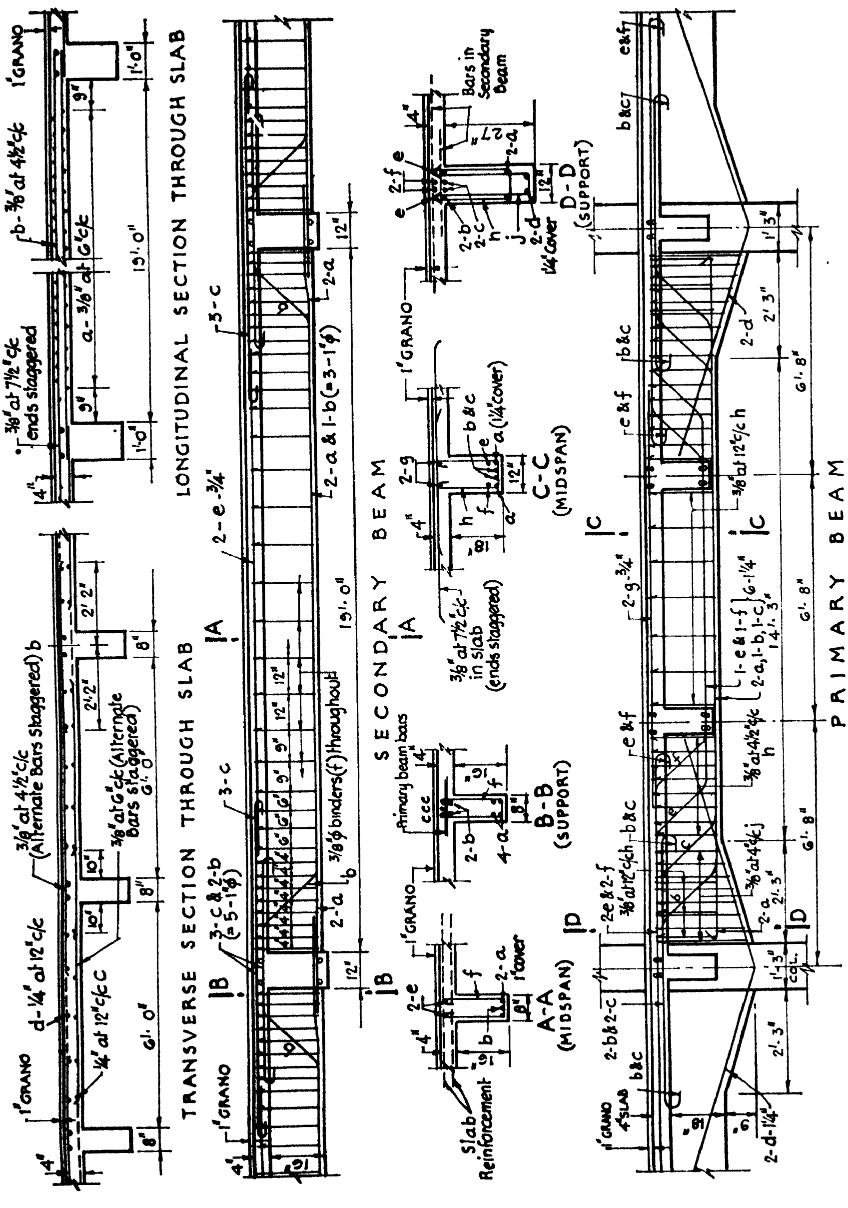
TO BE RESISTED BY BINDERS = 19,500 − 11,100 = 8,400 LB., $V = \dfrac{8400}{15} = 560$

AT APPROX. 2 FT. 6 IN. FROM ℄ OF SUPPORT:—

$Q = 2047 \times \frac{15}{2} = 15{,}350$ LB. $V = \dfrac{15{,}350}{15} = 1020$

1 - 1" AT 45° SINGLE SYSTEM + BINDERS $\frac{3}{8}''$ AT 4" c/c.

For Calculations, see pages 349 and 351.



For Details, see page 350.

PRIMARY BEAMS

ℓ = 20 FT. (FULLY CONTINUOUS)

CLEAR SPAN (15" COL.) = 18'-9"

18 IN. X 12 IN. (NET)

LOADS. CONCENTRATED (NET). (AT THIRD POINTS)

DEAD LOAD = 2 No. x 547 x 19 = 20,700 LB. PER SPAN
LIVE " = 2 No. x 1500 x 19 = 57,000 " " "
TOTAL 77,700 " " "

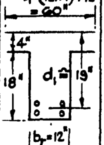
UNIFORMLY-DISTRIBUTED. (ON WIDTH OF BEAM) DEAD: SLAB, ETC. = 62 LB./FT.
BEAM RIB = 225 " "
TOTAL = 287 X 18·75 = 5,400 " " "
LIVE: 18·75 X 224 X 1·0 = 4,200 " " "

TOTALS — TOTALS 511 LB./FT. = 87,300 " " "

BENDING MOMENTS. APPLY B.M. COEFFICIENTS IN TABLE (21) ASSUMING BEAM TO BE EQUIVALENT TO CENTRAL SPAN OF FIVE CONTINUOUS SPANS.

SUPPORTS — DEAD LOAD. CONCENTRATED 0·106 x 20,700 = 2190
DISTRIBUTED 0·080 x 5,400 = 432
LIVE LOAD. CONCENTRATED 0·148 x 57,000 = 8450
DISTRIBUTED 0·111 x 4,200 = 467 IN.-LB.
TOTAL (UNADJUSTED) 11,539 x 20 x 12 = 2,775,000
DEDUCT 15% = 416,000
2,359,000

MID-SPAN. $b \not> (12 \times 4) + 12 = 60''$



$\ell_a = 19 - \frac{4}{2} = 17''$

$d_n = 0{\cdot}43 \times 19 = 8{\cdot}2''$

MID-SPAN — DEAD LOAD. CONCENTRATED 0·061 x 20,700 = 1260
DISTRIBUTED 0·046 x 5400 = 248
LIVE LOAD. CONCENTRATED 0·115 x 57,000 = 6560
DISTRIBUTED 0·086 x 4200 = 381
TOTAL (UNADJUSTED) 8429 x 20 x 12 = 2,035,000
ADD DEDUCTION FROM SUPPORT B.M. (MAX. ADJUSTMENT) = 416,000
TOTAL (ADJUSTED) 2,451,000

$1\frac{1}{4}$ IN. COVER (MIN.)

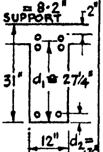
MID-SPAN.

$$A_{st} = \frac{2451{,}000}{20{,}000 \times 17} = 7{\cdot}21 \text{ SQ IN.}$$

$$M_{rc} \triangleq \frac{1000 \times 60 \times 17 \times 4\,(16{\cdot}4 - 4)}{2 \times 8{\cdot}2} = 3{,}080{,}000 \text{ IN.-LB. OK.}$$

$6 = 1\frac{1}{4}''$ BARS AT BOTTOM

SUPPORT



$\ell_{ac} = 0{\cdot}86 \times 27\frac{1}{4}'' = 24\frac{1}{2}''$

$\ell_{as} = 27\frac{1}{4} - 1\frac{7}{8} = 25\frac{3}{8}''$

ℓ_a(MEAN) $\triangleq 25''$

$d_n = 0{\cdot}43 \times 27\frac{1}{4}'' = 11{\cdot}7''$

SUPPORTS. B.M. PER INCH WIDTH $= \frac{2{,}359{,}000}{12} = 197{,}000$ IN.-LB./IN.

d_1 REQUIRED WITH $A_{sc} = A_{st}$ [FROM TABLE (70B)] = 22" (> 19" APPROX.)

PROVIDE SPLAYS SAY 9" x 27"

PER FORMULAE IN TABLE (66)

$$M_s = 2{,}359{,}000 - \left[184 \times 12 \times (27\tfrac{1}{4})^2\right] = 2{,}359{,}000 - 1{,}635{,}000 = 724{,}000 \text{ IN.-LB.}$$

$$f_{sc} = 14 \times 1000 \left(\frac{11{\cdot}7 - 1\frac{7}{8}}{11{\cdot}7}\right) = 11{,}750 \text{ IN.-LB.}$$

$$A_{sc} = \frac{724{,}000}{11{,}750 \times 25\frac{3}{8}} = 2{\cdot}44 \text{ SQ. IN.}$$

$2 - 1\frac{1}{4}''$ BARS AT BOTTOM

$$A_{st} = \frac{1{,}635{,}000}{20{,}000 \times 24\frac{1}{2}} + \frac{724{,}000}{20{,}000 \times 25\frac{3}{8}} = 3{\cdot}33 + 1{\cdot}45 = 4{\cdot}78 \text{ SQ. IN.}$$

$8 - 1\frac{1}{4}''$ BARS PROVIDED AT TOP

SHEARING RESISTANCE. [TABLE (81)]

$$Q \text{ (AT FACE OF SUPPORT)} = \frac{87{,}300}{2} = 43{,}650 \text{ LB.}$$

$$q = \frac{43650}{12 \times 25} = 146 \text{ LB./SQ IN. } (> 100;\ \text{REINF}^{T}.\ \text{REQ}^{D}.)$$

$1 - 1\frac{1}{4}''$ BAR AT 45° SINGLE SYSTEM = 17,350 LB.
TO BE RESISTED BY BINDERS = 43,650 − 17,350 = 26,300 LB.

$$V \text{ REQUIRED} = \frac{26{,}300}{25} = 1050 \quad (\tfrac{3}{8}'' \text{ AT } 4'' \text{ c/c} = 1105)$$

$1 - 1\frac{1}{4}$ AT 45° SINGLE SYSTEM AND $\frac{3}{8}''$ BINDERS AT 4" c/c

AT EDGE OF SPLAY (AND APPROXIMATELY THE SAME UP TO THIRD POINT)

$$Q = 43{,}650 - (2{\cdot}25 \times 511) = 42{,}500 \text{ LB.}$$

$$q = \frac{42{,}500}{12 \times 17} = 208 \text{ LB./SQ. IN. } (> 100 \not> 400;\ \text{REINFORC}^{T}\ \text{REQUIRED})$$

ARRANGEMENT OF INCLINED BARS EQUIVALENT TO $1\frac{1}{2}$ x SINGLE SYSTEM

$1 - 1\frac{1}{4}''$ BAR AT 45° = $1\frac{1}{2}$ x 17,350 = 26,025 LB.
TO BE RESISTED BY BINDERS = 42,500 − 26,025 = 16,475 LB.

$$V \text{ REQUIRED} = \frac{16{,}475}{17} = 970 \quad (\tfrac{3}{8}'' \text{ AT } 4\tfrac{1}{2}'' \text{ c/c} = 982)$$

$1 - 1\frac{1}{4}''$ AT 45° SPACED AT 15" c/c AND $\frac{3}{8}''$ BINDERS AT $4\frac{1}{2}''$ c/c.

BETWEEN SECONDARY BEAMS AT THIRD POINTS.

$$Q = 511 \times (\tfrac{1}{2} \times 6) = 1533 \text{ LB.} \quad q < 100 \text{ (NO REINFORCEMENT)}$$

NOMINAL BINDERS $\frac{3}{8}''$ AT 12" c/c.

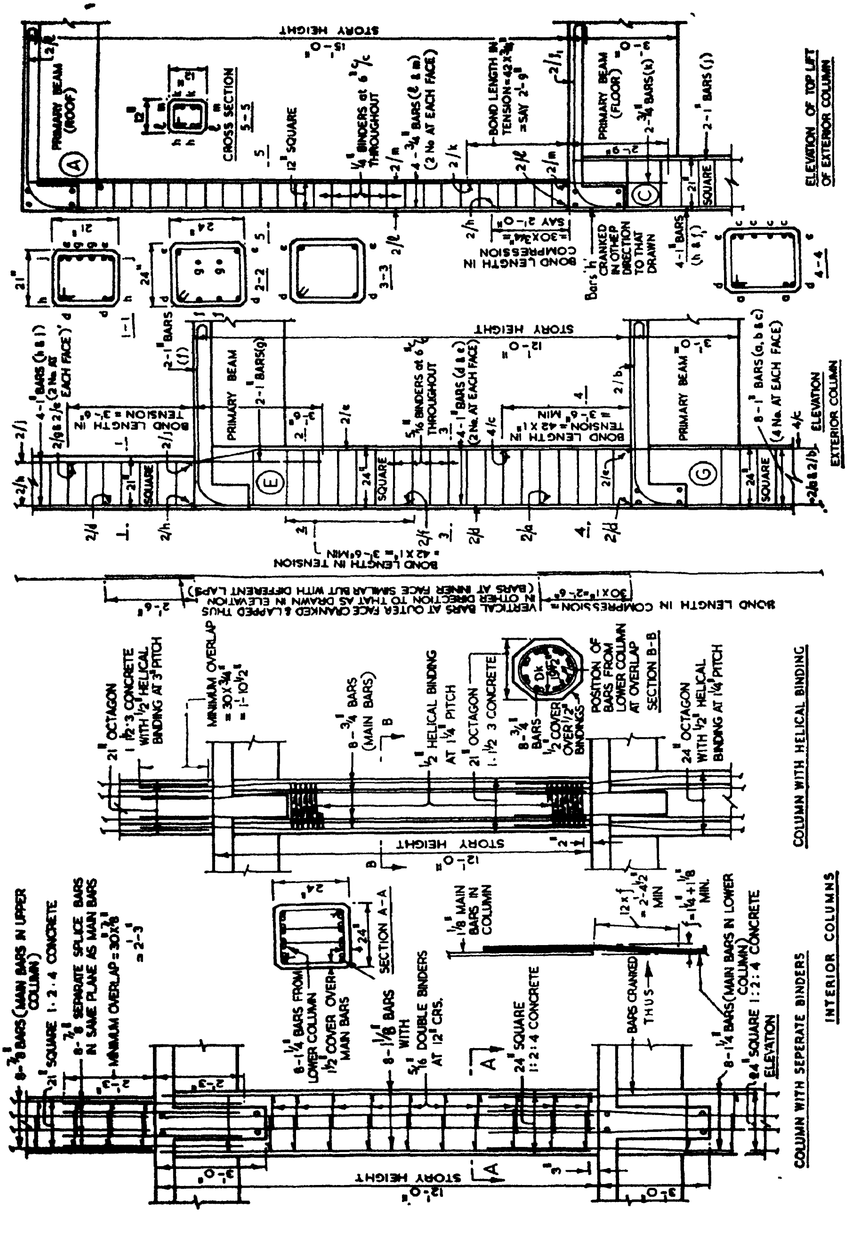
NOTES ON SLABS AND BEAMS

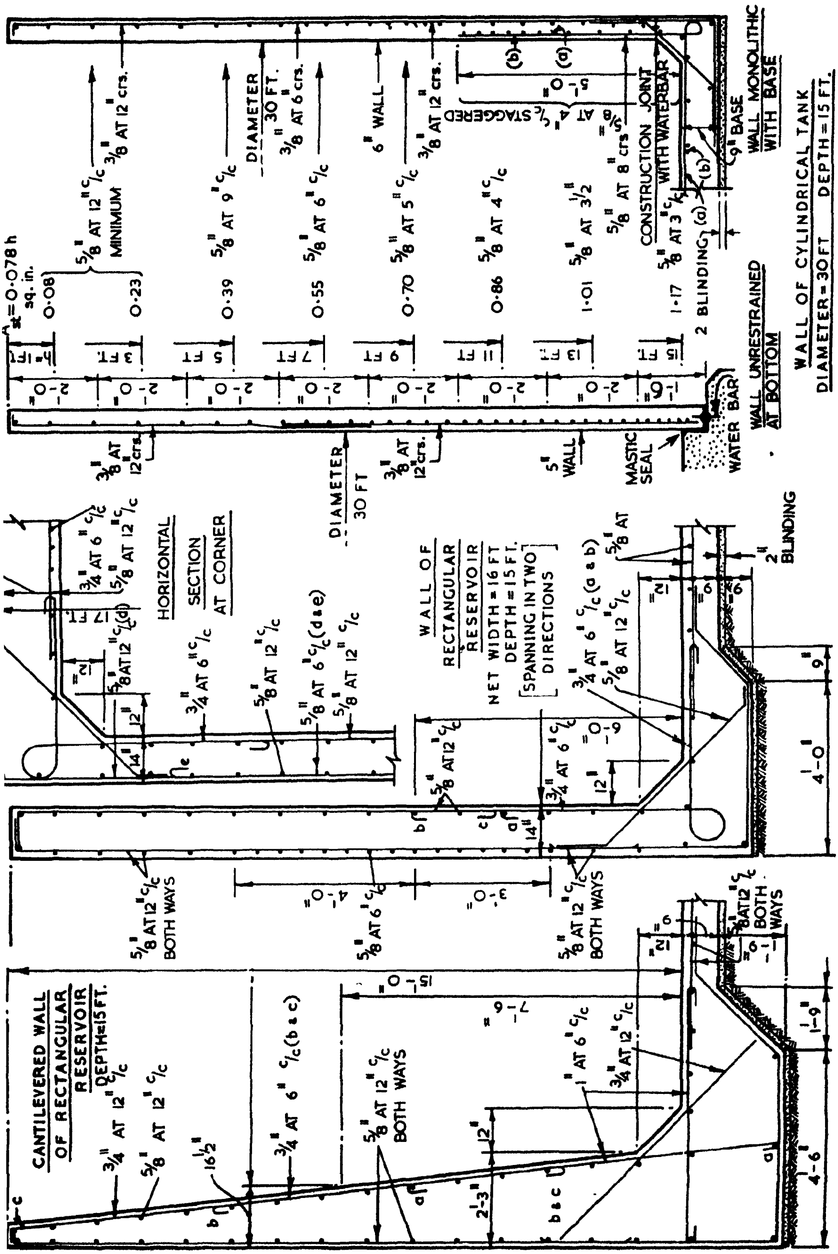
BOND LENGTHS AND ANCHORAGES OF BARS TO BE CHECKED FROM TABLE (62)

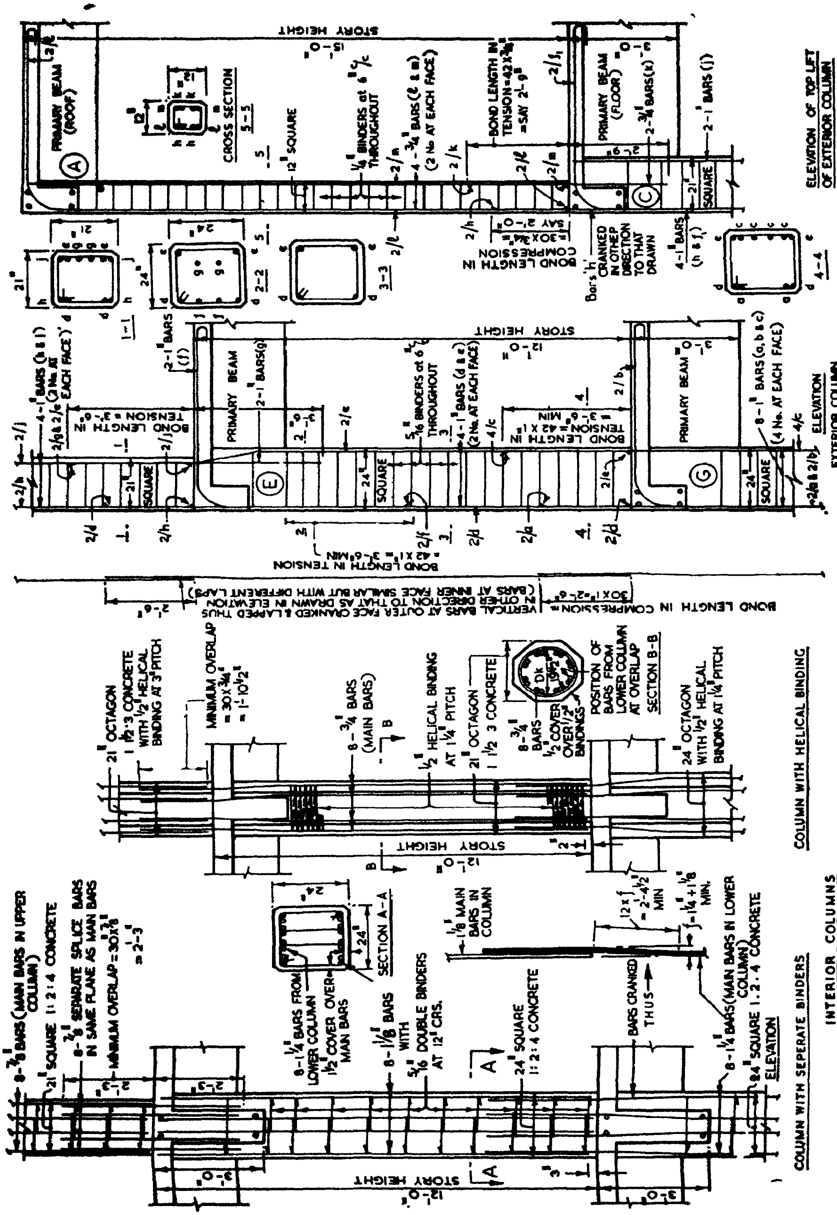
POINTS OF BENDING UP (OR STOPPING OFF) BARS TO BE CHECKED FROM TABLE (69) FOR SLAB AND SECONDARY BEAMS AND FROM B.M. DIAGRAM FOR PRIMARY BEAMS.

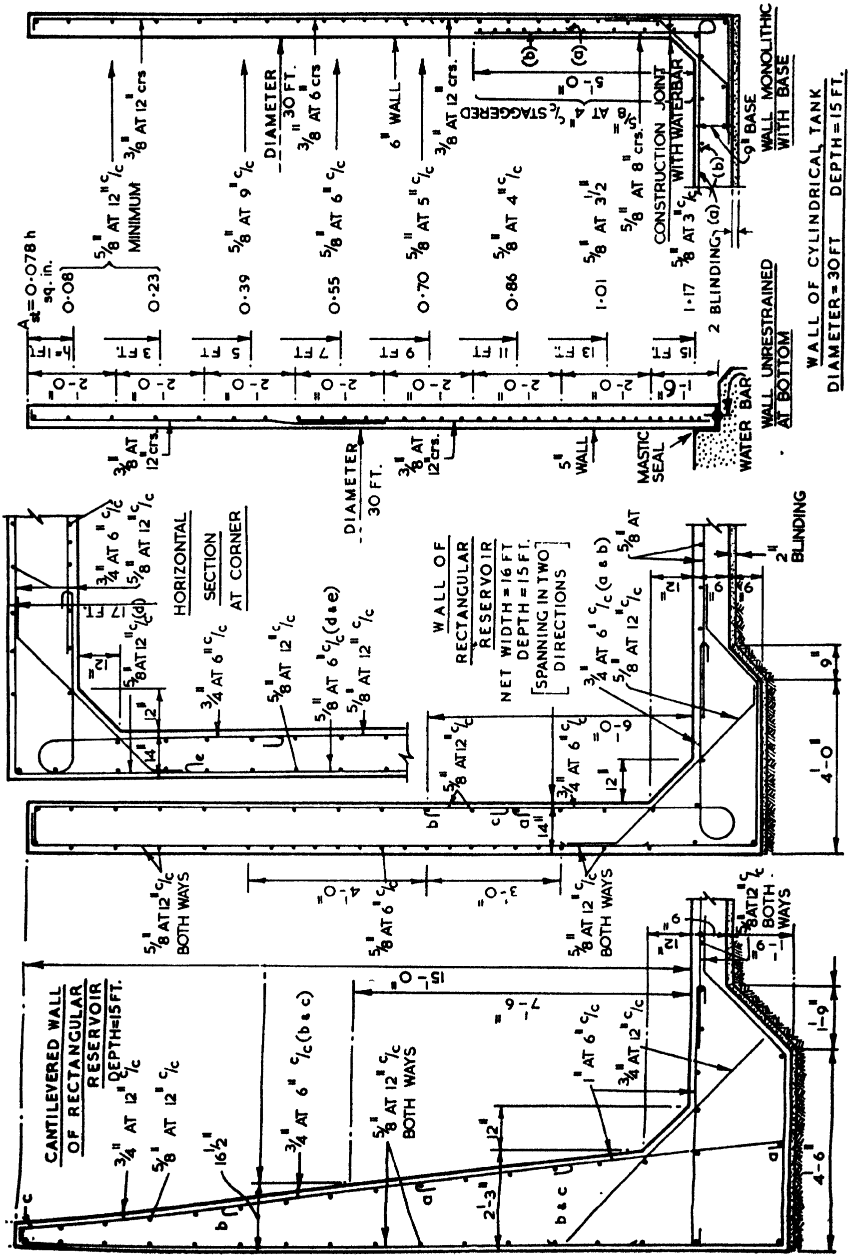
WHEN DETAILED ARRANGEMENTS OF BARS HAVE BEEN DRAWN OUT, ACTUAL VALUES OF d_1, ℓ_a ETC. SHOULD BE DETERMINED AND, IF DIFFERING MUCH FROM ASSUMED VALUES, CALCULATIONS SHOULD BE REVIEWED, AND REINFORCEMENT ALTERED IF NECESSARY.

INTERIOR AND EXTERIOR COLUMNS.—DETAILS.



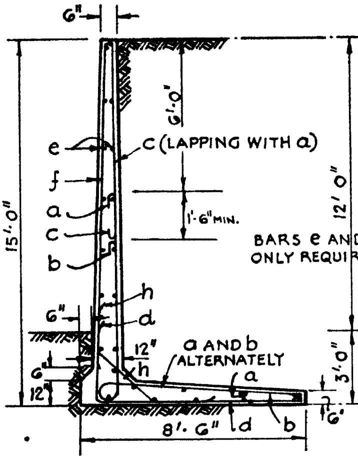

CANTILEVERED WALL OF RECTANGULAR RESERVOIR DEPTH=15 FT.
3/4" AT 12" c/c
5/8" AT 12" c/c
3/4" AT 6" c/c (b & c)
5/8" AT 12" c/c BOTH WAYS
1" AT 6" c/c
3/4" AT 12" c/c
5/8" AT 12" c/c BOTH WAYS
5/8" AT 6" c/c
5/8" AT 12" c/c BOTH WAYS
WALL OF RECTANGULAR RESERVOIR
NET WIDTH = 16 FT
DEPTH = 15 FT.
[SPANNING IN TWO DIRECTIONS]
3/4" AT 6" c/c (a & b)
5/8" AT 12" c/c
5/8" AT 12" c/c
3/4" AT 6" c/c
HORIZONTAL SECTION AT CORNER
3/4" AT 6" c/c
5/8" AT 12" c/c
5/8" AT 12" c/c
5/8" AT 6" c/c (d & e)
5/8" AT 12" c/c
2" BLINDING
4'-0"
4'-6"
1'-9"
15'-0"
7'-6"
3'-0"
4'-0"
2'-3"
DIAMETER 30 FT
WALL UNRESTRAINED AT BOTTOM
WALL MONOLITHIC WITH BASE
WALL OF CYLINDRICAL TANK
DIAMETER = 30 FT DEPTH = 15 FT.
Ast = 0.078 h sq. in.
0.08
0.23
0.39
0.55
0.70
0.86
1.01
1.17
5/8" AT 12" c/c MINIMUM
5/8" AT 9" c/c
5/8" AT 6" c/c
5/8" AT 5" c/c
5/8" AT 4" c/c
5/8" AT 3 1/2"
5/8" AT 3" c/c
3/8" AT 12" crs.
3/8" AT 6" crs.
3/8" AT 12" crs.
5/8" AT 8" crs
5/8" AT 4" c/c STAGGERED
6" WALL
5" WALL
MASTIC SEAL
WATER BAR
CONSTRUCTION JOINT WITH WATERBAR
9" BASE
2 BLINDING


ELEVATION OF TOP LIFT OF EXTERIOR COLUMN
ELEVATION EXTERIOR COLUMN
COLUMN WITH HELICAL BINDING
COLUMN WITH SEPERATE BINDERS
INTERIOR COLUMNS

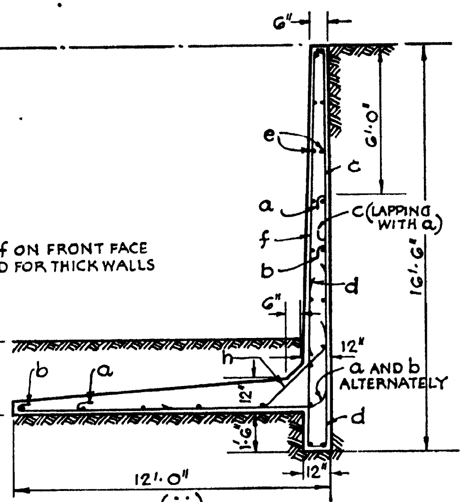

CANTILEVERED WALL OF RECTANGULAR RESERVOIR DEPTH=15FT.
WALL OF RECTANGULAR RESERVOIR
NET WIDTH = 16 FT
DEPTH = 15 FT.
[SPANNING IN TWO DIRECTIONS]
HORIZONTAL SECTION AT CORNER
WALL UNRESTRAINED AT BOTTOM
WALL MONOLITHIC WITH BASE
WALL OF CYLINDRICAL TANK
DIAMETER = 30 FT DEPTH = 15 FT.

RETAINING WALLS.—DETAILS.

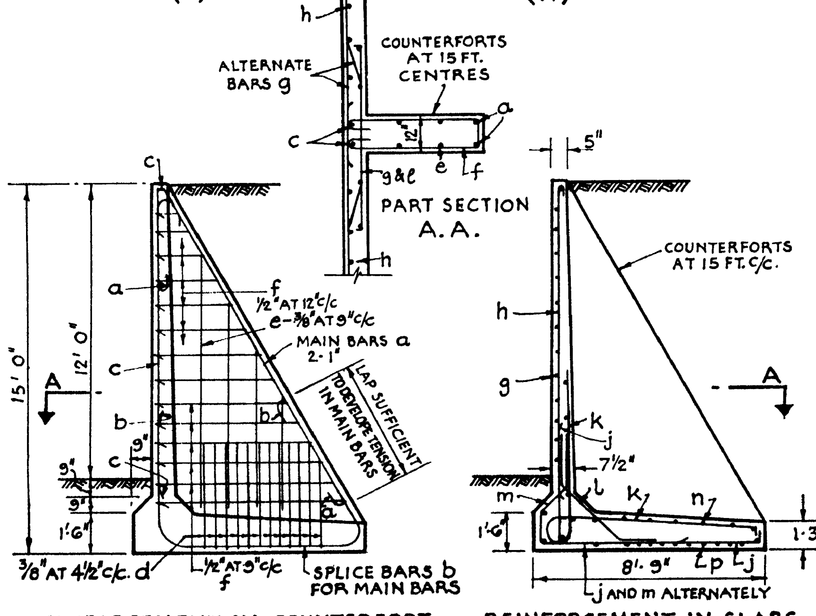
For Calculations, see pages 357 and 358.



(i)



(ii)



REINFORCEMENT IN COUNTERFORT. REINFORCEMENT IN SLABS.

(iii)

For Details, see page 356.

HEIGHT (NET) = 12 FT. DEPTH OF BASE BELOW GROUND IN FRONT OF WALL = 3 FT.
RETAINING LARGE SANDY GRAVEL. SURCHARGE = 200 LB./SQ. FT.

DESIGN DATA

[PER TABLE (11)] ω = 120 LB./CU. FT. θ = 35 DEG. K_2 = 0·27; $p = (0{\cdot}27 \times 120h) + (0{\cdot}27 \times 224) = 32h + 61$ LB./SQ. FT.

CONCRETE 1 : 2 : 4 p_{cb} = 1000 LB./SQ. IN. q = 100 LB./SQ. IN.

MILD STEEL REINFORCEMENT: p_{st} = 18,000 LB./SQ. IN. $1\frac{1}{2}$ IN. COVER ON EARTH FACE; $\frac{3}{4}$ IN. COVER ON FRONT FACE

[PER TABLE (68)] p_{st}/p_{cb} = 18. n_1 = 0·455. a_1 = 0·848. Q_c = 193. (m = 15)

DESIGN (i) CANTILEVERED WALL — BASE EXTENDING UNDER RETAINED BANK

STEM

AT TOP OF SPLAYS h = 13·5 FT. P_o [TABLE (100)] $= (32 \times 13{\cdot}5^2 \times \frac{1}{2}) + (61 \times 13{\cdot}5)$ = 2930 + 825 = 3755 LB.

$M_{AA} = (2930 \times \frac{13{\cdot}5}{3}) + (825 \times \frac{13{\cdot}5}{2})$ = 18,770 FT.-LB./FT.

[PER TABLE (71)] $d_1 = 0{\cdot}072\sqrt{18{,}770}$ = 9·85 IN. + SAY $\frac{1}{2}$ IN. + $1\frac{1}{2}$ IN. COVER = — 12 IN. WALL AT BOTTOM (d_1 = 10")

$A_{st} \not< 0{\cdot}0109 \times 18{,}770$ = 1·49 SQ. IN./FT. — 1 IN. BARS a & b AT 6 IN. C/C AT BACK FACE

AT 6 FT. FROM TOP: $P_o = (32 \times 6^2 \times \frac{1}{2}) + (61 \times 6)$ = 577 + 366 = 943 LB.

$M = [(577 \times \frac{6}{3}) + (366 \times \frac{6}{2})]\,12$ = 27,000 IN.-LB./FT.

$d = (\frac{12-6}{13{\cdot}5})(13{\cdot}5 - 6) + 6$ = 9·3 IN. $d_1 = 9{\cdot}3 - 1\frac{1}{2}$ − SAY $\frac{3}{8}$ = 7·4 IN.

$Q = \frac{27{,}000}{12 \times 7{\cdot}4^2}$ = 41; f_{cb} < 500 [PER TABLE (68)] IF f_{st} = 18,000 LB./SQ. IN.

$A_{st} = \frac{27{,}000}{18{,}000 \times 0{\cdot}9 \times 7{\cdot}4}$ = 0·224 SQ. IN./FT. SAY — $\frac{5}{8}$ IN. BARS c AT 12 IN. C/C AT BACK FACE

DISTRIBUTION BARS: HORIZONTAL — $\frac{0{\cdot}15}{100} \times 12 \times 12$ (MAX) = 0·216 SQ. IN./FT.
SAY $\not<$ 0·108 SQ. IN./FT. EACH FACE — $\frac{3}{8}$ IN. BARS e AND f AT 12 IN. C/C.
VERTICAL (FRONT FACE) DITTO

STABILITY. ASSUME WIDTH OF BASE L = 8 FT. 6 IN. H = 15 FT.

[PER TABLE (100)] $P = (32 \times 15^2 \times \frac{1}{2}) + (61 \times 15)$ = 3600 + 915 = 4515 LB./FT.
$Py = (3600 \times \frac{15}{3}) + (915 \times \frac{15}{2})$ = 24,870 FT.-LB./FT.

NEGLECT ACTIVE PRESSURE IN FRONT OF WALL.

MOMENTS ABOUT HEEL OF WALL

				FT.	
STEM	0·5 × 13·5 × 150 LB.	=	1010 LB.	× 1·25	= 1,260 FT.-LB.
	0·5 × 13·5 × ½ × 150	=	505	× 0·83	= 480
BASE	0·5 × 8·5 × 150	=	637	× 4·25	= 2,700
	0·5 × 2·0 × 150	=	150	× 1·0	= 150
	0·5 × 6·5 × ½ × 150	=	244	× 4·17	= 1,020
SPLAYS	2 × 0·5 × 0·5 × ½ × 150	=	38	× 1·0 AV.	= 40
EARTH	7·0 × 13·75 × 120	=	11,550	× 5·0	= 57,750
SURCHARGE	7·0 × 224	=	1568	× 5·0	= 7,850
		W =	15,702 LB.	Wx =	71,180 FT.-LB.

$x = \frac{71{,}180}{15{,}702}$ = 4·5 FT. APPROX. F. OF S. AGAINST OVERTURNING = $\frac{71{,}180}{24{,}870}$ = 2·85

RESISTANCE TO SLIDING. (μ = 0·4) F. OF S. AGAINST SLIDING = $\frac{0{\cdot}4 \times 15{,}702}{4{,}515}$
(NEGLECTING PASSIVE RESISTANCE) = 1·4 APPROX. — BASE 8 FT. 6 IN. WIDE O.K.

GROUND PRESSURE $e = \frac{24{,}870}{15{,}702} + \frac{8{\cdot}5}{2} - 4{\cdot}5$ = 1·33 FT. ($< \frac{8{\cdot}5}{6}$ = 1·42 FT.)

[PER TABLE (104)] $\frac{p_{max.}}{p_{min.}} = \frac{15{,}702}{8{\cdot}5}(1 \pm \frac{6 \times 1{\cdot}33}{8{\cdot}5}) = \frac{3600 \text{ LB./SQ. FT.} = 1{\cdot}6 \text{ TON/S. FT.}}{110 \text{ ,, ,, ,,}}$ — O.K. FOR SANDY GRAVEL

DESIGN (ii) CANTILEVERED WALL — BASE PROJECTING IN FRONT OF WALL

STEM. DESIGN CALCULATIONS AS FOR DESIGN (i). — 12-IN. WALL AT BOTTOM

STABILITY. ASSUME WIDTH OF BASE L = 12 FT. WITH "HEELRIB". H = 16·5 FT.

[PER TABLE (100)] $P = (32 \times 16{\cdot}5^2 \times \frac{1}{2}) + (61 \times 16{\cdot}5) - (32 \times 4{\cdot}5^2 \times \frac{1}{2})$
= 4360 + 1008 − 324 = 5044 LB./FT.
$Py = (4360 \times \frac{16{\cdot}5}{3}) + (1008 \times \frac{16{\cdot}5}{2}) - (324 \times \frac{4{\cdot}5}{3})$ = 31,814 FT.-LB./FT.

MOMENTS ABOUT TOE OF WALL

STEM	0·5 × 16·5 × 150 LB.	=	1235 LB.	× 11·75 FT.	= 14,500 FT.-LB.
	0·5 × 13·5 × ½ × 150	=	505	× 11·33	= 5,730
	0·5 × 3·0 × 150	=	225	× 11·25	= 2,540
BASE	0·5 × 11·0 × 150	=	825	× 5·5	= 4,530
	0·5 × 11·0 × ½ × 150	=	413	× 7·35	= 3,040
SPLAY	0·5 × 0·5 × ½ × 150	=	19	× 10·83	= 206
EARTH	11·0 × 2·25 × 120	=	2970	× 5·5	= 16,300
		W =	6182 LB.	Wx =	46,846 FT.-LB.

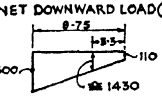
$x = \frac{46{,}846}{6182}$ = 7·58 FT. F. OF S. OVERTURNING = $\frac{46{,}846}{31{,}814}$ = 1·47

RESISTANCE TO SLIDING. ACTIVE PRESSURE OUTWARDS = 4360 + 1008 = 5368 LB.
PASSIVE PRESSURE INWARDS = $(\frac{120}{0{\cdot}27})(\frac{4{\cdot}5^2}{2})$ = 4500 LB.
FRICTIONAL RESISTANCE (μ = 0·4) = 0·4 × 6182 = 2473 ,, } 6973 LB.
F. OF S. AGAINST SLIDING = $\frac{6973}{5368}$ = 1·3 — BASE 10 FT. WIDE O.K.

GROUND PRESSURE. $e = \frac{31{,}814}{6{,}182} + \frac{12{\cdot}0}{2} - 7{\cdot}58$ = 3·57 FT. ($> \frac{12{\cdot}0}{6}$ = 2·0 FT.)

[PER TABLE (104)] $p_{MAX.} = \frac{4 \times 6182}{3\,[10 - (2 \times 3{\cdot}57)]}$ = 2880 LB./SQ. FT. = 1·3 TON/SQ. FT.

For Details, see page 356.

DESIGN (iii) COUNTERFORTED WALL	STABILITY. DIMENSIONS AND ARRANGEMENT OF BASE ARE COMPARABLE WITH DESIGN (I) THEREFORE F. OF S. AGAINST OVERTURNING AND SLIDING WILL BE NOT LESS THAN IN DESIGN (I) AND GROUND PRESSURE WILL BE SIMILAR.	COUNTERFORTS AT 15 FT. CENTRES
WALL SLAB $L_H \simeq 14{\cdot}25$ FT. (CONT.) $L_V \simeq 13$ FT. (CANTILEVER) $\frac{L_H}{L_V} = 1{\cdot}1$	WALL PANELS. B. M. COEFFICIENTS. TRIANGULARLY-DISTRIBUTED PRESSURE: $p = 32 \times 13 = 416$ LB./SQ.FT. [PER TABLE (40)] VERT. (BOTTOM) $= 0{\cdot}042$ HORIZ. (SIDES) $= 0{\cdot}037$ } $\times L_V = 0{\cdot}45 \times 13 = 6$ FT. " (MID.) $= 0{\cdot}025$ } UNIFORMLY-DISTRIBUTED PRESSURE: INSPECTION OF RELATIVE "EFFECTIVE" SPANS INDICATES THAT PRACTICALLY ENTIRE PRESSURE ($p = 61$ LB./SQ.FT.) IS TAKEN BY HORIZONTAL SPAN.	(= MID-HEIGHT)
HORIZL. SPAN (MID-HEIGHT) $d = 6\frac{3}{4}''$ AT SIDES: ($1\frac{1}{2}''$ COVER) $d_1 = 4\frac{15}{16}''$ $l_a = 0{\cdot}85 \times 4\frac{15}{16} = 4{\cdot}2''$ AT MID-SPAN ($\frac{3}{4}''$ COVER) $d_1 = 5\frac{11}{16}''$ $l_a = 4{\cdot}8''$	BENDING MOMENTS (MAX.). IN.-LB./FT. VERTICAL SPAN: $M_V = 0{\cdot}042 \times 416 \times 13^2 \times 12 = 36{,}500$ HORIZONTAL SPAN: AT SIDES. (AT MID-HEIGHT) $M = 0{\cdot}037 \times 416 \times 14{\cdot}25^2 \times 12 = 37{,}600$ } 49,800 $+ \frac{1}{12} \times 61 \times 14{\cdot}25^2 \times 12 = 12{,}200$ } AT MID-SPAN. $M = 0{\cdot}025 \times 416 \times 14{\cdot}25^2 \times 12 = 25{,}300$ } 31,400 $+ \frac{1}{24} \times 61 \times 14{\cdot}25^2 \times 12 = 6{,}100$ } THICKNESS.— CRITICAL B.M. = 49,800 IN-LB./FT. (AT MID-HEIGHT) (TENSION ON INNER FACE): $d \simeq 6\frac{3}{4}$ IN. [PER TABLE (72) + $\frac{3}{4}$ IN. EXTRA COVER]	WALL SLAB 6 IN. AT TOP $7\frac{1}{2}$ IN. AT BOTTM
VERTL. SPAN. AT BOTTOM $d_1 = 7\frac{1}{2}''$ ($1\frac{1}{2}''$ COVER) $d_1 = 5\frac{11}{16}''$ $l_a = 4{\cdot}8''$	REINFORCEMENT. HORIZONTAL: AT OUTER FACE, MID SPAN, (AT MID-HT.) $A_{st} = \frac{31{,}400}{18{,}000 \times 4{\cdot}8} = 0{\cdot}363$ SQ.IN./FT. AT INNER FACE AT SIDES $A_{st} = \frac{49{,}800}{18{,}000 \times 4{\cdot}2} = 0{\cdot}650$ " " VERTICAL: AT INNER FACE AT BOTTOM $A_{st} = \frac{36{,}500}{18{,}000 \times 4{\cdot}8} = 0{\cdot}422$ " "	5/8 IN. BARS AT 10 IN C/C (BARS g) AT 5 IN. C/C (BARS g & l) AT 8 IN. C/C (BARS k)
BASE SLAB EFFY LONGL SPAN $= 14{\cdot}5$ FT. (CONT.) [CANTILEVER $\simeq 6{\cdot}75$ FT.] $d = 15''$ (MIN.) ($1\frac{1}{2}''$ COVER) $d_1 = 13\frac{5}{16}''$ MIN.	BASE SLAB. NET DOWNWARD LOAD (LB./SQ.FT.): (AV. / 6" FROM EDGE) SURCHARGE = 200 / 200 EARTH $= 13{\cdot}875$ (AV.) $\times 120$ = 1665 / 1650 $16\frac{1}{2}''$ (AV.) SLAB = 206 / 190 TOTAL: 2071 / 2040 DEDUCT AV. EARTH PRESSURE = 1430 / 310 NET: 641 / 1730 (CRITICAL)  B.M. AT EDGE $\ngtr \frac{1730 \times 14{\cdot}5^2 \times 12}{12} = 375{,}000$ IN.-LB./FT. $\times$ SAY $\frac{2}{3} = 250{,}000$ IN.-LB./FT. TO ALLOW FOR SOME CANTILEVER EFFECT. =	BASE SLAB 18 IN. AT ROOT 15 IN. AT EDGE (BARS k).
	REINFORCEMENT $Q_c = \frac{250{,}000}{12 \times (13\frac{5}{16})^2} = 120$; [PER TABLE (68)] $18{,}000/750 = 24$; $a_1 = 0{\cdot}872$ $A_{st\,(MAX.)} = \frac{250{,}000}{18{,}000 \times 0{\cdot}872 \times 13\frac{5}{16}} = 1{\cdot}21$ SQ.IN./FT. (= 5/8 IN. BARS AT 3 IN. C/C) VERTICAL BARS TYING INTO COUNTERFORT. AV. TENSION $= 641 \times 14 = 9000$ LB./FT. $A_{st} = \frac{9000}{18{,}000} = 0{\cdot}50$ SQ. IN. PER FT.	5/8 IN. BARS AT 6 IN. C/C OVERLAPPING AT CRITICAL SECTIONS TOP & BOTTOM (BARS n & p) 3/8 IN. U-BARS (d) AT $4\frac{1}{2}$ IN. C/C.
AT BOTTOM $d_1 \simeq 94''$ $l_a = 94 - \frac{7\frac{1}{2}}{2} \simeq 90''$ "SPAN" $\simeq 14$ FT. $d_s = 7\frac{1}{2}''$ $b = (12 \times 7\frac{1}{2}) + 18 = 108''$ sin (SLOPE OF MAIN BARS) $\simeq 0{\cdot}9$ $d_n = 0{\cdot}455 \times 94 = 43''$ $b_r = 12''$	COUNTERFORTS. TOTAL NET PRESSURE (ABOVE BASE) ON 15-FT. BAY OF WALL $\simeq 15\,P_o$ [IN DESIGN (i)] $= 15 \times 3755 = 56{,}300$ LB. TO ALLOW FOR PART CANTILEVER ACTION OF WALL SLAB ASSUME NET LOAD ON COUNTERFORT $= \frac{2}{3} \times 56{,}300$ LB. $\simeq 40{,}000$ LB $= Q$ ASSUMED TRIANGULARILY DISTRIBUTED B.M. $= \frac{40{,}000 \times 14 \times 12}{3} = 2{,}240{,}000$ IN.-LB. $A_{st} = \frac{2{,}240{,}000}{18{,}000 \times 90 \times 0{\cdot}9} = 1{\cdot}535$ SQ.IN. M_{rc} [PER FORMULA IN TABLE (66)] $= \frac{(86 - 7\frac{1}{2})\,90 \times 1000 \times 108 \times 7\frac{1}{2}}{2 \times 43} = 66{,}500{,}000$ IN.-LB. $q = \frac{40{,}000}{12 \times 90} = 37$ LB./SQ.IN. (NO SHEAR REINFORCEMENT REQUIRED) HORIZONTAL BINDERS TYING WALL SLAB INTO COUNTERFORT. AT MID-HEIGHT: $p = (32 \times 13) + 61 = 477$ LB./SQ.FT. $\times 14 = 6800$ LB. AV. $A_{st} \ngtr \frac{6800}{18{,}000} = 0{\cdot}37$ SQ. IN./FT.	WIDTH = 12 IN. 2 - 1-IN BARS (a) O.K. $\frac{1}{2}$ IN. BINDERS f AT 12 IN. (AV.) C/C.

INDEX

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